

DET KONGELIGE DANSKE
VIDENSKABERNES SELSKABS SKRIFTER

SYVENDE RÆKKE

NATURVIDENSKABELIG OG MATHEMATISK AFDELING

TIENDE BIND



KØBENHAVN

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OVER

DET KONGELIGE DANSKE VIDENSKABERNES SELSKABS MEDLEMMER

September 1913



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STUDIES
ON THE
CYCLOSTOMATA OPERCULATA

BY
G. M. R. LEVINSEN

WITH 7 PLATES AND 2 FIGURES IN THE TEXT

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURV. OG MATHEMATISK AFD. X. 1



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PREFACE.

Many years ago I undertook a study of the Danish cretaceous *Bryozoa* the results of which have not yet been published, and among the species studied by me were also a small number belonging to the family *Melicerititidae* (Eleidae d'Orb.) to the members of which d'ORBIGNY ascribes a calcareous operculum the presence of which, however, has been denied by all the later authors who interpret the supposed operculum as a closure-plate of the same nature as that which has been found both in the *Cyclostomata* and the *Cheilostomata*. I came however to the result that d'ORBIGNY was right in his interpretation of the named structure, and I have published some remarks on this subject in my preliminary communication »Studies on Bryozoa«¹⁾. As later I wished to make a more comprehensive study of this interesting group I extended my investigations over a number of foreign species, and in this effort I have been supported by several colleagues abroad. In the first instance I owe a debt of gratitude to Dr. F. CANU the author of so many valuable works on fossil Bryozoa, who has not only sent to me a large number of French species defined by him, but also helped me to acquire materials containing cretaceous bryozoa from a number of French localities. A similar material from a few other French localities has been sent me by Mr. G. DOLLFUS, and to Mr. M. FILIOZAT who has made a special study of the cretaceous Bryozoa from Vendôme I owe the possession of a number of species from that locality. For the gift or loan of specimens I am also indebted to Mr. A. W. WATERS, Dr. E. PERGENS, Prof. Dr. G. STEINMANN, Bonn and Prof. Dr. H. WEGNER, Münster, and lastly I have been able to acquire a collection of cretaceous Bryozoa from the Chatham chalk, by the aid of Mr. W. GAMBLE. To all these gentlemen I offer my sincere thanks.

Zoological Museum of Copenhagen.
August 16, 1912.

G. M. R. LEVINSSEN.

¹⁾ 16.

Historical Introduction.

GOLDFUSS¹⁾ was the first author, who described a species belonging to this division namely *Ceriopora gracilis* while the first genus *Meliceritites* has been instituted by ROEMER²⁾ for the three species *Mel. gracilis* (Goldf), *Mel. Roemeri* (Hag) and *Mel. porosa* Roemer. The name *Meliceritites* is derived from *Melicerita*, a genus founded by MILNE EDWARDS³⁾ for a fossil species of the present genus *Cellularia* (*Cellaria*), *Cel. Charlesworthii*, and the main character upon which the genus was founded is the arrangement of the zooecia in continuous transverse series, a character which at the present state of systematic knowledge cannot be regarded as sufficient for the institution of a genus, and therefore the name *Melicerita* must be regarded only as a synonym to *Cellularia*. ROEMER's genus is defined in the following manner: »Runde Stämme deren sechsseitige Zellen mit einer Ecke und nicht wie bei allen bisher beschriebenen Arten, mit einer Seite nach oben gerichtet sind und die gemeinschaftliche Scheidewand zweier Zellen der höheren Reihe tragend«. After this definition he adds the following apparently contradictory information: »Gleiche Zellenbildung findet sich auch bei *Eschara* und hat MILNE EDWARDS hier darauf die Gattung *Melicerita* gegründet«. As stated above however it is not the form of the zooecial areas but the arrangement of them in transverse series which has induced MILNE EDWARDS to found a new genus for this species. It is easy to understand that ROEMER has derived the name of his new genus from *Melicerita* as the three species described by him agree with *Mel. Charlesworthii* in the possession of rhombic-hexagonal concave zooecial areas, surrounded by distinct ridges and, besides, arranged in transverse series, and as he has not seen the long tubular part of the zooecia hidden within the colony his only motive to divide *Meliceritites* from *Melicerita* seems to be the different form of the colony, which in the latter is a two-layered plate while in the former it is ramose with rounded branches.

V. HAGENOW⁴⁾ is the first author who refers these forms to a special division which he named *Salpingina* and characterized in the following manner: »Angewachsene verästelte Polypenstöcke, mit langen Röhrenzellen, welche sich kurz vor der Mündung trompetenartig, fast sackförmig erweitern, und äusserlich mit einer kalkigen oder hornartigen Membran verschlossen sind. In dieser Membran befindet sich die kleine, verschieden gestaltete Mündung, die warscheinlich bei Allen mit einer Klappe versehen war und die bei einigen Arten noch erhalten ist.« At another place he names this »Klappe« operculum: »Ein Operculum ist bei Einigen, vielleicht bei Allen vorhanden«. To this division he refers the two genera *Escharites* and *Inversaria* the former of which also comprises ROEMER's *Meliceritites*. Apart from the circumstance

¹⁾ 1. p. 35, pl. X, figs 11 a—c. ²⁾ 6, p. 18, pl. V, fig. 13. ³⁾ 2, p. 345, pl. 12, fig. 19. ⁴⁾ 5, p. 55.

that HAGENOW's division only embraces freely growing species it is founded on the examination of a few badly conserved species of which only a single *Escharites gracilis* with certainty can be referred to the *Eleidae*, and as species belonging to this division have never been found in the tertiary formation the following statement of the author distinctly shows that his *Salpingina* also embraces common cyclostomatous species: »Alle sind fossil und gehören dem Jura, der Kreide und der Tertiärbildung an«.

The presence of an operculum in the *Salpingina* should be a real difference from the *Cyclostomata*, but there is no evidence that the author has seen a real operculum in any of the species referred by him to this division. As a species provided with such an operculum he names *Escharites (Felicea) velata* Hag., but this species belongs to quite another division, the *Ceidae*, which are not provided with an operculum, and what HAGENOW has seen is only a closure-plate.

D'ORBIGNY¹⁾ in 1852 founded a division to which he gives the name *Centrifuginés operculinés* and which he defines as follows: »Cellule centrifuginée toujours pourvue d'un opercule. Colonie très variable dans sa forme composée de cellules généralement peu saillantes, mais toujours pourvues d'un opercule: partie testacée ou calcaire, s'ouvrant comme une porte pour laisser sortir l'animal«. As to the operculum he later adds: »D'ailleurs s'il pouvait encore rester quelques doutes à cet égard la présence de ces opercules encore restés en place dans beaucoup d'espèces fossiles, vient entièrement les lever et donner la preuve que cet opercule existait«. To this division D'ORBIGNY refers two families, the *Eleidae* and the *Myriozoumidae*, the last of which contains the extinct genus *Foricula* and the recent genus *Myriozoum*. The last named genus, however, belongs to the *Cheilostomata*. The above quotation leaves no doubt that D'ORBIGNY has founded his division *Eleidae* on the presence of an operculum, and it is a curious fact that all the modern authors with the exception of the present though accepting this division, at the same time deny that its member possesses an operculum, explaining the calcareous plate which may be found closing the aperture of more or less zooecia as a closure-plate. Not a single author even mentions this statement of D'ORBIGNY.

HAMM²⁾ in 1881, founded a very artificial division, the *Stigmatoporina* to which he besides two inoperculate cyclostomatous genera *Stigmatopora* Hamm (-*Hammia* Grey) and *Cyrtopora* Hag. also referred the operculate genus *Meliceritites*. He characterizes this division as follows: »Die Zellen sind ähnlich wie die der Tubuliporiden beschaffen. Statt dass sie aber in der mittleren Axe des Stammes entspringen legen sie sich rings um ein senkrecht stehendes, im Querschnitt rundliches Bündel von langen cylindrischen unter einander parallelen Röhren«. He divides the *Stigmatoporina* in two groups the second of which (*Meliceritites*) he characterizes by the trumpet-shaped distal enlargement of the zooecia: »... zweitens in solche, deren Zellen lang, anfangs sehr dünn sind und erst an der Mündung sich plötzlich trompetenförmig erweitern«. As we have seen HAGENOW has al-

¹⁾ 7, p. 605. ²⁾ 8, p. 45.

ready used this character for his *Salpingina*, but HAMM has overlooked that the narrow tubes of the axial bundle are only the proximal parts of the zooecia, and as we shall see later, this error is repeated by MARSSON. As to the operculum spoken of by D'ORBIGNY he expresses the following opinion¹⁾: »Dieser Deckel ist indessen kein dem beweglichen Deckel der chilostomen Bryozoen analoges Organ, sondern eine einfache Kalkwand, mittelst deren sich das Thier bei zunehmendem Alter oder um sich gegen äussere Einflüsse zu schützen, in die Zelle einkapselt; er ist also ein blosses biologisches Produkt. Dem entsprechend findet sich derselbe nicht nur bei den Operculés von D'ORBIGNY, sondern ebenso bei den Clausidae D'Orbg., Diastopora, Terebellaria, Osculipora, Heteropora etc. also bei Formen von sehr verschiedener Natur. Zur Systematik darf er daher nicht verwendet werden«.

MARSSON²⁾, in 1888, united the *Ceidea* and the *Eleidea* into a division *Metopoporina*, and the *Eleidea* he characterized as follows: ». . . die Hauptzellen bei den cylindrischen Arten auf der Aussenseite eines centralen cylindrischen Bündels langer, Röhrenzellen entspringend, (ob auch bei den flachen, blattartigen Formen die Hauptzellen aus einer Schicht von Nebenzellen entspringen, bedarf einer weiteren Untersuchung), auf der Oberfläche des Stocks dicht an einander gedrängt, mit mehr oder weniger rhombisch-sechseckigen Stirnseiten ausmündend, die im vorderen Theile eine dreiseitige Mündung tragen«. In the diagnose of *Meliceritites* he mentions the trumpet-shaped expansion of the zooecia, and the manner in which he speaks about the operculum seems to show that he interpretes it as a preliminary calcareous covering which later disappears: »Mündung . . . anfangs durch eine Kalkmembran geschlossen und mehr hervortretend«.

PERGENS³⁾ in 1890, separated the two groups, the *Ceidae* and the *Eleidae*, united by MARSSON in his *Metopoporina*, and the latter of them, to which he gives the name *Meliceritina* he defines in the following manner: »Les zoécies se dilatent vers l'extrémité en forme de trompette; l'orifice n'occupe qu'une partie du diamètre transversal des zoécies et est situé à leur partie distale. Les ovicelles sont situées à l'intérieur des colonies, entre les zoécies, ou occupant leur place, et communiquant avec l'extérieur par une ouverture triangulaire à base proximale et à pointe plus ou moins allongée«. The structures which PERGENS as D'ORBIGNY and MARSSON regards as ovicells are as WATERS has pointed out very like the Cheilostomatous avicularia, and the real gonozoocia have not been seen by PERGENS, who, besides, speaks about the supposed ovicells as if there were two different kinds, some placed within the colony and others taking the place of zooecia. This however is not the case. PERGENS as HAMM and MARSSON means that the zooecia of *Meliceritites* take their origin from a central bundle of narrow tubes, and like these authors he also denies the presence of a real operculum⁴⁾: »les Operculés renferment deux familles: les *Myriozoumidæ* et les *Eleidæ*. Le type de la première est réellement pourvu d'un opercule, c'est donc un Cheilostome; les secondes n'ont pas d'opercule, mais souvent

¹⁾ 8, p. 9. ²⁾ 10, p. 45. ³⁾ 9, pp. 325, 391. ⁴⁾ 9, p. 320.

une membrane calcaireuse transversale, caractère qu'elles partagent avec des *Heteropora*, des *Entalophora* et une quantité d'autres genres. —

WATERS¹⁾ who was the first to point out the likeness between the »cellules accessoires« or »cellules ovariennes« of D'ORBIGNY and the Cheilostomatous avicularia is inclined to think, that the *Eleidae* have been provided with a chitinous operculum. He says about that, referring to transverse sections of *Meliceritites Royana*: »In transverse sections (figs 5 and 11) a contraction formed by a curved plate is seen on each side just below the opening. Possibly an operculum has an attachment here, but of this I have not been able to satisfy myself«. He does not mention D'ORBIGNY's calcareous opercula, and, therefore, no doubt regards them in the same way as the other authors, namely as closure-plates. On the whole WATERS seems inclined to think that the *Eleidae* are nearer related to the *Cheilostomata* than to the *Cyclostomata*.

GREGORY²⁾ who regards the *Eleidae* only as a family of his suborder *Cyclostomata tubulata* defines this family as follows: »*Cyclostomata tubulata* in which the

¹⁾ 12, p. 48, pl. VI.

²⁾ GREGORY (14, p. 285) when speaking about D'ORBIGNY's institution of the family *Eleidae* has quite misunderstood this author. He says as follows: »D'Orbigny founded this family in 1853 for a series of Bryozoa with anomalous characters, of which the most striking was the presence of a series of modified zoecia that he described as »cellules accessoires«. These accessory structures are of two types, large superficial marsupial chambers, and cells with triangular or elongated apertures and a platform parallel to the surface of the zoarium. The marsupial chambers were described by d'Orbigny as »cellules ovariennes«, and they are clearly gonocysts or gonæcia. The nature of the second set is more important. D'Orbigny recognized that they sometimes occur on the same specimen as »cellules ovariennes«, and therefore cannot be ovarian. He suggested that they may be male cells or sperm-cells. Nevertheless Pergens and Marsson subsequently described them as ovicells. Every reader must understand the above statement in that manner 1) that D'ORBIGNY has founded the family *Eleidae* chiefly on account of its possessing modified zooecia (»cellules accessoires«), and that he has only found such modified zooecia in the *Eleidae*. Further 2) that D'ORBIGNY has pointed out the presence of marsupial chambers (gonozoecia) in the *Eleidae*, and 3) that he has interpreted some of the modified zoecia found in the *Eleidae* as »cellules mâles«. As to the first point the above quotation of D'ORBIGNY shows that he has founded the division *Centrifugines operculinés* not on account of its possessing modified zooecia, but because it possessed calcareous opercula, and for both the families belonging to this division he only mentions in somewhat different terms that they sometimes possess »cellules accessoires« which structures, besides, he has found not only in this division, but also in the *Cheilostomata* and the *Cyclostomata*. As to the two other points D'ORBIGNY has nowhere mentioned or pictured an Eleid gonozoecium, and what he says about »cellules mâles« only concerns the *Cheilostomata*. The structure of the avicularia being at that time unknown D'ORBIGNY is evidently most inclined to look upon the modified zooecia as gonozoecia (»cellules ovariennes«), but as in ESCHARA¹⁾ they are found together with zooecia wearing ooecia he cannot escape regarding the latter as »cellules ovariennes«, a view held further by all his predecessors, and therefore he asks whether they may not here be regarded as »cellules mâles«. In the representation he gives of the *Bryozoa centrifuginés*, a division which besides some *Ctenostomata* comprises the present *Cyclostomata*, the *Eleidae*, the *Myriozoumidae* and the *Ceidae*, he says about these modified zooecia²⁾: Dans quelques genres

¹⁾ 7, p. 99. ²⁾ 7, p. 587.

apertures are lateral and subterminal, and which frequently have avicularia and spines. The marsupial chambers are gonocysts or gonoecia. Many of the zooecia may be closed by thin calcareous films«. The author here uses the term »lateral« in quite an unusual way; namely as a synonym to »subterminal«, and when he speaks about »spines« he no doubt means the small projections which in many species are found where three dividing ridges meet each other, and which by other authors have been called »tubercles«. However, he only uses this expression in the family diagnose, and nowhere in the descriptions of the single species. GREGORY distinguishes between gonocysts and gonoecia, and as gonocysts he designates the pyriform greatly enlarged gonozooecia which are provided with a small terminal aperture. He ascribes gonoecia to the genus *Nodelea*, but in *N. durobrivensis* he has found a gonocyst, and the gonoecium which he ascribes to *N. semiluna* is really an avicularium. To the genus *Foricula* he ascribes »a gonæcium or gonocyst«, but the pyriform gonæcium (? gonocyst), which he assigns to *F. pyrenaica* is also an avicularium. He is inclined to think that the *Eleidae* have been provided with chitinous opercula, and the calcareous plates which in so many zooecia cover the aperture he interpretes in the same manner as his predecessors, namely as closure-plates.

The present author¹⁾, in 1902, in a preliminary paper expresses the view that D'ORBIGNY has been right in ascribing opercula to the *Eleidae* at the same time pointing out the difference between the operculum and the closure-plate.

LANG²⁾, in 1906, distinguishes between »closed zooecia and normal zooecia« and uses the presence or absence, the frequency or rarity of the former as a specific character. He says about the closed zooecia: »Like the avicularia their physiological significance is a matter of conjecture«.

The Morphology.

The Zooecia.

The zooecia³⁾ have essentially the same form and structure as those of the *Cyclostomata* being very long slender tubes, each of which rises from the proximal

nous voyons, mais très-rarement, des cellules différentes des autres, beaucoup plus grandes, et que nous regardons ici comme des cellules ovariennes servant à la reproduction des oeufs (pl. 735, 736, 741, 761, 777). Quelques genres seulement offrent de véritables vésicules ovariennes distinctes des cellules et placées du côté opposé, destinées aussi à produire les oeufs (pl. 770). While the cellules ovariennes pictured in pl. 761, 770 and 777 (belonging to *Multisparsa Luciana*, *Hornera lichenoides* and *Reptomultisparsa diluviana*) are real cyclostomatous oecia those figured in pl. 735, 736 and 741 are Eleid avicularia«. To GREGORY, therefore, is due the credit of being the first author who has found gonozoecia in the *Eleidae*.

¹⁾ 16, p. 28. ²⁾ 19.

³⁾ In the descriptions of the species the names »zooecia«, »heterozooecia« and »kenozooecia« are used to designate that part of the named zooids, which is visible in the surface of the colony.

part of another tube, and only appearing on the surface of the colony with their distal parts, but while in most *Cyclostomata* the distal part of each zooecium keeps its character as a narrow cylindrical tube whether it projects with a shorter or longer free end or only with its frontal convex surface, I have hitherto only found this to be the case in a single species of this division, namely in *Meliceritites Dollfussi* Perg. (pl. IV, fig. 23, pl. V, figs. 9, 10.) in which the zooecia are provided with a shorter or longer freely projecting terminal part. As a rule the superficial or terminal part of the zooecia enlarges into a rather broad hexagonal, rhombic or quadrangular area, the zooecial area, and the single areas are in most cases divided from each other by a meshwork of distinct ridges. These areas are very much like those found in the coilostegous *Cheilostomata*, for inst. in *Onychocella*, *Micropora*, *Membranicellaria* and *Cellularia* (*Cellaria*), and ROEMER¹⁾ therefore says: »Gleiche Zellenbildung findet sich auch bei *Eschara*«. The greatest likeness these areas show to those found in the genus *Cellularia* as has been pointed out in the historical introduction, but this likeness is a quite superficial one as in this genus the meshwork of ridges has nothing to do with the dividing walls and do not correspond to the single zooecia which have a quite different form²⁾. In the *Meliceritidae* contrary the ridges are the edges of the walls dividing the single zooecial areas which may easily be seen by grinding away a part of the surface of the colony, there being left a system of hollows corresponding in size to the single areas. This meshwork of ridges which is only absent in some species presenting a convex surface, f. inst. in *Meliceritites micropora* (pl. III, fig. 10), is very often in the points of intersection provided with more or less prominent tubercles the presence and development of which is dependent on the manner in which the concave surfaces of the zooecial areas join the marginal ridges. The more the transition of the surface into the ridge is a gradual one the more developed are the tubercles (pl. II, figs. 10, 11, 23, pl. III, fig. 17) while on the contrary they are feebly developed or quite lacking in such species (pl. III, fig. 20, pl. V, figs. 1, 7, 9, 12.) where the ridge is more distinctly defined from the surface. While two contiguous concave surfaces gradually rising towards each other must form a roof-shaped ridge, three or four must where they meet, form a pyramid, but only in a few cases the single surfaces of these pyramids are distinctly defined (pl. V, fig. 19.) being generally rounded. In the circumference of an hexagonal area (pl. II, fig. 11) there can be found six tubercles three of which may be called »præoral« being placed immediately distally to an aperture while the other three each of which has its place between two apertures may be called »interoral«. When the zooecial areas are rhombic there can only be found præoral tubercles.

While concave or saddle-shaped zooecial areas surrounded by distinct ridges and sometimes provided with more or less distinct tubercles are also present in a number of cyclostomatous species, f. inst. in *Peripora pseudospiralis* Mich., *Spiropora*

¹⁾ 6, p. 18. ²⁾ 18, p. 209—212, pls. VII VIII.

macropora d'Orb. *Entalophora ramosissima* d'Orb. and *Ent. madreporacea* Goldf., the *Melicerititidae* always seem to show a distinct difference from the *Cyclostomata* therein that the zooecial tube the larger part of which is very narrow possesses a great and rather sudden distal enlargement while in the *Cyclostomata* the much wider zooecial tube only gradually widens distally without attaining the great distal enlargement. A transverse section of a cyclostomatous colony therefore shows a number of hollows gradually increasing in size from within outwards, (pl. VII, figs. 24, 25), while a corresponding section of a *Meliceritites* distinctly shows a contrast between numerous inner very small hollows and a single or double outer circle or series of much larger ones (pl. VII, figs. 10, 19, 23, 30). We see from these transverse sections that the above enlargement takes place especially in the direction from within outwards, and its presence in the *Melicerititidae* may possibly be explained from the fact that they have possessed an operculum, and a calcareous too, the relative great weight of which must have required strong occlusor muscles. We have seen in the historical introduction that HAMM, HAGENOW, MARSSON and PERGENS use the trumpet-shaped enlargement of the zooecia in the *Eleidae* as a systematic character. D'ORBIGNY does not mention it, but in his work he has given numerous figures of transverse sections which distinctly show the above contrast between the *Cyclostomata* and the *Melicerititidae*.

The aperture is placed in the distal part of the zooecium which in many species is more or less protruding. It is always provided with a straight or almost straight proximal margin, and the two lateral margins which are in most cases more or less convex, more seldom somewhat incurved (pl. II, fig. 23.) or almost straight, either run together in a distal curve or form a distal angle (pl. II, figs. 1, 23, pl. IV, fig. 22). It takes up a greater or smaller part of the zooecial area, which in a few species (pl. VI, figs. 12, 13, pl. VII, fig. 4) it almost fills, the suboral part of the area being very small. The aperture is in most species surrounded by a more or less developed peristomial thickening a greater or smaller distal part of which is formed by the marginal ridge but in many cases the proximal part of this thickening may be absent or only developed in old zooecia. Sometimes the apertures of contiguous zooecia are divided from each other by broad pillar-like swellings, representing both the dividing ridge, the lateral parts of the peristomial thickening (pl. II, fig. 11), and sometimes also the interoral tubercle (pl. III, fig. 18). The præoral tubercle is not rarely developed in the shape of a beak-like projection (pl. I, figs. 1, 2, pl. V, figs. 3, 4, pl. VI, fig. 13). We have seen in the historical introduction that D'ORBIGNY characterizes this division by the presence of a calcareous operculum while all the later authors interpret d'ORBIGNY's operculum as a closure-plate. Some of these authors, however, think that these forms have possessed a chitinous operculum, and the presence of an operculum seems, besides, to be a natural consequence of the assumption, that certain individuals of the colony must be explained as avicularia an avicularium being a modified zooecium provided with a strongly developed and modified operculum. There can be no doubt that D'ORBIGNY is right

in his interpretation of the named calcareous plate. The operculum is an arched calcareous plate provided with more or less distinct radiating striæ starting from the middle of the proximal margin, and in contrast to the closure-plate found in all the species examined it shows a distinct free margin, while the closure-plate is soldered together with the margin of the aperture, being really a growth starting from this margin and gradually extending over the aperture. Sometimes may be found closure-plates the middle part of which is not yet closed (pl. IV, fig. 6, 15). The very presence of the radiating striæ in the operculum is sufficient to show that we have here to do with an independent structure and not with a closure-plate, as such striæ which we f. inst. know from the zooecia of the *Cheilostomata* indicate that the calcification has taken place in radiating belts. At a time when the operculum was still in a membranous state, the calcification therefore started from the middle of the proximal margin and extended in radiating belts gradually outwards and distally. As the other calcareous surfaces the operculum shows a number of more or less distinct pores which in some species are disposed in two distally converging series (pl. I, fig. 1), while in others they are placed in the ends of small claviform projections which show a flabelliform arrangement (pl. I, fig. 11). In *Melic. undata* the operculum shows a number of 1-4 curved impressions (pl. IV, fig. 11, 12, 16). The operculum fits into the aperture in two different manners. In a number of species the margin of the aperture is in the same manner as a door-frame or a window-frame provided with a more or less developed depression, the "oral ledge" decreasing in breadth towards the proximal margin and destined to support the margin of the operculum, when the zooecium is closed (pl. I, fig. 13; pl. III, figs. 10, 11, 23; pl. IV, figs. 18, 22) while in other species the margin of the aperture is only obliquely sloping inwards. The difference between these two cases, however, is not always easy to see, especially when the state of preservation is not good. — As in most *Cyclostomata* the zooecial areas as well as the other surfaces are provided with numerous fine pores, but in a few species the pores are only to be found in the opercula, being in the zooecial and heterozooecial areas replaced by more or less numerous pits (*Melic. punctata*, *Melic. pyrenaica*).

The Heterozooecia.

D'ORBIGNY is the first author who has described and pictured the heterozooecia of the *Meliceritidae*, which he designates as "cellules accessoires" or "cellules ovariennes" the two terms being generally used by him as synonyms.

While MARSSON and PERGENS still look upon these individuals in the same manner as D'ORBIGNY, namely as gonozooecia WATERS is of the opinion that they must be regarded as avicularia, and this view has been followed by the later authors. CANU names them "eleocellaires".

The heterozooecia consist as the zooecia of a long narrow tubiform part hidden within the colony and a superficial part, the heterozooecial area, which is more or

less different from the zooecial area, the aperture being especially of another form or of another size than the zooecial aperture. As a rule it is more elongate, and very often much larger, but in some cases it is much smaller and at the same time of a quite similar form. At the whole there is seen similar differences in the form of the aperture as are found in the avicularia of the *Cheilostomata*. The simplest form of heterozooecia is found f. inst. in *Mel. angulosa* (pl. II, figs. 8, 10), *Mel. Dollfusi* (pl. IV, fig. 23), *Mel. sarissata* (pl. II, fig. 1), *Mel. punctata* (pl. V, fig. 7) and *Mel. hexagona* (pl. V, fig. 3). In the aperture there may be discerned between the proximally situated "inner aperture" distally limited by a curved or angularly bent line and a generally larger or longer distal concave portion, which corresponds to the "oral ledge" of the zooecial aperture, and therefore must be designated in the same manner.

A sagittal section through a heterozooecium shows that the presence of this distinctly bounded oral ledge is due to the circumstance that the inner distal surface of the zooecial tube suddenly alters its direction forming an obtuse angle with the proximally situated part, and the edge thus formed just makes the distal boundary line of the inner aperture. In the heterozooecia of *Mel. Filiozati* n. sp. (pl. VI, figs. 7, 9) and *Mel. squamata* (pl. V, figs. 14–16), and in the large heterozooecium of *Mel. Roemeri* (pl. V, fig. 18) the oral ledge consists of a deeper median part and two more or less developed lateral thickenings while in a larger number of species f. inst. in *Mel. magnifica* (pl. I, figs. 3–5, 7), *Mel. lamellosa*, *Mel. pentagonum* (pl. IV, fig. 22) and *Mel. gothica* (pl. II, fig. 24) these thickenings have been developed into two inwards sloping triangular processes, the "lateral processes" which conceal the larger part of the oral ledge and the inner aperture.

The thickness of these two processes (pl. I, fig. 10) which may be coherent distally decreases towards the median line where they are divided from each other by a more or less narrow longitudinal fissure their free proximal margin being divided by a transverse fissure from the distal margin of the suboral area. When the state of preservation is not good the outer boundary of the two processes is not distinct, and the opening formed by the two fissures may be taken as the space once filled by the mandible. The aperture in all the species of this group is of an elongate triangular form. A fourth group of heterozooecia which have been found in *Mel. Steenstrupi* (pl. III, figs. 11, 12), *Mel. Canui* n. sp. (pl. III, figs. 19–27), *Mel. durobrivensis* (pl. IV, figs. 1–4) and *Mel. Roemeri* (pl. V, figs. 21–24) have that in common with the first mentioned, that the aperture lacks both lateral thickenings and lateral processes but these apertures have a form very like to or at least not very different from the zooecial aperture, and most of them are, moreover, much smaller than the latter. Also the heterozooecia themselves are as a rule much smaller than the zooecia, and the smallest heterozooecia found in *Mel. durobrivensis* only attain the sixth part of the length of the zooecia. A few of the heterozooecia found in *Mel. Canui* have the apertures provided with a comparatively narrow oral ledge, but as a rule the latter takes up the larger part of the aperture and is per-

forated either with a small rounded or with a narrow triangular or fissure-like opening. While the form of the apertures is constant in *Mel. durobrivensis* and *Mel. Steenstrupi*, in *Mel. Canui* and *Mel. Roemeri* it is subject to a rather great variation both as to form and size, and in the latter species the largest of them are ligulate and stretched beyond the distal angulate border of the zooecium. In *Mel. durobrivensis* these heterozooecia are found not only interspersed among the zooecia, but also constituting an incrusting base (pl. IV, fig. 19), and they seem here to play a similar role as the corresponding kenozooecia of a *Retepora*-colony¹⁾.

In most species I have found the aperture of more or less heterozooecia closed by a calcareous mandible (pl. I, figs. 1, 2; pl. II, figs. 3, 16, 18; pl. III, figs. 3, 20; pl. IV, figs. 1, 23; pl. V, figs. 5, 8, 17; pl. VI, fig. 2) which has a similar arched surface as the opercula and often shows more or less distinct radiating striæ. Sometimes it is as many mandibles of cheilostomatous avicularia provided with a hooked beak, being at the same time strongly arched not only from side to side but also proximally distally (pl. II, fig. 3).

In *Mel. Canui* and *Mel. durobrivensis* the semi-elliptical mandible is provided with distinct radiating striæ, and chiefly differs from the zooecial operculum in being much smaller. Lastly it might be of interest to compare these heterozooecia with those found in the *Cheilostomata*, and in order to make the difference between them more conspicuous we shall choose for comparison such presenting a maximum of outer likeness, f. inst. those found in a *Thalamoporella*-species²⁾ and in *Mel. angulosa* (pl. II). Besides the likeness in the form of the aperture we may in both discern between an inner aperture and a distal concavity, but while the latter in *Meliceritites* is the distal inner surface of the zooecial tube it is in *Thalamoporella* formed by a free lamina (a cryptocyst) which rises from the lateral and the distal walls within the free margin. Besides the difference which the heterozooecia of the *Meliceritidae* show from those of the *Cheilostomata* in being long slender tubes the greatest part of which is hidden within the colony they present another constant difference from the latter therein that the aperture is always limited by a continuous calcified frame while in the *Cheilostomata* it is limited proximally by a membranous area of different extent. The group of heterozooecia above spoken of in which the larger part of the original aperture has been concealed by two triangular laminae shows a certain likeness to the heterozooecia of certain species of *Onychocella*³⁾ and *Rhagasostoma* in which the cryptocyst lamina has attained its greatest development, being only provided with a small perforation for the occlusor muscles, but the two laminae are two thick processes from the lateral parts of the oral ledge, and the two mutually vertical fissures which may be compared to the perforation in the avicularia of *Onychocella* and *Rhagasostoma* belong both to the opercular area while the proximal part of the latter perforation is placed within the suboral area. Lastly we shall remind of the different structure of the mandibles.

¹⁾ 18, p. 290, pl. X. ²⁾ 18, pl. VI a. ³⁾ 7, pl. 673, figs. 1, 4, 8.

In opposition to the *Cheilostomata* in which heterozooecia may also arise by a process of budding from the surface of the zooecia (dependent heterozooecia) the *Melicerititidae* only possess independent heterozooecia, and it is therefore a mistake when GREGORY in *Foricula aspera*¹⁾ interprets a pair of long narrow pits seated on each side of the aperture as avicularia.

As a result of the above comparison I must agree with the opinion set forth by GREGORY²⁾ that the avicularia in both divisions have developed independently, and therefore are only parallel, not homologous structures.

The Kenozooecia.

Kenozooecia or bryozoids without an aperture (dactylethrae Greg) have been found in 12 of the 31 species examined. As a rule they have the same form and size as the zooecia, but as they lack an aperture they are provided with a uniform flat or concave frontal area within the marginal ridges. In *Mel. Steenstrupi* they are exceptionally much smaller than the zooecia. They are generally found interspersed among the zooecia (pl. VI, fig. 19) in greater or smaller numbers, but in a few species they take up together with a number of heterozooecia either the whole "dorsal" surface or a large part of it (pl. III, figs. 11, 14), and at the whole they are in most cases accompanied by heterozooecia, whether they appear in groups among the zooecia or take up a large part of the surface of the colony.

The Gonozooecia.

The gonozooecia (pl. VII) which have hitherto only been found in a small number of the species examined are zooecia provided with a large (1—2 mm. long), more or less convex terminal expansion, an ooecium which in most cases has a pyriform outline, but in the same species, nay in the same colony the form may be subject to great variation, being sometimes roundedly triangular, sometimes ovate or even circular. The distal end of the ooecium is provided with a transversely ovate, somewhat infundibuliform, frontally directed aperture, and proximally it generally ends in a shorter or longer tail-shaped process. When the thick porous frontal wall is removed, the flat or somewhat concave inner wall in most of the ooecia examined presents a uniform smooth surface which is only interrupted at the boundary between the caudal process and the rest of the cavity by a rhombic zooecial area surrounded by prominent ridges and closed by a concave calcareous film. Distally to this area which no doubt belongs to the gonozooecium may sometimes be seen a few more or less indistinct rhombic impressions of the same form and size as the zooecial areas, and when the inner wall of the ooecium has been dissolved by the use of strongly diluted acid, there appears a mosaic of rhombic zooecial areas each of which is closed by a concave calcareous film.

¹⁾ 14, p. 358 ²⁾ 14, p. 288

In the ooecia of *Mel. lamellosa* (pl. VII, figs. 20—22) however, only a few zooecial areas are seen in the proximal part while in the rest of the cavity the zooecia are only represented by a number of more or less open tubes. In opposition to the ooecium of *Mel. magnifica* figured in Pl. VII, fig. 14, in which the inner wall has covered a number of undeveloped zooecia and heterozooecia I have found another small ooecium of the same species in which the corresponding individuals are provided with completely developed apertures. Also the gonozooecium the larger part of which is seen proximally to the tail-shaped process of the ooecium is provided with a completely developed aperture. A third different case I have found in a number of open ooecia belonging to the same species, all of which were placed not far from the growing edge in a superficial layer of growth (pl. VII, fig. 17). Here there projects in the proximal part of the ooecium a number of zooecia and heterozooecia of very different development while the rest of the inner wall presents a uniform smooth surface, the dissolution of which by the use of diluted acid denudes a layer of completely developed zooecia and heterozooecia belonging to an older part of the colony. In the ooecium figured in Pl. VII, fig. 17 there is seen in the proximal part a number of open zooecial areas, and the gonozooecium is in the same state of development, while that figured in fig. 18 shows a number of almost completely developed zooecia and heterozooecia. Proximally to the latter is seen a small triangularly rounded aperture, but the tail-shaped process seems to have been broken off.

By the aid of the above facts we may form the following picture of the development of the ooecia. When the gonozooecium begins to expand into the ooecium the inner wall of the latter covers a number of more or less developed zooids, which of course get chequed in their development while the adjacent zooids are able to continue their growth, which explains that the lateral margins of the ooecia are partly covered by a number of zooecia and heterozooecia. When the gonozooecium belongs to a superficial layer of growth and is placed near to the growing edge, the ooecium first covers the undeveloped zooids placed distally to the gonozooecium and thereafter a number of zooids belonging to the older part of the colony. The presence of freely prominent zooids in the proximal part of the ooecium must no doubt be explained in that manner that they have originally been covered by a bulging part of the inner wall of the ooecium, which has later been destroyed. A curious fact is the different state of development shown by the gonozooecia even in the same species.

The aperture of the ooecium leads into a short atrium partially divided from the rest of the ooecial cavity by a low ring-shaped ridge (pl. VII, figs. 21, 22, 28, 29) which from the inner wall passes obliquely forwards to the frontal wall. By the use of a great magnifying power its surface is seen to be provided with small projections which stand out as free irregular teeth on its distal margin (pl. VII, fig. 28). HARMER¹⁾ has pointed out a similar atrium in the ooecia of

¹⁾ 12 a, p. 170, pl. XII, fig. 10.

Crisia, and in *Cr. ramosa* Harmer he has found it partially separated from the rest of the cavity by a calcareous valve which however is not developed on the frontal wall. I have found a similar valve in *Cr. hamifera*¹⁾ n. sp., but in *Cr. eburnea*, (pl. VII, figs. 11, 12) *Cr. denticulata* and *Cr. aculeata* it is replaced by a similar continuous ring as that found in the ooecia of *Meliceritites*. It is provided with more or less finely ramose processes the character of which is distinctly different in the three species.

The Regeneration.

Besides the regeneration of the polypide which is no doubt a common feature in all *Bryozoa* a regeneration of the whole individual has been shown to take place²⁾ in a number of species both in the *Ctenostomata* and the *Cheilostomata*, and such a regeneration I have also found in most species of the present division. While in the *Ctenostomata* this regeneration takes place in that manner that the old individual drops off, and a new one takes its place, in all species the zooecium of which is more or less calcified the new individual develops within the old zooecium, and that such a complete regeneration has taken place is evident from the fact that a new aperture is seen within the old one. In the *Cheilostomata* I have shown that this regeneration takes place in such a manner that the different forms of bryozoids may replace each other, and the same case I have found also in the *Meliceritidae*. The regeneration in this division has hitherto been overlooked or interpreted as a certain form of closure, f. inst. by GREGORY who in *Mel. durobrivensis* speaks about a closure by means of an inverted funnel-shaped cap pierced by a pore. Here we have to do with a regeneration of a new heterozooecium within an old zooecium. As I am later to give information of the regeneration in the single species described I shall here only give a short summary of the main features of this process.

In the *Meliceritidae* as in the *Cheilostomata* we can discern between the following four forms: ¹⁾ the regeneration of a new zooecium within an old one (pl. III, figs. 2, 7; pl. IV, figs. 4, 8, 11, 14; pl. V, figs. 2, 4, 19; pl. I, figs. 3, 4, 16); ²⁾ the regeneration of a new zooecium within a heterozooecium (pl. II, figs. 5, 7, 12; pl. V, fig. 14); ³⁾ the regeneration of a new heterozooecium within an old one (pl. I, fig. 4; pl. II, figs. 6, 9, 20, 22; pl. III, figs. 6, 7; pl. V, fig. 15) and ⁴⁾ the regeneration of a new heterozooecium within an old zooecium, which is perhaps the most common form of regeneration (pl. I, figs. 18, 19; pl. III, figs. 1, 3, 4, 5, 27; pl. IV, figs. 3, 7, 10, 14). When a new large heterozooecium is regenerated within an old one it may fill the old aperture completely (pl. II, figs. 20, 22) or only a part of it, and in the latter case the space between the two apertures is gradually filled by a clo-

¹⁾ This species which is taken at lat. 33° 9' N., long. 129° 18' W. is in the dorsal surface provided with a number of hook shaped appendages each of which through a corneous joint is fixed to the proximal part of an internode "17.

sure-plate. As a rule this closure begins with the formation of three slender processes (pl. II, fig. 9) starting one from the end of the new aperture and the two others from the lateral margins of the old one. They unite about half the way between the end of the new and that of the old aperture, and the three open spaces are later filled, each by a calcareous lamina. A similar closure also takes place when a new zooecium is regenerated within a large heterozoecium (pl. II, fig. 5). In the *Cyclostomata* a complete regeneration seems to be very rare, and I have hitherto only been able to find a few indistinct cases in *Entalophora madreporacea* and *Homera lichenoides*.

The Closure.

The closure of old zooecia by means of a calcareous film is a well-known fact both in the *Cheilostomata* and the *Cyclostomata*, and it seems to be more common in the latter than in the former division. The real operculum of the *Meliceritidæ*, which has been correctly interpreted by D'ORBIGNY has by all later authors been regarded as a closure-plate, and with the exception of WATERS who mentions a peculiar form of closure in *Meliceritites Royana* no author seems to have noticed a real closure in this division. I have found a closure in almost all the species examined, not only in the zooecia, but also in the heterozoecia, and as a rule the zooecia are closed by a concave or sometimes flat calcareous film (pl. I, figs. 4, 14, 16; pl. II, fig. 7; pl. III, figs. 7, 26; pl. IV, figs. 6, 7, 15; pl. V, figs. 2, 19; pl. VI, figs. 4, 10), which in the species provided with an oral ledge either starts from the inner margin of the latter or at a somewhat deeper level; and in that case the aperture is lastly closed in its proximal and middle part (pl. II, fig. 7; pl. III, fig. 25). In the other species the closure starts from the margin of the aperture and gradually extends towards the centre (pl. IV, fig. 15). In *Mel. magnifica* (pl. I, fig. 7) and *Mel. plana* D'ORB. (pl. V, fig. 12) the closure takes place by means of 3—5 processes starting from the margin of the aperture and later coalescing into a cover perforated by 3—5 holes, which are gradually closed. A third form of closure I have found in *Mel. palpebrosa* (pl. VII, fig. 1, 4—9). In opposition to what is found in the operculum the closure-plate never presents a flabelliform striation.

The Colonies.

In opposition to the rich diversity of colonial forms or forms of growth shown by the *Cyclostomata* the number of colonial forms presented by the *Meliceritidæ* is very small, and we can only discern between incrusting disciform colonies, one- or two-layered laminose fronds, and ramose colonies with cylindrical branches. The laminose colonies sometimes form hollow expansions f. inst. in *Mel. Vieilbanci* or a reticulate network, f. inst. in *Retelea pulchella* D'Orb. In many species the colo-

nies of which are incrusting or freely ramose there appears in the course of time new layers of zooecia over the old ones, and the old colonies therefore get many-layered.

According to D'ORBIGNY the formation of new layers takes place in three different manners. In the ramose colonies f. inst. in *Multelea magnifica* they are said to start from the proximal part of the colony whence they gradually and regularly extend distally. In the disciform colonies each layer may either f. inst. in *Semimultelea cupula* and *Sem. gradata* be formed by a single subcolony starting from the centre and extending towards the margin or f. inst. in *Reptomultelea tuberosa* and *Clausimultelea tuberosa* or the surface of the colonie may at the same time present a greater or smaller number of small disciform sub-colonies which at last must come in contact or fuse together. I have examined a large number of fragments of *Mel. magnifica*, and I have come to the result that the superficial layers are not formed in such a regular manner as D'ORBIGNY means, the fragments examined presenting in different parts of the surface a number of independent layers or patches of zooecia. Pl. VII, fig. 16, shows a fragment of a colony the one surface of which presents three different layers or sub-colonies. One surrounds the proximal part of the rudiment while another arising from the space between the two branches extends both upwards and downwards, and a third, a small round patch is seen to the link side between the two larger ones. The opposite surface of the fragment presented still two others. While I have not been able to find the ancestrulae of the new layers in *Mel. magnifica* I have seen a number of them in small fragments of *Mel. tuberculata* D'ORB. (pl. VI, fig. 3) and *Mel. Filiozati* n. sp. (pl. VI, fig. 7). As can also be seen in the figure of a young sub-colonie of *Semimultelea gradata* given by D'ORBIGNY such an ancestrula is only represented by the aperture, the rest of the zooecium being covered by the new zooecia which have arisen from it. But while this aperture in D'ORBIGNY's figure is seen in the centre of a small distinct sub-colony the margin of which is formed by undeveloped zooecia, the named fragments each presents a uniform continuous surface formed by zooecia and heterozooecia among which are seen a number of ancestrular apertures, some of which may often be placed so near to each other that two such apertures are only divided by the breadth of a zooecium. Each aperture which is obliquely ascending is placed in the centre of a small deepening, and the zooecia and heterozooecia surrounding two or more such apertures placed near to each other, may be more or less irregularly arranged, but I have never seen such an aperture making the centre of a distinct sub-colony, and the zooecia arising from the different ancestrulae seem to have accommodated themselves pretty well to each other during their growth. A fragment of this species 5 mm long and 3 mm broad presents 8 such ancestrular apertures, and another of a similar size 6. A single time I have seen a short cylindrical zooecium placed vertically between four zooecia and a kenozooecium, and it must no doubt be regarded as an ancestrula destined to take part in the formation of a new superficial layer.

Affinities.

As to the affinities of the present division there can be no doubt that the *Melicerititidae* are *Cyclostomata*, and in the first place this is distinctly shown by the form and development of the zooecia, these being long slender tubes each of which arises from the proximal part of another zooecium. The presence of numerous fine pores is also a cyclostomatous character, and rhombic or hexagonal zooecial areas divided by prominent marginal ridges may also be found in a number of *Cyclostomata*. The gonozooecia are provided with similar ooecial expansions as are found in the *Cyclostomata*, and the superficial layers of growth above spoken of are also found in a number of cyclostomatous species, but never in the *Cheilostomata*. The only two characters which might speak in favour of Cheilostomatous affinities are the presence of an operculum and of heterozooecia. Apart from the fact that the latter as the common zooecia are long slender tubes they differ from the cheilostomatous heterozooecia in possessing calcareous mandibles and in lacking a membranous suboral area. The opercula are also calcareous while the opercula in the great majority of the *Cheilostomata* are chitinous, a calcareous operculum being only present in a few cheilostomatous species. Therefore we must regard the presence of opercula and of heterozooecia as a case of parallel development.

We here propose to divide the *Ordre Cyclostomata* in two subordres, the *Cyclostomata inoperculata* and the *Cyclostomata operculata*, and the latter may be defined in the following manner.

Cyclostomata operculata.

Cyclostomata the zooecial tubes of which are much widened distally appearing on the surface of the colony as hexagonal, rhombic or quadrangular mostly concave areas, in most cases divided from each other by a meshwork of ridges, in the knots of which are very often seen more or less developed tubercles. The subterminal aperture which has a straight or almost straight proximal margin is provided with a convex calcareous operculum showing more or less distinct radiating striae. In most species are found heterozooecia the aperture of which is very often of a similar form as that found in the cheilostomatous avicularia. They always lack a membranous subopercular area and are provided with a calcareous mandible.

Family *Melicerititidae* Pergens.

Eleidae d'Orbigny.

The family *Melicerititidae* has been divided by D'ORBIGNY in 11 and by GREGORY in 10 genera, and the generic characters have been taken partly from the form and mode of growth of the colony partly from the presence or absence of heterozooecia (avicularia). As to my opinion none of these characters are sufficient to serve as base for a generic division, and I have not yet been able to distinguish

between well divided groups based on structural diversities I prefer at present to acknowledge only a single genus, *Meliceritites*. The most significant structural diversity is to my opinion the presence or absence of an oral ledge, but when the state of conservation is not a good one it is not easy to see whether the aperture is provided with a feebly developed oral ledge or not. Also the diversities found in the heterozooecia may perhaps be of systematic significance, but to decide these different questions it should be necessary to possess a large and well-conserved material.

Meliceritites magnifica d'Orbigny.

Multelea magnifica d'Orbigny, Bryoz. crét. p. 649, pl. 740, figs. 1—9.

Meliceritites magnifica Pergens, Revision d. Bryoz. p. 397.

Meliceritites royana Waters, Annals Nat. Hist. [6] VIII 1891, p. 51, pl. VI, figs. 2, 4—6, 11.

Multelea magnifica Gregory, Cret. Bryoz. p. 316.

(pl I, figs. 3—10, pl. VII, figs. 13—19.)

The *Zooecia* which are divided by distinct marginal ridges are small and have when freely developed a more or less regular rhombic outline, but in most cases each zooecium is enclosed between two heterozooecia which greatly influence both the size and the form of the subopercular area, and in the zooecia enclosed between the proximal halves of two heterozooecia a large part of this area is covered by the distal part of a proximal heterozooecium. No distinct tubercles. The aperture which takes up the whole breadth of the zooecium in the distal part may be contained two or rarely three times in the length of the zooecium, but in most cases it is longer than the subopercular area. It is longer than broad, half-elliptical or roundedly triangular and surrounded by a raised peristome, sometimes provided with a small distal projection. The anter. of the aperture is provided with a distinct but rather narrow oral ledge, and the convex operculum which sometimes shows a distinct flabelliform striation presents a more or less distinct triangular depression which from the proximal margin extends more or less far distally.

The *Heterozooecia* which are much larger than the zooecia are of a lengthened rhombic or hexagonally rhombic form with the four lateral margin more or less incurved. The opercular area which ends in a rounded apex may be very much protuding and obliquely ascending, and the inner aperture is concealed by a much concave covering (the »lateral processes«) the proximal half of which is provided with a narrow median fissure and the proximal margin of which by a similar transverse fissure is divided from the more or less protuding somewhat thickened distal margin of the concave subopercular area. The heterozooecia in this species are much more numerous than in any other hitherto described.

The *Closure* takes place in different ways. In many zooecia the aperture is closed by a concave lamina but in others (figs. 6, 7) I have seen a closure-plate of a similar appearance as the concave covering found in the opercular area of the heterozooecia, namely presenting two narrow fissures forming right angles with each other. Perhaps the latter form of closure is only the beginning of the concave

lamina. A second form of closure is that pointed out by WATERS. It starts by the growing forth from the margin of the aperture of 2—5 processes which later coalesce after which the smaller apertures between the processes are gradually filled out. Both forms of closure may be found in the same colony, but as a rule one of them is predominant.

The heterozooecia are closed by a filling out of the two fissures found in the opercular cover. —

The *Ooecia* are of very different form and size.

Kenozooecia have not been found.

Regeneration is found both of the zooecia and of the heterozooecia, but with the exception of a few indistinct cases always in that manner that old zooecia have been regenerated by new zooecia and old heterozooecia by new heterozooecia. There has been as many as three regenerations in a single zooecium or heterozooecium.

The *Colonies* are erect with cylindrical branches, and when old are surrounded by a number of superficial layers of new zooecia, which may arise in very different parts of the branches. The zooecia and heterozooecia show a disposition to arrange themselves in transverse series in such a manner that two series of zooecia are followed by a single series of heterozooecia, and in the most regular colonies these series form more or less complete, more or less unmixed, ring-shaped belts surrounding the branches. Sometimes, however, the arrangement of the zooecia and heterozooecia may be more or less irregular, and the heterozooecia may sometimes be present in sparse numbers, irregularly distributed among the zooecia (var. *royana* Wat.). The zooecia placed between the distal halves of the heterozooecia are provided with a small narrow subopercular area, while in the other series of zooecia the subopercular area is as a rule much broader, but the proximal part of it is concealed by the distal end of the proximal heterozooecia.

Critical remarks. In D'ORBIGNY's figure 2 is seen distally to each transverse series of heterozooecia a transverse series of zooecia which differ from those placed proximally to the heterozooecia by the possession of a very small half-elliptical aperture. The zooecia of this series, however, do not differ from the other in the form of the aperture, and D'ORBIGNY's error must no doubt be explained in that way that he has seen in such a series a number of regenerated zooecia (see pl. 1, fig. 3), the structure of which he has misunderstood. For the rest D'ORBIGNY regards the heterozooecia as »cellules ordinaires» and the zooecia as »cellules accessoires». — In longitudinal sections of many-layered colonies of this species WATERS means to have seen that the zooecia of the external layers arise from a plate covering the aperture of the subjacent zooecia. I do not agree with WATERS in that question, and to my opinion a longitudinal section of such a colony only shows that the inner wall of the external zooecia has quite coalesced with the frontal wall of the subjacent ones. That the operculum or the closing plate of an old zooecium should be able to give rise to a new zooecium is not very probable, and that each zooecium of the external layer should grow forth from a subjacent one does not correspond

with the fact that the surface of such a colony shows a number of smaller or larger patches or layers in which the single zooecia are arranged round a centre. In transverse sections of colonies of the same species WATERS has further seen »a contraction formed by a curved plate« placed »on each side just below the opening«. I have never been able to find the two curved projections figured by WATERS but sometimes two 'slender conical processes which arise from a transverse section of the above covering found in the opercular area of the heterozooecia. — Of this species I have examined a large number of specimens from Villedieu (Coniacian).

Meliceritites trifolium n sp.

? *Multelea semiluna* d'Orbigny, Bryoz. crét. p. 646, pl. 739, figs. 8—11.

(Pl. 1, figs. 17—19.)

The *Zooecia* which are divided by distinct marginal ridges are small, rhombic or hexagonal, not so much longer than broad, and their distal part is as a rule not entirely taken up by the aperture. There may be found more or less distinct tubercles. The surface which may be concave, flat or even a little convex is more or less distinctly ascending toward the half-elliptical aperture the length of which may be contained two or two and a half times in the length of the zooecium. The aperture is provided with a distinct but rather narrow oral ledge and a distinct peristomial thickening, and the operculum is much convex and provided with distinct radiating striae.

The *Heterozooecia* which may be found in very different numbers are of a similar form and size as the zooecia; but the surface is much more concave and not ascending toward the aperture. The latter has a similar covering as that found in the heterozooecia of *Mel. magnifica*. It is provided with a narrow median fissure, and by a similar transverse fissure it is divided from the opercular area.

Ooecia have not been found.

Kenozooecia have not been found.

The *Closure* is effected in the zooecia by means of a concave lamina and in the heterozooecia by a filling out of the fissures.

The *Regeneration*. The only form of regeneration which I have seen distinctly in this species is the formation of new heterozooecia in old zooecia. In that case the aperture of the latter is taken up by a large arched projection, the frontal and proximal part of which is provided with a similar aperture as that found in the heterozooecia, only much smaller. In old colonies I have seen a very great number of the zooecia transformed in that manner.

The *Colonies* are free with cylindrical branches, and in some of them I have seen superficial layers of different extension.

In the presence of a similarly developed oral ledge, in the structure of the heterozooecia and in the form of the colony this species shows affinity to *Mel. magnifica*. —

I have examined a number of fragments from Bruillé Poncé. (Turonian).

Meliceritites angulosa d'Orbigny.

- Nodelea angulosa d'Orbigny Bryoz. crét. p. 610, pl. 735, figs. 4—5 (non 6).
 — ornata d'Orbigny op. cit. p. 612, pl. 736, figs. 12—16.
 — transversa d'Orbigny op. cit. p. 613, pl. 736, figs. 5—8.
 — pulchella d'Orbigny op. cit. p. 613, pl. 736, figs. 1—4.
 — ogivalis d'Orbigny op. cit. p. 624, pl. 737, figs. 8—10.
 — semiclausa d'Orbigny op. cit. p. 619, pl. 678, fig. 7, pl. 736, fig. 16.
 Multinodelea tuberosa d'Orbigny op. cit. p. 615, pl. 736, figs. 9—11, 13—15.
 Meliceritites undata Gregory (non d'Orbigny) op. cit. p. 340, pl. XV, figs. 3, 4, pl. XVI, fig. 3.
 (Pl. II, figs. 4—22.)

The *Zooecia* which are divided from each other by more or less developed marginal ridges are in most cases of a more or less distinct hexagonal form, and the semielliptical aperture is always provided with a distinct, but more or less developed oral ledge, and with a distinct peristomial thickening. It takes up in most cases the whole breadth of the zooecium in the distal part; but while in some cases it is longer than the subopercular area in others it is shorter, and sometimes its length may be contained twice in the length of the latter. More or less developed tubercles. The operculum is convex and provided with radiating striae.

The *Heterozooecia* which are present in very sparse number and are very variable in form and size are dispersed singly or in pairs among the zooecia which they always surpass in length. Most of them have about the double length of the zooecia, but sometimes they are only a little longer. In opposition to the heterozooecia of *Mel. magnifica* and *Mel. trifolium* the inner aperture is always visible, not being concealed by »lateral processes«, but the form of the opercular area is subject to a very great variation, being dependent not only on the different relation between its length and its (largest) breadth, but also on the form of the lateral margins, and lastly on the manner in which the latter run together to form the distal end of the area. The relation named varies between 4:3 and 5:2, and the lateral margins may sometimes in the greater part of their length be parallel or almost so, sometimes more or less convergent and sometimes more or less incurved. As to the last named difference the lateral margins run together to form an almost semicircular curve while in other cases they make an almost rectangular bend in order to form the feebly curved distal end. Also the relative length of the opercular and the subopercular area is subject to variation, and the former may sometimes be three times as long as the latter while in other cases it is only a little longer. In most cases it attains the double length. In a number of heterozooecia I have found an arched calcareous mandible which sometimes shows a similar striation as that found in the operculum.

Kenozooecia have not been found.

The *Ooecia* are of different form and size.

The *Closure* of the zooecia is effected by a concave lamina starting from the free edge of the oral ledge and developing in such a way that at a certain point

of time the closure-plate is perforated by a fissure-like opening seated in its proximal and median part (fig. 7).

The *Regeneration*. In a few cases I have found zooecia regenerated in old ones and more frequent a regeneration of the heterozooecia, sometimes by a new heterozooecium sometimes by a zooecium (figs. 5, 7). Sometimes the new heterozooecium may fill out the whole aperture of the old one (fig. 20), but if that is not the case the space between the old and the new aperture is gradually closed by a calcareous lamina which however is not formed as a unity, but as more (as a rule three) plates filling out the spaces between as many calcareous processes. Fig. 9 shows the three calcareous processes, and in fig. 5, which presents a zooecium regenerated in a heterozooecium is seen a similar case in which the spaces between the three processes have been filled out. In fig. 7 is seen a more irregular case and in the case presented in fig. 22 the filling out of the narrow space between the old and the new aperture has been prepared by the formation of short connecting processes.

The *Colonies* are free with cylindrical branches, and in time increase in thickness by the formation of superficial layers several of which may be seen at the same time in different parts of the branch.

This species is subject to a very great variation in all respects, and I have tried in vain to divide the material examined in more different forms. We may discern between the following two chief-varieties which are however very far from being sharply limited.

Var. *latirostris*. The subopercular area is longer than or as long as the aperture, longer than broad; the oral ledge is narrow, and its height in the distal part does not attain a third part of the height of the aperture. The marginal ridges are narrow and the tubercles are small. The distal half of the heterozooecial aperture is broad.

Var *angustirostris*. The subopercular area is shorter than the aperture broader than long. The oral ledge is broad and its height in the distal part attains the third part of the height of the aperture. The marginal ridges and the tubercles are strongly developed.

I have examined colonies from Villedieu, Fécamp, Couture, Vendôme, St. Patern, Evreux and from Chatham.

Meliceritites semiluna d'Orbigny.

Nodelca semiluna d'Orbigny, Bryoz. crét. p. 611, pl. 735, figs. 9—11.

— — Gregory, Cret. Bryozoa p. 307.

(Pl. VI, figs. 4—6.)

The *Zooecia* which have a more or less distinct rhombic or hexagonal outline and the subopercular area of which is as a rule convex or flat, and more or less distinctly ascending towards the aperture are rarely divided by indistinct marginal ridges, and as a rule there are no tubercles. The aperture which is about as long

as the subopercular area and takes up the whole breadth of the zooecium in its distal part is provided with a strongly developed, as a rule angularly bent oral ledge, the height of which in its distal part may be contained about three times in the height of the whole aperture. The form of the latter varies between half-elliptical and roundedly triangular, the lateral margins being in their proximal half sometimes almost parallel sometimes more or less converging distally. The peristomial thickening is rarely distinctly developed being in most cases coalesced with the convex subopercular areas of the adjacent zooecia, and the apertures of the zooecia in the same transverse series are in most cases divided by columnar projections formed wholly or partly by the lateral parts of the peristomial thickening. The proximal part of the peristome forms a more or less prominent lip. The operculum is very convex, but in such a manner that its proximal half is provided with a triangular flat, obliquely ascending area from the margins of which the surrounding parts are gradually descending. It is provided with distinct radiating striae.

The *Heterozooecia* which are about half as long as the zooecia are sparingly spread over the surface of the colony, singly or in pairs, the two heterozooecia belonging to a pair being either placed side by side or divided from each other by a single zooecium. The strongly projecting obliquely ascending aperture is as long as broad or a little longer and has a somewhat variable form, the two lateral margins being more or less converging distally. No »lateral processes«.

Ooecia have been found.

No *Kenozooecia*.

The *Closure* takes place by means of a concave or in most cases flat lamina which arises a little within the free edge of the oral ledge from which, therefore, it is distinctly defined. A similar closure I have seen also in a heterozooecium.

The *Regeneration*. I have seen cases of regeneration both of zooecia and of heterozooecia.

The *Colonies* are free, with cylindrical branches, and in some of the fragments examined I have found superficial layers.

Mel. semiluna is nearly related to *Mel. angulosa* and may perhaps be regarded as a constant variety of this species, from which it is most easily discerned by the lack of or the feeble development of the marginal ridges and the tubercles, the form of the operculum and the mode of closure. I have seen a number of fragments from Bruillé-Poncé (Turonian).

Meliceritites palpebrosa nov. nom.

Meliceritites semiluna d'Orbigny, Bryoz. crét. p. 623, pl. 736, fig. 20—21.

non — — Gregory, Cretac-Bryoz. p. 343.

(Pl. VII, figs. 1—9.)

The *Zooecia* are chiefly represented by their large half-elliptical apertures which as a rule take up most of the surface of the colony. Marginal ridges and tubercles

are not developed. The aperture is provided with a very distinct, but rather narrow oral ledge, but a peristomial thickening is very seldom distinct, being in most cases coalesced with the surface of the small, triangular mostly convex subopercular area the height of which in most cases is contained about two times in the height of the aperture. The apertures placed in the same transverse series are in most cases divided from each other by columnar projections wholly or partly formed by the lateral parts of the peristomes. The operculum is convex with distinct radiating striae.

No *Heterozooecia*.

No *Kenozooecia*.

Ooecia have not been found.

The *Closure* as a rule takes place in a very singular manner. It starts by an enlargement of the oral ledge, and at the same time the distal and the proximal margins of the latter get connected by a number of vertical columnar projections divided by rounded pits. As this distal closure-plate gradually increases in size and in thickness the pits increase in length, at the same time diminishing in breadth, and the oral ledge thus gets transformed into a strongly arched structure very much like a cheilostomatous ooecium or an eye-lid. According to their age these distal closure-plates are either smooth or their surface presents a series of more or less distinct impressions, the vestiges of the original pits. Somewhat later than the distal also a proximal closing plate begins to develop, starting from the proximal margin of the aperture. In opposition to the distal plate the latter is as a rule concave or flat, and sometimes directed obliquely inwards. By and by the originally large aperture is transformed into a narrow fissure-like opening which is lastly filled out. This process of closure, however, shows a great variation even in the same fragment, the two plates being developed in very different degree, and in some cases the distal one may be very small (figs. 1, 9). In a few zooecia I have found the closure effected in the usual way by means of a concave or flat lamina (fig. 2).

Regeneration has not been found.

The *Colonies* are free with cylindrical branches, and I have seen no superficial layers.

I have seen rather numerous specimens of this species from Villedieu.

Meliceritites Lorieri d'Orbigny.

Vincularia Lorieri d'Orbigny, Bryoz. Crét. p. 61, pl. 601, figs. 18—20.

Meliceritites semiclausula Pergens, Revision d. Bryoz. p. 394.

non *Meliceritites semiluna* d'Orbigny, Bryoz. Crét., p. 623, pl. 736, figs. 20—21.

non *Meliceritites semiclausula* Gregory, Cretac. Bryozoa, p. 328, pl. XIV, figs. 1—3.

(Pl. IV, fig. 18.)

The *Zooecia* which are at the utmost as long as broad and in most cases a little broader than long are not divided by distinct marginal ridges, and the peri-

stomial thickenings are either quite confluent with the suboral areas of the adjacent zooecia or indistinctly divided from the latter. Sometimes there may be found indistinctly defined tubercles between the zooecia of the same transverse series. The aperture which is broader than high has a broadly rounded anter and a well-developed sharply defined oral ledge which only decreases very little in height towards the proximal margin. A very much convex operculum has only been found in a small number of zooecia. The triangular flat suboral area is obliquely descending towards the aperture.

The *Heterozooecia* are very rare, and in each of the four fragments examined which have a length of 12–20 mm they have only been found in a number of 1–4. They are more than twice as long as the zooecia and are of a similar form as the heterozooecia figured in pl. II, fig. 19 and belonging to *Mel. angulosa*. The two lateral margins, however, are much more incurved, and the aperture therefore is almost completely hour-glass-shaped, the two dilatations being about of the same form and size and being connected by a very narrow median part, the breadth of which is contained about four times in the breadth of the proximal margin. The suboral area has the same form and size as in the zooecia. Each heterozooecium is bordered by two pairs of zooecia, and those belonging to the distal pair has a similar, but still more oblique position as in those seen in pl. II, fig. 18.

Ooecia have not been found.

No *Kenozooecia*.

The *Closure* takes place in great measure by the aid of a flat or somewhat concave lamina placed at the rule at a much deeper level than the oral ledge.

A *Regeneration* has not been found.

The *Colonies*. I have examined four incomposite cylindrical fragments each provided with 1–2 lateral branches. The zooecia, the apertures of which are generally placed very near to each other, are arranged in distinct transverse series, each containing about 20 zooecia.

Le Mans (*Cenomanian*).

In the specimens examined a number both of the closure-plates and of the opercula have undergone a more or less complete decalcification, and several of these structures are represented only by thin chitinlike membranes, which are left unaltered after a fragment has been dissolved in muriatic acid.

Meliceritites Canui n. sp.

(Pl. III, figs. 20–27.)

The Zooecia which are divided by distinct marginal ridges, are more or less regular hexagonal, twice as long as broad, and the large about half-elliptical aperture which only in its distal half takes up the whole breadth of the zooecium is a little shorter than the concave or mostly saddle-shaped subopercular area which is strongly ascending towards the well-developed peristome. The aperture is provided

with a distinct, but narrow peristomial thickening, and with a well-developed oral ledge the height of which in its distal part may be contained four or five times in the height of the whole aperture. The operculum is convex and provided with very fine radiating striae. No distinct tubercles.

The *Heterozooecia* have about the same form and size as the zooecia being only a little narrower, and the chief difference between them and the zooecia is that they are provided with a very small aperture which is either of a somewhat similar form or more elongate. They are provided with a well-developed oral ledge and with a more or less distinct peristomial thickening, but their height is somewhat variable and may be contained four or five times in the height of the whole heterozooecium. The operculum has a similar structure as that found in the zooecia.

The *Kenozooecia*. A somewhat variable number of kenozooecia of the same form and size as the heterozooecia are mixed with the heterozooecia, and these two forms of zooids take up about the one half of the surface in larger or smaller portions of the fragments examined.

Ooecia have not been found.

The *Closure*. I have seen a few zooecia closed by a concave lamina, and a larger number of wholly or partially closed heterozooecia. The closure starts from the edge of the oral ledge, and proceeds in that way that the last part of the aperture which is closed is a small median opening distally to the proximal margin.

The *Regeneration*. I have seen a few zooecia regenerated in old ones.

The *Colonies* are free, but I have only seen a number of small (long 4 mm) unbranched cylindrical pieces, each of which contains c. 10 longitudinal series of bryozoids.

Fécamp (Middle Senonian).

Meliceritites gracilis Goldfuss.

Ceriopora gracilis Goldfuss, Petref. Germ., vol. 1, p. 35, pl. X, figs. 11a—c.

non *Meliceritites gracilis* Roemer, Verstein. nordd. Kreideg., p. 18, pl. V, fig. 13.

non — — Canu, Bull. Soc. Géol. de France, 3e série, t. XXV. 1897, p. 752, pl. XXII, figs. 1—2.

Meliceritites gracilis Gregory, Cret. Bryoz., vol. 1, p. 324, figs. 38a—b.

(figs. a, b.)

The *Zooecia* which are divided by more or less distinct marginal ridges are as a rule rhombical or hexagonally rhombical, and the triangular aperture which takes up the whole breadth in the distal part of the zooecium and rarely attains the half length of the latter is a little longer than broad and provided with a well-developed oral ledge. There is a well-developed peristomial thickening the poster of which forms a prominent lower lip which is often left as a transverse bridge after the rest of the frontal wall has disappeared. The frontal wall is distinctly concave. An operculum has only been found in a single zooecium,

Heterozooecia and *Ooecia* have not been found.

The *Kenozooecia*. Only a single time two small kenozooecia have been found in an angle between two branches.

The *Closure* which has been found in many zooecia is effected by means of a flat or concave calcareous plate which as a rule starts from the free edge

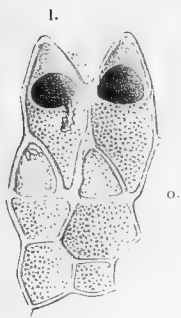


fig. a.

l. oral ledge.

o. operculum.

k. kenozooecium.

of the oral ledge and therefore fuses together with the latter. Sometimes, however, it starts at a deeper level, and in that case the marginal depression is distinctly divided from the closure-plate.

The *Colonies* are freely branched with cylindrical branches, in which the zooecia are arranged in more or less regular, alternating transverse series each of which in the primary or incomposite colony contains 14—16 zooecia. Gradually the colonies get multi-layered, and in one of the original specimens of GOLDFUSS there is found four different layers the outmost of which presents a number of different centra.

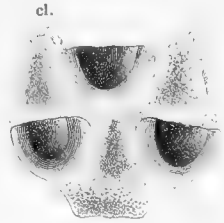


fig. b.

cl. closure-plate.

This species is as a rule badly preserved, and most zooecia have either lost the larger part of their frontal wall or this has been in different degree covered by incrustations. These circumstances together with the frequent closure of the zooecia are the causes why only very few apertures present the characteristic oral ledge the length of which attains more than the third part of the length of the whole aperture.

Of this species I have been able to investigate not only the original specimens of GOLDFUSS kept in the palæontological museum of Bonn (Prof. G. STEINMANN), but also a number of exemplars from the palæontological museum of Münster (Prof. H. WEGNER), which have been found in a conglomerate of gypsum at Essen. Under the name of *Ceriopora gracilis* Goldf. the latter were mixed with a number of species belonging to different families and genera, and among these I found besides another species of *Meliceritites* a species of *Entalophora* to which I must refer the figure which accompanies ROEMER's description of *Mel. gracilis*. While the aperture of the latter species is longer than broad, and the zooecia are as a rule about twice as long as broad the aperture in ROEMER's figure is broader than long and the zooecia only half a time longer than broad. The description, however, corresponds better to the original specimens than to the figure. I provisionally propose the name *Entalophora Roemeri* for this species of which I here give the following description. The hexagonal zooecia which are only half as long as broad are provided with a very concave frontal area and divided by strongly developed marginal ridges. The aperture which takes up the larger part of the breadth in the distal part of the zooecium, and together with the peristome about half the length of the whole zooecium is triangularly rounded, broader than high and pro-

vided with a strongly developed peristomial thickening, the proximal part of which forms an obliquely or even vertically ascending under lip. The fragments examined are elongate clavate, rounded or a little compressed and accreasing gradually in thickness towards the tip which is about double as thick as the proximal end. The fragment to which the zooecia belong, which are figured in Pl. VII (figs. 25—26), has a length of 10 mm and the thickness of the tip is 4 mm.

I think that the two specimens figured by GREGORY both must be referred to *Mel. gracilis* Goldf., but in that case they are figured in the inverted position, and the supposed aperture in GREGORY's figures seems to be identical with the opening seen in the proximal part of my figure b and which is due to a partial destruction of the frontal wall.

Meliceritites gothica nov. nom.

Meliceritites gracilis Marsson (non Goldfuss), Bryoz. Rügen p. 46, pl. IV, fig. 8.
(Pl. I, figs. 11—14).

The *Zooecia* which are divided by distinct, more or less developed marginal ridges, are hexagonally rhombic, and the very concave suboral area is strongly, sometimes almost vertically ascending towards the aperture which does not take up the whole breadth in the distal part of the zooecium. The tubercles are distinct but developed in very different degree. The aperture which is provided with a distinct peristomial thickening and a distinct, but rather narrow oral ledge has the form of a gothic arch, the lateral margins in their distal part running together to form a more or less distinctly angulate terminal portion while in their proximal part they are somewhat converging proximally or parallel. The supraoral tubercle forms a more or less developed beak-shaped projection. The convex operculum is provided with a distinct flabelliform striation, and, besides, with a number of small claviform projections, which are arranged in a similar manner and seem to be perforated.

The *Heterozooecia* are as a rule found in groups of 2—7, rarely singly, and they larger groups are generally mixed with a number of kenozooecia. They are as long as or somewhat longer than the zooecia and provided with a narrow, more or less projecting aperture of very different length the lateral margins of which are almost parallel or very little converging distally. The fissure-like opening which has a proximal triangular enlargement is bordered by two obliquely descending lateral thickenings.

Ooecia have not been found.

The *Kenozooecia* which are of the same form and size as the zooecia are found singly or in groups and as a rule together with heterozooecia.

The *Closure* is effected by means of a flat or concave lamina (fig. 14).

The *Regeneration*. There has been found both a regeneration of new zooecia in old ones, of new heterozooecia in old zooecia and rarely of new heterozooecia in old ones.

The *Colonies* are incomposite, and the cylindrical fragments examined contain c. 10 zooecia in each transverse series.

I have examined a rather scarce number of fragments from Rügen and from Möen, and while in the specimen from Rügen the aperture is only as long as the suboral area in those from Möen it is more than half a time longer.

Melicerititis gothica, var. *acuminata* n.

(Pl. II, figs. 23—24.)

The Zooecia are hexagonal, and the aperture which is not far from being double as long as broad has the form of an elongate triangle the lateral margins of which are somewhat convex in their proximal part and a little incurved in their distal half. The distal part of the oral ledge is strongly developed, and may sometimes be almost half as long as the whole aperture. All other features as in the main form.

Very common in the chalk (upper Senonian) from Tullstorp (Sweden) and from Svinklöven (Jutland).

Meliceritites pentagonum n. sp.

(Pl. IV, fig. 22.)

The *Zooecia* which are divided from each other by well-developed marginal ridges are more or less distinctly hexagonal, and in most cases about half a time as long as broad. As a rule distinct tubercles. The large aperture which does not take up the whole breadth of the zooecium in its distal part is more or less distinctly pentagonal, the two distal sides of the pentagone being sometimes represented by a curve while the two proximal paired sides are always distinctly converging proximally. It is provided with a well-developed oral ledge the two halves of which form with each other an acute or sometimes right angle. The oral ledge which is very narrow within the two paired proximal sides does not as in other species attain its largest height in the middle line, but in the two lateral halves. There is a well-developed peristomial thickening, the distal strongly projecting part of which belongs to the marginal ridges, and this part together with the supra-oral tubercle often forms a more or less prominent beak-like projection. All the hitherto examined specimens have lost their opercula.

The *Heterozooecia*, which are spread among the zooecia in groups of 2—9 are of somewhat variable size, the smallest of them, however, not being much smaller than the zooecia. They are provided with a long narrow proximally gradually widening aperture, the narrow opening of which is bordered by two lateral processes. The aperture is of very different length and in the largest of them it is longer than the concave suboral area and takes up the whole breadth of the heterozooecium in its distal third part.

Ooecia have not been found.

Kenozooecia of the same form and size as the zooecia are often found together with the heterozooecia in a number of 1—3.

The *Closure* takes place by means of a concave lamina.

The *Regeneration*. Besides a regeneration of new zooecia in old ones there is found very commonly a regeneration of heterozooecia in zooecia.

The *Colonies* are incomposite, and the examined cylindrical fragments contain 10—12 zooecia in each transverse series.

Numerous specimens from the chalk (upper Senonian) of Tullstorp (Sweden).

Meliceritites Roemeri v. Hagenow.

Ceripora Roemeri v. Hagenow, Monogr. d. Rügensch. Kreide-Verst. (N. Jahrb. f. Mineral. 1839, p. 285, pl. V, figs. 7a—b.

Meliceritites (Ceripora) Roemeri Roemer, Verst. nordd. Kreidegeb. 1841, p. 18.

Vaginopora Roemeri v. Hagenow, Geinitz Grundr. d. Versteinerungskunde, 1846, p. 602, pl. XXIII b, fig. 20.
non *Ceripora velata* v. Hagenow, Monog. Rüg. Kreide-Verst. (N. Jahrb. f. Mineral.) p. 285, pl. V, fig. 6.

Nodelea propinqua Marsson, Bryozoen Rügen, p. 47, pl. V, fig. 1.

non *Meliceritites gracilis* Marsson, Bryozoen Rügen, p. 46, pl. IV, fig. 8.
(pl. V, figs. 18—24.)

The *Zooecia* which are in most cases only half as long as broad are hexagonal and divided from each other by a meshwork of distinct ridges in the knots of which there is found more or less developed often prismatic tubercles. The half-elliptical aperture which may be a little longer than broad and does not take up the whole breadth of the zooecium in its distal part is as a rule much shorter than the concave suboral area. It is provided with a well-developed peristomial thickening, and in especially well preserved zooecia there is seen a distinct, but rather narrow oral ledge. All the specimens examined have lost their opercula.

The *Heterozooecia*. There is found two different forms of heterozooecia the larger of which is avicularia-like and provided with a long narrow aperture. They are present in very scarce number, and in most of the fragments examined there has been found only a single one, rarely two or three. They are always larger than the zooecia, and the longest of them have a length a little more than twice that of the zooecia. The aperture which is longer than the suboral area and three or four times as long as the breadth in their middle part is provided with a median depression bordered by two narrow lateral thickenings. The other form of heterozooecia is found in large numbers among the zooecia, rarely singly, but as a rule in groups of 2—10, in most cases mixed with a number of kenozooecia. They have the same form and size as the zooecia from which they differ in the form of the apertures which, however, is subject to great variation. The same is the case also with their size as their length may be contained five to two and a half times in the length of the zooecium. In opposition to the zooecial apertures they have a narrow, feebly developed, peristomial thickening and a broadly rounded

distal margin, but while the smaller of them are nearly circular apart from the straight proximal margin the larger of them are more or less ligulate with almost parallel or proximally somewhat converging lateral margins. While those heterozooecia that are provided with the smaller apertures like the zooecia have an angularly bent distal end, the distal end of the others corresponds to a smaller or larger distal part of the ligulate aperture. The larger part of the aperture is filled by a concave lamina which no doubt corresponds to the oral ledge in the zooecial aperture, but in most cases this lamina has been expanded by a more or less advanced closure, and there is only left a fissure-like or narrow triangular opening. Sometimes the whole aperture is filled, and only in a few cases there has been found a half-elliptical or semi-circular opening (fig. 19).

The *Kenozooecia* which have the same form and size as the zooecia are as a rule found together with a number of heterozooecia, but they are not so frequent as the latter.

The *Closure* is effected by means of a concave lamina but it is much more common in the heterozooecia than in the zooecia.

The *Regeneration*. A regeneration of new zooecia in old ones is not frequent, but in most fragments there is found a number of old zooecia the aperture of which have been filled by heterozooecia with a small aperture (fig. 19).

The *Colonies* are uni-layered, and the cylindrical branches bear alternate transverse rows of c. 15 zooecia.

I have found numerous specimens in the chalk from Tullstorp (upper Senonian) Sweden.

When GREGORY refers. v. HAGENOW's and ROEMER's descriptions of *Mel. Roemeri* to *Mel. gracilis* Marss. he relies upon the authority of MARSSON, who, however, is wrong in his supposition. In HAGENOW's figure of the former species the length of the aperture is contained about thrice in the length of the zooecium, and that is just the case in most exemplars of *Mel. propinqua*, while in *Mel. gracilis* Marss. the aperture is as long as or even longer than the suboral area. But also the very short and incomplete descriptions agree with *Mel. propinqua* and not with *Mel. gracilis*. v. HAGENOW designates the apertures as „Kreisrunden oder abgerundet dreieckigen . . . Poren“, but these terms cannot in any way be used about the elongate distally somewhat pointed apertures of *Mel. gracilis*, and when both authors speak about zooecia in which the aperture is only represented by a fine pore, they evidently speak about the heterozooecia. Such a heterozooecium is seen in HAGENOW's figure.

Meliceritites squamata Marsson.

Marsson, Bryozoen Rügen p. 47, pl. IV, fig. 9.
(pl. V, figs. 13 -17.)

The *Zooecia* which at the utmost may be half a time longer than broad are in most cases as broad as long and even a little broader. They are hexagonal and

divided by distinct ridges, of which the two distal ones may often be more or less curved. The tubercles are either lacking or very feebly developed. The more or less strongly protruding, half-elliptical aperture which at the utmost takes up the half breadth in the distal part of the zooecium is a little longer than broad and about as long as the concave or saddle-shaped suboral area which rises obliquely towards the aperture. In well preserved zooecia there may be found a narrow oral ledge.

The operculum is radially striated, and its pores seem to be perforations of small rounded elevations.

The *Heterozooecia* which are present in rather scarce numbers are of very different size, the smaller of them being only as long as the zooecia while the larger may attain more than the double length of the latter. While the former have an elongate triangularly rounded aperture, the length of which is contained 3—2 times in the length of the whole heterozooecium the latter have an elongate ligulate aperture, which is longer than the suboral area. Distally to the triangular opening is seen a longitudinal depression, bordered by two marginal thickenings.

Ooecia have not been found.

Kenozooecia of the same form and size as the zooecia are found in most fragments, generally in groups of 2—10.

The *Closure* takes place by means of a concave lamina.

The *Regeneration*. There is not rarely found a regeneration of new zooecia in old ones, and when the new aperture is not sharply divided from the old, its presence may be inferred from that it is much smaller and much more protruding than the other. Another form of regeneration which is perhaps more frequent is that the aperture of an old zooecium is filled by a small heterozooecium, the roundedly triangular aperture of which is as a rule obliquely protruding, and I think that MARSSON statement: „Die Zellen besitzen an der Spitze oft auch ein kleines, zuweilen von einer Pore durchbortes Wärzchen“ must be referred to this form of regeneration. In a few cases I have seen a zooecium regenerated in a large heterozooecium (fig. 14) and a new large heterozooecium in an old one (fig. 15).

The *Colonies* are incomposite, and the cylindrical branches have c. 10 zooecia in each transverse row.

Numerous specimens have been found in the chalk (upper Senonian) from Møen (Denmark) and from Tullstorp (Sweden).

Meliceritites Filiozati n. sp.

Clausimultelea tuberculata d'Orbigny, Bryoz. cré. p. 656, pl. 784, figs. 12—15.
(Pl. VI, figs. 7—10.)

The *Zooecia* which are in most cases twice as long as broad and more or less regularly rhombic, are divided by well-developed marginal ridges, and the suboral area is as a rule a little convex. No tubercles. The half-elliptical aperture, the

length of which is very often contained twice in the length of the suboral area is provided with a distinct but rather narrow oral ledge, and together with the narrow peristomial thickening very often takes up the whole breadth in the distal part of the zooecium. The convex operculum which has been lost in most zooecia presents a distinct flabelliform striation.

The *Heterozooecia* which are spread singly among the zooecia in rather scarce number are as long as or even longer than the zooecia. The aperture which in most cases is a little longer than the convex suboral area, is about half as long as broad, and the two almost parallel lateral margin run together distally in a very broad curve. We may discern between two obliquely ascending lateral thickenings and a deep median somewhat hour-glass-shaped depression, in the proximal half of which is seen the half-elliptical inner aperture.

Ooecia have not been found.

Kenozooecia of a similar form and size as the zooecia are scattered among the latter in rather scarce numbers singly or more together.

The *Closure* takes place by means of a flat lamina which arises proximally to the oral ledge.

A *Regeneration* has not been seen.

The *Colonies*. I have examined a number of laminate fragments consisting of 3—4 layers of zooecia, and to judge from these the colony seems to have had a somewhat similar structure as that figured in d'ORBIGNY's pl. 784, fig. 12 and referred to *Clausimultelea tuberculata*. But while in this figure the sub-colonies which compose the different layers partly appear as sharply defined small discs, I have found no such in the few fragments of the present species, which present ancestrulae, but the small sub-colonies, the ancestrulae of which are sometimes placed very near to each other in a number of 2—7, are fused together, partly by the aid of Kenozooecia of very different form and size. The only visible part of such an ancestrula is the obliquely ascending aperture, and as the zooecia immediately surrounding it have an ascending frontal wall, each ancestrular aperture is the centre of a more or less deep depression, which may be elongate when two ancestrular apertures are placed very near to each other (fig. 7). In a single fragment I have found a short, cylindrical erect zooecium (fig. 8) arising between four zooecia and a kenozooecium, and I must regard it as an ancestrula destined to give rise to a new sub-colony.

A number of fragments from Fécamp (Middle Senonian).

Meliceritites tuberculata d'Orbigny.

? *Clausimultelea tuberculata* d'Orbigny, Bryoz. Crét. p. 656, pl. 784, figs. 12—15.
(pl. VI, figs. 1—3.)

The *Zooecia* which are divided by distinct ridges and generally present a convex or saddle-shaped distally obliquely ascending suboral area are of a rather

variable form and size, but in most cases they are about twice as long as broad and sometimes much longer. No tubercles. The aperture which is provided with a distinct but rather narrow oral ledge and a narrow peristomial thickening presents some variation in form and dimensions, but is generally half as long as broad, and the two lateral margins are somewhat converging proximally. The convex operculum shows a distinct flabelliform striation. —

The *Heterozooecia* which are spread among the zooecia singly or more together are about of the same length as the latter, and the beak-shaped more or less projecting distal end is as a rule much shorter than the concave suboral area. The narrow triangular obliquely ascending aperture is provided with an almost fissure-like opening bordered by two inwardly sloping lateral thickenings. In a single case I have found a calcareous mandible (fig. 2).

Ooecia have not been found.

The *Kenozooecia*, which have a similar form and size as the zooecia are in large numbers spread among the zooecia.

The *Closure* takes place by means of a concave lamina.

The *Regeneration*. In the fragments examined I have only seen the regeneration of a new heterozooecium from an old one (fig. 1).

The *Colonies*. I have examined a few laminate fragments consisting of a number of layers, and in one of them the zooecia are as in the above fragments of *Mel. Filiozati* arranged in groups around a number of ancestrulae.

The basin of Paris (Danian), Fécamp (Middle Senonian).

When I refer the present species and not *Mel. Filiozati* to *Clausimultelea tuberculata* d'Orb. it is because it agrees better with the figure given by d'ORBIGNY, not only in the form of the apertures but also in the large number of the kenozooecia. For the rest d'ORBIGNY's description as also his figure bear witness to a very superficial examination as the zooecia according to this author, quite in opposition to what is really the case, are only represented by their apertures. Also the relation between the apertures and the kenozooecia in the figure leaves no doubt, that the latter is constructed and not made according to nature.

Meliceritites pyrenaica d'Orbigny.

Foricula pyrenaica d'Orbigny, Bryoz. Crét. p. 658, pl. 741, figs. 16—18.

(Pl. VI, figs. 11—21.)

The *Zooecia* which are never divided by marginal ridges are subject to a very great variation both in respect to the form and size of the aperture and to the form and relative extension of the suboral area which never presents distinct pores, but always a small number of more or less developed pits. The aperture which in the best preserved zooecia shows a distinct or even well-developed oral ledge is in most cases semi-elliptical or makes a larger part of an ellipse, but it may also be semi-circular, quadrangularly rounded, and some times much broader than high (fig. 11).

The distal part which is not rarely provided with a more or less distinct beak-shaped projection (figs. 13, 14, 16) is sometimes angularly bent from side to side (figs. 13, 14), while in most cases it is broadly rounded, and within the proximal margin there is generally found a more or less distinct broad low projection, the median part of which is provided with an impression or indentation (figs. 12, 13, 17, 21). The operculum which has been found only in a very small number of zooecia shows a flabelliform striation and a number of pores. The suboral area which is always feebly developed is rarely broad and short, and in that case it is provided with a pit on each side (fig. 19). In most cases it is long and narrow and provided with 2—4 pits arranged in different manner, and sometimes the peristomes of the neighbouring zooecia come in contact with each other in such a manner that the suboral area is only represented by a proximal and a distal pit (fig. 12). The rich deposition of calcareous matter which takes place in this species may go on in a very different manner, and while in some colonies the pits increase both in number and in size, and the suboral area therefore shows a rich areolation (fig. 16), in others the pits are gradually effaced, and the suboral areas are transformed into an inter-lacing net-work of convex pillars (fig. 15).

The *Heterozooecia* which are present in very scarce number and not even found in all the fragments examined are as a rule a little longer than the zooecia. The more or less projecting aperture is triangularly rounded, a little longer than broad and the two lateral margins converge to form a rather broadly rounded distal curve. No. lateral thickenings. The suboral area is provided with similar pits as those found in the zooecia, and there may be found a projection within the proximal margin (fig. 21).

Ooecia have not been found.

No *Kenozooecia*.

A *Closure* has not been seen with certainty in any of the fragments examined.

The *Regeneration*. In all the specimens examined a larger or smaller number of the apertures are much more projecting than the others, and when we have to do with a larger fragment which presents the original distal end tolerably intact we find that the number of these projecting apertures increases towards the proximal end, the surface of which is chiefly or exclusively composed of them. A consequence hereof is that the proximal end of such a fragment is much thicker than the distal, and for inst. in one from Villedieu which has a length of 13 mm., the distal end has a thickness of 1,5 mm. but the proximal end of 2,5 mm. The named prominent apertures are very often distinctly or even much larger than the common ones and of a different form (fig. 17), and their arrangement is always more or less irregular as a larger or smaller number of them have another direction than the common apertures. Sometimes we even find specimens in which these apertures are placed in all possible directions (fig. 20). I cannot doubt but that we have here to do with a process of regeneration which differs from that commonly found in this division therein that the aperture of the new zooecium proceeds so far be-

yond that of the old one, and at the same time the rich deposition of calcareous matter speedily effaces the limits between the two apertures and prevents us from deciding how many times a certain zooecium has been regenerated. In a specimen from Tours (fig. 11) the apertures are directed obliquely downwards, and some of the zooecia show a strongly developed dorsal surface provided with a number of deep pits.

Numerous specimens from Villedieu and Tours.

Meliceritites dichotoma (?) d'Orbigny.

Semielea dichotoma d'Orbigny, Bryoz. Crét. p. 637, pl. 638, figs. 6—8, pl. 738, figs. 10—11.
(Pl. V, figs. 1, 2.)

The *Zooecia* which are twice as long as broad are not divided by distinct marginal ridges, and the distally ascending suboral area is in most cases longer than the aperture which together with the well-developed peristomial thickening takes up the whole breadth of the zooecium in its distal part. No tubercles. The aperture which is as broad as long or even broader than long is provided with a well-developed oral ledge, and the two lateral margins either run together in a broad distal curve or in a curve more or less angularly bent. The convex operculum shows a distinct flabelliform striation. —

The *Heterozooecia* which are spread among the zooecia, rarely singly, mostly in groups of 2—8 have the same length as the zooecia, and the somewhat projecting distal half is provided with a triangular aperture, about twice as long as broad. All the heterozooecia examined were closed by a somewhat concave lamina.

Ooecia have not been found.

No *Kenozooecia*.

The *Closure* takes place by means of a flat or concave lamina which in the zooecia sometimes has started from the edge of the rim, sometimes at a deeper level. —

The *Regeneration*. I have seen many cases of regeneration both of new zooecia in old ones, of new heterozooecia in old ones and of heterozooecia in old zooecia. I have examined a hollow compressed fragment from Tours lent me by Dr. Pergens.

Meliceritites armata n. sp.

Hornera Steenstrupi Pergens (partim), Bryoz. de Faxe, p. 218, pl. XIII 2 a, 2 b.
(Pl. III, figs. 14—16.)

The *Zooecia* (long 0.8 mm.) which are divided by well-developed marginal ridges are about twice as long as broad, and the concave or saddle-shaped suboral area is obliquely ascending towards the somewhat projecting distal end. No tubercles. The aperture, the length of which in most zooecia is contained about three times in the length of the whole zooecium is about as broad as high and provided

with a strongly developed peristomial thickening. There is found a distinct, but narrow oral ledge, and the lateral margins as a rule converge a little proximally. The convex operculum is provided with a flabelliform striation.

The *Heterozooecia*, which in most fragments are found in a number of 2—10 either singly or two together are as a rule not spread among the zooecia, but together with the kenozooecia form a longitudinal belt which takes up about the third part of most fragments examined. They have a similar form and size as the zooecia, but are provided with a triangular distally rounded aperture, not twice as long as broad, the length of which is contained about three times in the length of the whole heterozooecium. It is provided with two lateral thickenings.

The *Kenozooecia* which have the same form and size as the zooecia together with the heterozooecia form transverse series, each containing 3 (2—4) zooids.

Ooecia have not been found.

The *Closure* is effected by means of a concave lamina.

The *Regeneration*. I have seen a regeneration of new zooecia in old ones, and a corresponding regeneration of heterozooecia.

The *Colonies* (Diam. 0,8—1 mm.) are incomposite, and the cylindrical fragments have their zooids arranged in regular transverse series, which, however, in most cases are more or less oblique. The zooecia which as a rule are bordered by parallel lateral margins in the larger part of their length, generally are found in a number of six in each transverse row, but in rare cases the extension of the longitudinal belt formed by the kenozooecia and heterozooecia may be diminished by the increase of the number of zooecia in one or more transverse series, and sometimes it may even be divided in more parts, one or more transverse series forming closed rings round the branch. —

Numerous specimens from Faxoe and Rejstrup (Danian), Denmark.

Meliceritites Steenstrupi Pergens.

Hornera Steenstrupi Pergens (partim), Bryoz. de Faxe. p. 218, pl. XIII, figs. 3, 4.
(Pl. III, figs. 12, 13.)

The *Zooecia* (long. 0,5 mm.) which are divided by well-developed marginal ridges are in most cases more than twice as long as broad, and the suboral area is obliquely ascending towards the aperture which together with the well-developed peristomial thickening takes up almost the whole breadth in the distal part of the zooecium. No tubercles. The half-elliptical aperture lacks a distinct oral ledge, and its length is contained about two and a half times in that of the zooecium. The not very convex operculum shows a flabelliform striation.

The *Heterozooecia* which are much smaller than the zooecia and generally of an elongate sexangular form are never spread among the latter but together with a few kenozooecia form a longitudinal belt, which takes up a fourth to a sixth part of the extension of the fragments examined, and in which the zooids are arran-

ged in more or less oblique transverse series each containing 1—4 zooids. As in most of the latter the marginal ridges between the single zooids have vanished as also the apertures of the heterozooecia it is not easy to make up the relative number of the two forms of zooids, but the heterozooecia are at any rate much more numerous than the kenozooecia, and sometimes take up the whole belt especially when the latter is narrow. The very small aperture is almost circular — apart from the straight proximal corner —, and the larger part of it is filled by a concave lamina, the proximal part of which is perforated by a semicircular opening.

Ooecia have not been found.

The *Kenozooecia* which have the same form and size as the heterozooecia are spread among the latter in scarce number and seem as a rule to be placed in the middle of the longitudinal belt.

The *Closure* is effected by means of a concave lamina.

The *Regeneration*. Hitherto I have only seen a regeneration of new zooecia in old ones.

The *Colonies* (Diam. 0,4—0,8) are incomposite, and the cylindrical fragments show an arrangement of the zooecia in regular more or less oblique transverse series, each of which contains 6—11 zooecia. As a rule the apertures of the outermost zooecia in each transverse series are distinctly larger than the other.

Numerous specimens from: Faxoe and Rejstrup (Danian), Denmark. Of the two very distinct species which have been confounded under the name *Hornera Steenstrupi* the present is found in Faxoe in much larger numbers than the other (*M. armata*), and, therefore, I think it likely that it should keep the specific name given by PERGENS. According to the explanation of the plate this author refers the fragment figured to an old colony, no doubt because the marginal ridges between the heterozooecia and kenozooecia have been indistinct as is the case in most specimens of this species from Faxoe, while he has regarded the specimens of *Mel. armata* as young colonies of the same species because the corresponding areas are very distinct. PERGENS figure 4, however, shows sufficiently distinct the small narrow «dorsal» areas, and the smaller dimensions of the fragment figured is evident from the fact, that the figure, which has the same size as the figure 3 is pictured under a larger magnifying power. —

Meliceritites sarissata Gregory.

Reptomultelea sarissata Gregory, Cretac. Bryozoa, p. 322, pl. XVI, fig. 7.
(Pl. II, figs. 1—3.)

The *Zooecia* which are divided by very narrow more or less distinct marginal ridges, are generally rhombic, about twice as long as broad, and the convex or saddle-shaped suboral area, which is about of the length of the aperture, is strongly, almost vertically ascending towards the latter. No interoral tubercles. The aperture which together with the strongly developed peristomial thickening takes up the

whole breadth in the distal part of the zooecium is about a third part longer than broad, and provided with a distinct and sharply defined but narrow oral ledge. It is roundedly triangular with a pointed distal part, and the arched lateral margins which are somewhat incurved distally are more or less converging proximally. The præoral tubercle either forms a large rounded swelling or a more or less developed beak-shaped projection. The convex or saddle-shaped distally pointed operculum shows a flabelliform striation.

The *Heterozooecia* which are spread among the zooecia in rather scarce numbers are much longer than the latter. The aperture consists of a broad proximal part with distally converging lateral margins and a generally much longer (sometimes more than twice as long) narrow distal part with allmost parallel or very little converging lateral margins. The mandible is distinctly convex not only from side to side, but also proximally distally.

Ooecia have not been found.

The *Kenozooecia* have been found spread among the zooecia in scarce numbers singly or more together.

The *Closure*. A distinct closure has not been found.

The *Regeneration*. There has been found no regeneration in the fragment examined.

The *Colonies* are composite, and the fragment examined is a multilayered lamina fixed to a piece of flint from Chatham or Luton (middle chalk).

This species is evidently related to *Mel. sarthacensis*.

Meliceritites Sarthacensis d'Orbigny.

Reptelea Sarthacensis d'Orbigny, Bryoz. Crét. p. 640, pl. 604, figs. 9—10, pl. 738, fig. 15.

(Pl. I, figs. 1—2.)

The *Zooecia*, which are divided by distinct marginal ridges are more or less regularly rhombic, as a rule double as long as broad, and the suboral area which is often somewhat convex is as long as or a little shorter than the aperture. No lateral tubercles. The triangularly rounded aperture which is a little longer than broad has a narrow obliquely immersed oral ledge, and together with the well-developed peristomial thickening it takes up the whole breadth in the distal part of the zooecium. Its distal end is strongly projecting in the shape of a robust rounded beak (the præoral tubercle). The convex operculum shows a faint striation, and as a rule most of its pores are arranged in two distally converging longitudinal belts.

The *Heterozooecia* which are in most cases a little longer than the zooecia are spread among the latter in rather large numbers, and are provided with a triangular aperture which may sometimes be twice as long as broad, sometimes only a little longer. It has a broad proximal margin, and the two somewhat incurved lateral margin are strongly converging towards the narrow roundedly pointed distal

end. No lateral thickenings. In a large number of the heterozooecia I have found a calcareous mandible which is strongly arched not only from side to side, but also proximally distally.

Ooecia have not been found.

Kenozooecia of the same form and size as the zooecia have been found spread among the zooecia in very scarce numbers.

A *Closure* effected by means of a concave lamina has been found in a few zooecia.

A *Regeneration* has not been seen.

The *Colonies* are composite, and the only fragment examined is a hollow three-layered expansion from le Mans (Cenomanian).

Meliceritites micropora d'Orbigny.

Meliceritites micropora d'Orbigny, Bryoz. Crét. p. 624, pl. 737, figs. 4 -7.

— — Pergens, Revision d. Bryoz. p. 397.

Nodelea micropora Gregory, Cret. Bryoz. p. 313.

(Pl. III, fig. 10.)

The *Zooecia* (long 0,3—0,5) which are of very variable size are hexagonal-ovate, about twice as long as broad, convex and divided by distinct furrows in the bottom of which there may be found very narrow and indistinct marginal ridges. The half-elliptical aperture, the length of which is often contained about three times in the length of the zooecium, is not provided with a distinct peristomial thickening nor with a distinct oral ledge. The convex operculum shows a distinct flabelliform striation.

Heterozooecia have not been found.

The *Ooecia*. A single elongate ooecium has been found.

The *Kenozooecia*. A few of these zooids have been found among the zooecia.

A *Closure* of the primary zooecia has not been found.

The *Regeneration* takes place in great measure, but never in such a manner that the old aperture is filled by the new one. On the contrary the proximal half of the former is taken up by a concave lamina, the suboral area of the new zooecium, the distal half of which is strongly arched and provided with a small semicircular aperture, and the latter is at last closed by a concave lamina. The continued deposition of calcareous matter gradually effaces the limits between the old aperture and the suboral area of the new zooecium, and at a certain point of time only the rest of the small aperture and a more or less distinct depression proximally to the latter indicate that a regeneration has taken place.

The *Colonies* are incomposite, and the two fragments examined are cylindrical with 20—25 zooecia in each of the irregular transverse series.

Villedieu.

Meliceritites hexagona d'Orbigny.

Elea hexagona d'Orbigny, Bryoz. Crét. p. 633, pl. 738, figs. 1 4.
(Pl. V, figs. 3—5.)

The *Zooecia*, which are divided by well-developed marginal ridges, are rhombic or hexagonally rhombic, rarely twice as long as broad, and the half-elliptical or triangularly rounded aperture, which is as a rule longer than the suboral area together with the well-developed peristomial thickening takes up the whole breadth in the distal part of the zooecium. While the inter-oral tubercles may be developed in very different degree and may often be quite absent, the supra-oral one is large rounded and forms a robust beak-shaped projection. No distinct oral ledge. The convex operculum presents a distinct flabelliform striation.

The *Heterozooecia*, which are spread singly among the zooecia in rather scarce numbers are sometimes only a little longer than the latter, sometimes about twice as long. In the larger of them we may discern in the aperture between a broad proximal part, the lateral margins of which are converging distally, and a narrow distal part of different length with about parallel margins. In the shorter of them, however, the somewhat incurved lateral margins are gradually converging distally in their whole length. The mandible is arched both from side to side and proximally distally.

Ooecia have not been found.

No *Kenozooecia*.

A *Closure* effected by means of a flat or concave lamina has only been distinctly seen in a small number of zooecia.

The *Regeneration* takes place in great measure, and in old colonies many apertures are very much projecting because the zooecia have been regenerated several times. There has also been found a regeneration of new heterozooecia in old ones.

The *Colonies*. I have examined a number of thick ribbon-shaped two-layered fragments from Vendôme (zone with *Crania ignabergensis*).

Meliceritites plana d'Orbigny.

Semielea plana d'Orbigny, Bryoz. Crét. p. 638, pl. 738, figs. 12 14.
(Pl. V, fig. 11.)

The *Zooecia*, which are more or less regularly rhombic and sometimes more than twice as long as broad, are divided by distinct ridges, and the suboral area which is as a rule more or less convex is much longer; sometimes about twice as long as the aperture. No distinct tubercles. The half-elliptical or triangularly rounded aperture which is a little longer than broad together with the well-developed peristomial thickening takes up the whole or almost the whole breadth in the distal part of the zooecium. No distinct oral ledge. The convex operculum shows a flabelliform striation.

The *Heterozooecia*, which are spread among the zooecia singly or in groups, have the same length as these, but the obliquely ascending distal end is provided with a small elongate triangular aperture the length of which is contained four to five times in the length of the whole heterozooecium. The fissure-like opening is bordered by two inwards obliquely descending thickenings.

Ooecia have not been found.

Kenozooecia have not been found in the two small fragments examined.

The *Closure* starts by the formation of a number of processes which rise from the margin and grow together in a more or less irregular manner, thus forming at a certain point of time a calcareous cover perforated by 3—5 hollows which later get closed.

The *Regeneration*. I have seen a few cases of regeneration both of zooecia and of heterozooecia.

I have examined two small laminar fragments one of which has only the locality France while the other which I have bought from Mr. W. GAMBLE is from St. Antoine du Rocher. Under the name of *Semielea plana* Mons. FILIOZAT has sent me three small fragments of another species which with the same right as the present might be referred to *Semielea plana* d'Orb. The zooecium and the aperture have the same form and structure, but the closure is effected by means of a concave lamina, and in one of them I have found a heterozooecium a little longer than the zooecia and the aperture of which has about the same form as in that figured in pl. II, fig. 9.

Meliceritites cenomana d'Orbigny.

Nodelea cenomana d'Orbigny, Bryoz. Crét. p. 761, figs. 11—13.
(Pl. III, figs. 17, 18.)

The *Zooecia*, which are not twice as long as broad, are rhombic or hexagonally rhombic and divided from each other by the well-developed peristomial thickenings together with the tubercles which are as a rule well-developed and sometimes very large. The half-elliptical or triangularly rounded aperture, which together with the peristome takes up the whole breadth of the zooecium in its distal part, is half a time longer than broad, much longer than the suboral area, and the supra-zooecial tubercle forms a more or less developed, sometimes very robust beak-shaped projection. No distinct oral ledge. The convex operculum is provided with a flabelliform striation.

The *Heterozooecia* which seem to be very rare are much larger than the zooecia and provided with a very large quadrangularly rounded aperture which is about twice as long as the suboral area. No lateral thickenings.

Ooecia have not been found.

No *Kenozooecia*.

The *Closure* takes place by means of a concave or flat lamina placed some way within the aperture.

The *Regeneration*. I have found a few zooecia regenerated.

I have examined two small well conserved fragments from le Mans (Cenomanian), lent me by Dr. PERGENS, and in these a number of the closure-plates and of the opercula have undergone a similar more or less complete decalcification as that found in *Mel. Lorieri*.

Meliceritites lamellosa d'Orbigny.

Elea lamellosa d'Orbigny, Bryoz. Crét. p. 632, pl. 625, figs. 11—15.

— Pergens, Revision p. 398.

non *Cea lamellosa* d'Orbigny, Bryoz. Crét. p. 1007, pl. 787, figs. 11—13.

Elea lamellosa Gregory, Cretac. Bryoz. p. 299.

(Pl. III, figs. 1—9.)

The *Zooecia* which are as a rule rhombic or hexagonally rhombic and mostly twice as long as broad may be concave, flat or even a little convex and are divided from each other by more or less distinct ridges, the development of which may vary greatly even in the same colony. A more or less prominent tubercle may be developed not only at the distal end of the zooecium but also at the proximal corners of the aperture, and when the latter are placed in contiguous transverse series the tubercles of two adjacent apertures when sufficiently near to each other often fuse together into a single one. The development of the tubercles, however, is subject to great variation even in the same colony. The aperture the length of which is in most cases contained at least twice in the length of the whole zooecium takes up the whole breadth of the zooecium in its distal part and lacks an oral ledge, but is provided with a strongly developed peristomial thickening. It is half-elliptical or triangularly rounded and always longer than broad, but the relation between the length and the breadth is subject to rather great variation, and sometimes it is almost half a time as long as broad. The operculum shows a distinct flabelliform striation, and the greater part of its surface is more or less distinctly flattened.

The *Heterozooecia*, which are scattered over the colony in rather large numbers, partly singly, partly in groups up to four are in most cases longer and narrower than the zooecia, and their distal end is more or less obliquely ascending. It is provided with an elongate, roundedly triangular, distally protruding aperture, the lateral parts of which are covered by two very narrow, elongate triangular lateral processes between which is seen an opening in the shape of an inverted T.

Ooecia have been found.

Kenozooecia have not been found.

The *Closure*. I have not found a distinct case of closure in any zooecium, but in old heterozooecia.

The *Regeneration* takes place both in the zooecia and the heterozooecia, and in the former in a double manner, as an old zooecium may be replaced either by a new zooecium or by a heterozooecium, and in the first case there is seen a new oral margin within the old one. If an old zooecium is replaced by a heterozooecium (figs. 1, 3, 4, 5) the free, distal part of the latter gets another form than the corresponding part of the common heterozooecia, especially when it takes up the whole of the zooecial aperture, as in that case it must of course be much broader. The free part of such a heterozooecium is very much protruding, and one may discern between a posterior strongly convex, from side to side somewhat compressed, sometimes a little saddle-shaped surface and the frontal surface which as a rule forms a right angle with the frontal area of the old zooecium. We may compare this free part with a half somewhat compressed cone which rests on the surface of the cut and the basal surface of which wears the aperture. — Sometimes the named part does not take up the whole aperture (fig. 1) and in that case the difference of form is not so great. When a new heterozooecium is formed in an old one its free distal part, which is often vertically ascending partially covers the corresponding part of the old heterozooecium, the tip of which is seen protruding distally to it (figs. 5, 6, 7).

The *Colonies* have the form of free two- or more-layered laminae. Of this species I have examined or large number of fragments from Villedieu (Coniacian).

Meliceritites undata d'Orbigny.

Meliceritites undata d'Orbigny, Bryoz. Crét. p. 625, pl. 737, figs. 11—14.
non *Meliceritites undata* Gregory, Cret. Bryoz. p. 340, pl. XV, figs. 2—4, pl. XVI, fig. 3.
(Pl. IV, figs. 9—17.)

The *Zooecia*, which are divided by distinct more or less prominent ridges, are about twice as long as broad, and the suboral area generally presents a saddle-shaped concavity in the direction proximally distally. More or less developed tubercles. The half-elliptical aperture which is a little longer than broad lacks a distinct oral ledge, but is provided with a well-developed peristomial thickening, the lateral parts of which generally widen proximally, and these widened lateral parts belonging to two adjacent zooecia often coalesce with the interoral tubercle into a large rounded projection (fig. 17). The convex operculum is provided with a flabelliform striation, and with very few exceptions with 1—4 short arched, more or less distinct impressions turning the concavity proximally. When only a single impression is present it is seated in the proximal part.

The *Heterozooecia* which are scattered among the zooecia in rather small number, but not rarely two or more together, are of the same size as the latter, and their obliquely ascending more or less projecting distal end is provided with an aperture of somewhat different form and size, the length of which may be contained 3—6 times in the length of the whole heterozooecium. It is generally narrow

(figs. 12, 13) but sometimes of a semi-elliptical form (fig. 12). In the best preserved specimens I have found two narrow lateral processes.

Ooecia have not been found.

No *Kenozoecia*.

A *Closure* effected by means of a concave lamina has only been found in a few cases (fig. 15).

The *Regeneration* takes place in great measure, and in many of the fragments examined the larger part of the zooecia have been regenerated either by new zooecia (figs. 11, 16) or by heterozoecia (figs. 10, 14).

The *Colonies* are incomposite, and the cylindrical branches bear c. 14 zooecia in each transverse series.

Numerous specimens from Fécamp (middle Senonian).

Meliceritites Vieilbanci d'Orbigny.

Semielea Vieilbanci d'Orbigny, *Bryoz. Crét.* p. 636, pl. 637, figs. 7, 8.

Elea Vieilbanci Gregory, *Cretac. Bryozoa* p. 300, fig. 33, pl. 738, figs. 5—9.

(Pl. 1, figs. 15, 16.)

The *Zooecia*, the dividing ridges of which are often very indistinct, are generally about twice as long as broad, and their surface often presents a saddle-shaped concavity in the direction proximally distally. The broadly rounded, almost semicircular aperture, which is provided with a narrow and generally indistinct oral ledge has a strongly developed peristomial thickening and sometimes takes up the whole breadth of the zooecium in its distal part. Its length is contained 3—4 times in the length of the zooecium. The operculum is very convex and presents a distinct flabelliform striation.

Heterozoecia have not been found.

Ooecia have been found by Gregory.

No *Kenozoecia*.

The *Closure* is effected by means of a concave lamina (fig. 16).

The *Regeneration*. In most fragments examined a number of zooecia have been regenerated, but the protruding end of the new zooecium only in a few cases presents a half-elliptical aperture, the apertures being in most cases either perfectly closed or transformed into a narrow fissure which is no doubt the rest of the original aperture.

The *Colonies* are hollow free irregularly branched expansions with cylindrical or compressed branches. Some of the fragments examined show a beginning new layer in the form of one or more circular patches of different extension.

A number of fragments from Villedieu (Coniacian) and Bruillé-Poncé (Turonian).

Meliceritites durobrivensis Gregory.

Nodelea durobrivensis Gregory, Cret. Bryoz. p. 310, pl. XIV, figs. 4—13.

(Pl. IV, figs. 1—6, 8.)

The *Zooecia* which are divided by well-developed marginal ridges are generally rhombic or hexagonally rhombic, longer than broad, and the very concave suboral area takes up almost half the length of the whole zooecium. No distinct tubercles. The large aperture, which is surrounded by a strongly developed and very often obliquely ascending peristome, is a little longer than broad, and the lateral margins are generally distinctly converging proximally. No distinct oral ledge. The distal half of the aperture together with the peristome generally takes up the whole breadth of the zooecium. The very convex operculum is provided with a distinct flabelliform striation.

The *Heterozooecia*, which in most colonies are present in large numbers, rarely attain the size of the zooecia, and in most cases they are much smaller, sometimes only attaining half the length of the latter. They are rhombic very much concave and provided with a very small about half-elliptical aperture, the larger part of which is covered by a concave lamina, perforated by a narrow fissure, sometimes in the shape of an inverted T. In a number of them I have found an operculum of the same structure as that of the zooecia. Most of them are arranged in longitudinal series each consisting of 2—7 in such a manner that two succeeding heterozooecia are divided from each other by a pair of zooecia. In the more regular cases each two longitudinal series of heterozooecia are divided by two longitudinal series of zooecia, and, therefore, there may be seen on the surface of a fragment as many as 6 longitudinal series of heterozooecia. However, the arrangement as well as the size and the number of the heterozooecia is subject to great variation, and sometimes there may be seen groups of up to 14 adjacent heterozooecia.

Ooecia have been found.

No *Kenozooecia*.

A *Closure* by means of a concave lamina is rarely seen.

The *Regeneration* takes place in great measure, and the zooecia may be regenerated either by a new zooecium (figs. 4, 8) or by a heterozooecium (fig. 3). In the latter case the part of the heterozooecium enclosing the aperture may be more or less projecting, and the aperture of the new heterozooecium sometimes forms a right angle with the suboral area.

The *Colonies* are incomposite, and the fragments examined are cylindrical with 8—10 zooecia in each transverse series. I have examined a large number of fragments from Chatham (middle chalk) and Fécamp (middle Senonian).

Meliceritites durobrivensis, var. parviarmata Greg.

Meliceritites parviarmata Gregory, Cretac. Bryoz. p. 340, pl. XV, fig. 1.

(Pl. IV, figs. 7, 19—21.)

The *Zooecia*, which are divided by strongly developed marginal ridges are of rather different form and length, but in most cases they are twice as long as broad or even longer. No distinct tubercles. The aperture, the form of which is somewhat variable, may sometimes be almost half as long as broad, and the two lateral margins are in most cases more or less distinctly parallel. There is found a strongly developed peristomial thickening, and a very convex operculum with flabelliform striation. No distinct oral ledge.

The *Heterozooecia* which are present in large numbers are of very different form and size, the length of the smallest being contained 6—7 times in the length of the zooecia, while the larger of them may sometimes attain the length of the latter. Their distal end which is more or less projecting and sometimes forms almost a right angle with the suboral area contains an aperture of the same form as that of the zooecium, but the larger part of it is closed by a concave lamina which is provided with an opening in the shape of an inverted T. Their distribution is very different as they are sometimes placed between the apertures of a number of adjacent zooecia, while in other cases they are irregularly heaped together, partly around the distal end of a zooecium (fig. 7), partly between a number of zooecia, and in some fragments of colonies which have been growing on shells and which are provided with an incrusting base the larger part of the latter is formed by heterozooecia. While that portion of this incrusting layer which immediately surrounds the proximal part of the free stem is composed chiefly of zooecia its peripheral part almost entirely consists of heterozooecia which therefore here seem to play a similar role as the kenozooecia forming the incrusting base of a *Retepora*-colony. One of the incrusting bases examined not only covers the one surface of a small fragment of a shell, but also a large part of the opposite surface, and here forms an extension 10^{mm} long and 6^{mm} broad, in which there is only found 3 zooecia.

An *Ooecium* has been found in one of the incrusting bases.

No *Kenozooecia*.

A *Closure* of the zooecia by means of a concave lamina has only been found in a few cases (fig. 7).

The *Regeneration*. There is found a regeneration both of new zooecia in old ones and of heterozooecia in zooecia (fig. 7).

I have examined a few fragments from Chatham, Luton, Gillingham (middle chalk) and Evreux (middle Senonian).

Meliceritites punctata d'Orbigny.

Myriozoum punctatum d'Orbigny, Bryoz. Crét. p. 663, pl. 783. figs. 4—7.
(Pl. V, figs. 6—8.)

The *Zooecia*, which are not divided by marginal ridges, are provided with more or less, large and deep pits, the number of which increases with age, and in old zooecia they are divided from each other by a rich net-work of prominent ridges. The half-elliptical aperture which lacks a distinct oral ledge is provided with a generally strongly developed and much raised peristomial thickening. The convex operculum presents a flabelliform striation.

The *Heterozooecia*, most of which are longer than the zooecia, are spread among the zooecia in rather large numbers, either singly or two placed near together. The elongate aperture is in the smaller of them triangular the two lateral margins converging distally in their whole length, but in the larger the distal half is bordered by two almost parallel lateral margins. The concave surface distal to the inner aperture has no lateral projections or thickenings. In a few heterozooecia I have seen a calcareous mandible (fig. 8).

Ooecia have not been found.

No *Kenozooecia*.

A *Closure* by means of a concave lamina has only been seen in a few zooecia.

The *Regeneration*. While a distinct regeneration of new zooecia in old ones has not been seen, there can be no doubt that the form and position presented by some of the heterozooecia in a single of the colonies examined can only be explained from a regeneration of new heterozooecia in old ones. While the aperture of the common heterozooecia is about parallel to the axis of the colony the aperture of these heterozooecia forms an angle of up to 130 degrees with the surface of the latter, and in consequence hereof they are provided with two large, triangular or trapeziform lateral surfaces which show the same pitted appearance as the zooecia. How many times the regeneration has taken place in the single heterozooecia cannot be seen because of the rich deposition of calcareous matter which goes on over the surface of the whole colony and to which the pitted appearance is due.

The *Colonies* are incomposite, and the examined cylindrical fragments are provided with 10—14 zooecia in the transverse series.

I have examined a number of specimens from Vendôme (zone with *Crania ignabergensis*).

Meliceritites Dollfusi Pergens.

Meliceritites Dollfusi Pergens, Révision d. Bryoz. p. 395, pl. XIII, fig. 4.
Meliceritites lonsdalei Gregory, Cret. Bryoz. p. 335, pl. XV, figs. 5—9.
(Pl. IV, fig. 23.)

The *Zooecia* which are as a rule twice as long as broad are divided by more or less distinct marginal ridges, and the concave or saddle-shaped suboral area

ascends towards the strongly projecting distal part which rises in the form of a short tube. No tubercles. The aperture which lacks a distinct oral ledge and presents no peristomial thickening¹⁾ ranges between semicircular and triangularly rounded, and sometimes the distal end may be more or less distinctly pointed. The convex operculum presents a distinct flabelliform striation.

The *Heterozooecia* which may be twice as long as the zooecia are provided with a concave suboral area and a very elongate aperture, the breadth of which is contained about three times in the length. It attains its largest breadth at the distal margin, and the two lateral margins which are incurved in their middle part distally run together in a curve. There are no lateral thickenings. They are as a rule present in scarce numbers, but not rarely there may be found 2—4 placed near each other in the same transverse series, and in a single case I have found 7 forming an irregular, interrupted transverse row.

Ooecia have been found.

No *Kenozooecia*.

The *Closure* is effected by means of a concave lamina.

A *Regeneration* has not been found in the fragments examined.

The *Colonies* are incomposite, and the examined cylindrical fragments have 10—12 zooecia in each transverse series. One of them which is fixed to a fragment of a shell is provided with a basal expansion consisting of zooecia and heterozooecia of the same kind as those found in the free cylindrical part. The heterozooecia of this specimen (from Luton) are a little different from those found in the others as the lateral margins of the aperture are not incurved in the middle but converging distally.

Chatham, Gillingham, Luton (middle chalk) Fécamp (middle Senonian).

Meliceritites Dollfusi, var. *tubuliformis* n.

(Pl. V, figs. 9, 10.)

Of this form which I provisionally refer to *M. Dollfusi* I have only seen a single fragment 4^{mm} long and 1^{mm} broad. The most conspicuous difference is the length of the tubular distal part, which is about as long as the suboral area. Sometimes the frontal part of it is divided from the rest by two more or less distinct marginal ridges. The obliquely ascending aperture is half-elliptical or triangularly rounded, and the very convex operculum is distinctly striated. The Heterozooecia of which there is found three do not differ from those in *M. Dollfusi* and that seen in fig. 10 which seems to be shorter is not quite correctly figured as the distal part was broken off. — Evreux (middle Senonian).

¹⁾ When Gregory in this species speaks about "well raised" peristomes, he confounds a raised peristome with a freely projecting distal part of a zooecium.

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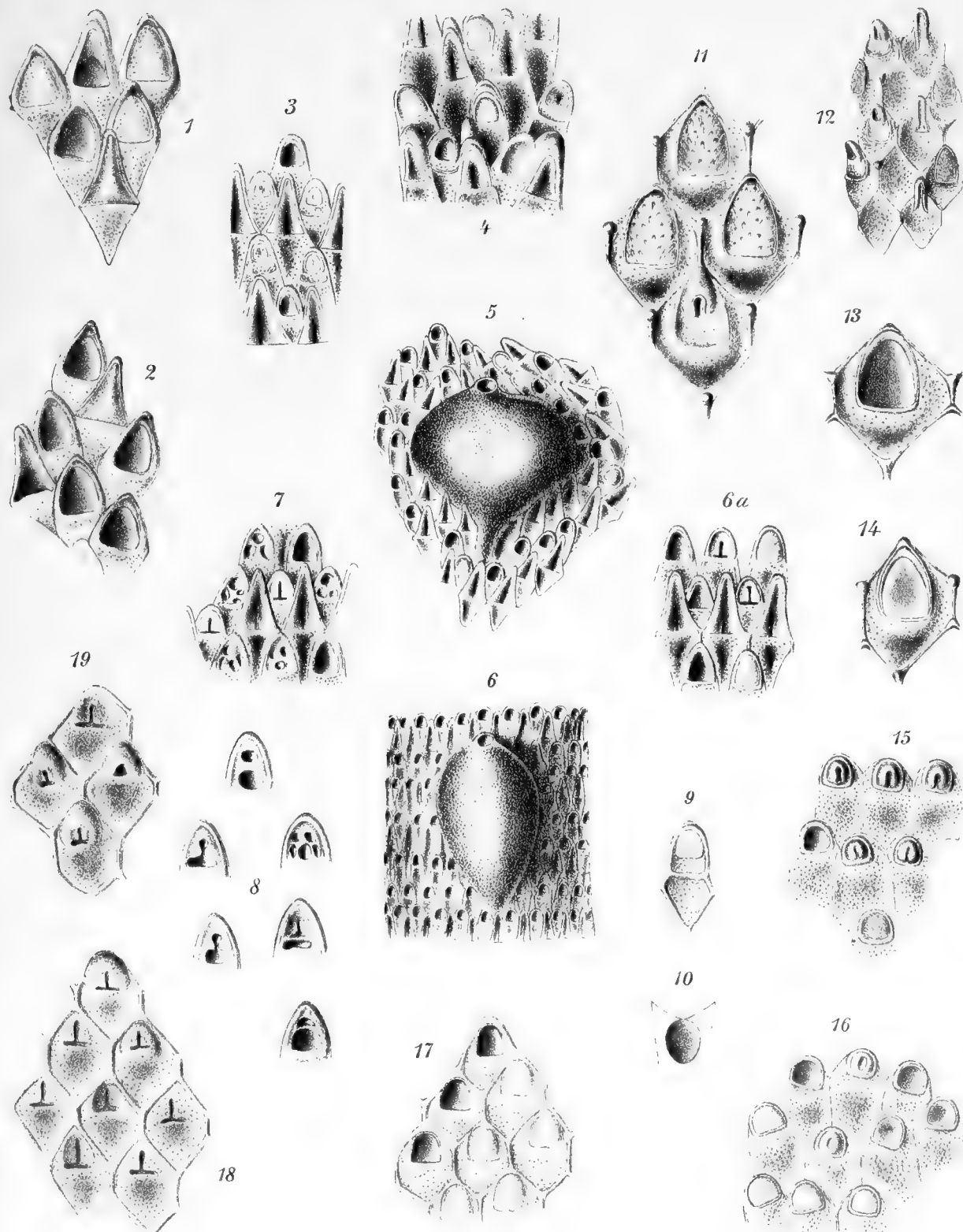
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Plate I.

(The numbering of the plates is to be changed into X 1.)

- Fig. 1. *Meliceritites sarthacensis* d'Orb. Distally is seen a heterozooecium which has been regenerated and proximally one provided with a calcareous mandible. $\times 34$.
- 2. The same species. There is seen two heterozooecia, one of them with a calcareous mandible. $\times 34$.
- 3. *Meliceritites magnifica* d'Orb. Of the six zooecia the four have been regenerated, and the apertures of the new zooecia have been closed by a concave lamina. (Not well executed). $\times 34$.
- 4. The same species. Four of the zooecia and three of the heterozooecia have been regenerated and some of them twice. The aperture of the new zooecium at the right side has been closed by a concave lamina. $\times 34$.
- 5. The same species with ooecium. $\times 20$.
- 6. The same species with another form of ooecium. $\times 20$.
- 6a. The same species. Two of the zooecia are provided with an operculum, and two are closed by a lamina presenting a \perp -shaped opening. $\times 34$.
- 7. The same species. There is seen two different forms of closure. $\times 34$.
- 8. The same species. Different forms of closure from the same colony. $\times 47$.
- 9. The same species. A regenerated zooecium with operculum. $\times 47$.
- 10. The same species. A transverse section through a heterozooecium to show the lateral processes covering the larger part of the aperture. $\times 47$.
- 11. *Meliceritites gothica* nov. nom. Three zooecia with opercula and one regenerated which has been replaced by a heterozooecium. $\times 34$.
- 12. The same species. There is seen three kenozooecia, three zooecia with opercula, three closed heterozooecia and three zooecia, all of which have been regenerated by means of heterozooecia. $\times 20$.
- 13. A zooecium of the same species. $\times 34$.
- 14. The same species. The aperture is closed by a flat lamina which has arisen from the free edge of the oral ledge. $\times 34$.
- 15. *Meliceritites Vieilbanci* d'Orb. Five of the zooecia have been regenerated. $\times 34$.
- 16. The same species. There are seen four zooecia with opercula, two regenerated, two open and two closed by a concave lamina. $\times 34$.
- 17. *Meliceritites trifolium* n. sp. Five zooecia with opercula. $\times 47$.
- 18. The same species. There are seen six heterozooecia, and two zooecia regenerated by means of heterozooecia. $\times 47$.
- 19. The same species. There is seen a heterozooecium, and three zooecia regenerated by means of heterozooecia. (In one of them the two lateral processes have been broken off.) $\times 47$.

* The present is the last plate lithographed by the excellent artist Mr. C. CORDIS, but the execution bears witness to that he has not been able to lay the last hand on it before his death.



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Plate II.

- Fig. 1. *Meliceritites sarissata* Greg. Two Kenozooecia and a heterozooecium. $\times 34$.
- 2. The same species. Zooecia with opercula. $\times 34$.
 - 3. The same species. A heterozooecium with mandible, surrounded by kenozooecia. $\times 34$.
 - 4. *Meliceritites angulosa* d'Orb. An ooecium. Villedieu. $\times 20$.
 - 5. The same species. Two heterozooecia of which the distal one has been regenerated by means of a new zooecium, and thereafter the remainder of the large aperture has been closed (compare with figs. 7 and 9). Villedieu. $\times 20$.
 - 6. The same species. Regeneration of a heterozooecium. The calcareous processes have not yet been formed distally to the new heterozooecium. Villedieu. $\times 34$.
 - 7. A new zooecium has been formed within an old heterozooecium, and the rest of the aperture has been closed, but the processes corresponding to those seen in figs. 5 and 9 are here very irregular. The zooecium is provided with a concave closure-plate which still has a small opening. Villedieu. $\times 34$.
 - 8. The same species. A heterozooecium. Villedieu. $\times 34$.
 - 9. Regeneration of a heterozooecium. Villedieu. $\times 34$.
 - 10. The same species. Fécamp. $\times 34$.
 - 11. The same species. Couture. $\times 20$.
 - 12. A zooecium in an old heterozooecium. Villedieu. $\times 34$.
 - 13. The same species. Chatham. $\times 34$.
 - 14. Two heterozooecia from the same colony, Couture. $\times 20$.
 - 15. The same species. At the left side of the heterozooecium is seen a zooid intermediate between a zooecium and a heterozooecium. Fécamp. $\times 20$.
 - 16. A heterozooecium with mandible. \times Villedieu. $\times 20$.
 - 17. The same species.
 - 18. Another heterozooecium with a mandible the distal part of which has been broken off.
 - 19. The same species. Villedieu. $\times 34$.
 - 20. The same species. Regeneration of a heterozooecium. Bruillé-Poncé. $\times 34$.
 - 21. The same species. Villedieu. $\times 20$.
 - 22. Regeneration of a heterozooecium. Villedieu. $\times 34$.
 - 23. *Meliceritites gothica*, var. *acuminata*. $\times 34$.
 - 24. The same species. A heterozooecium.

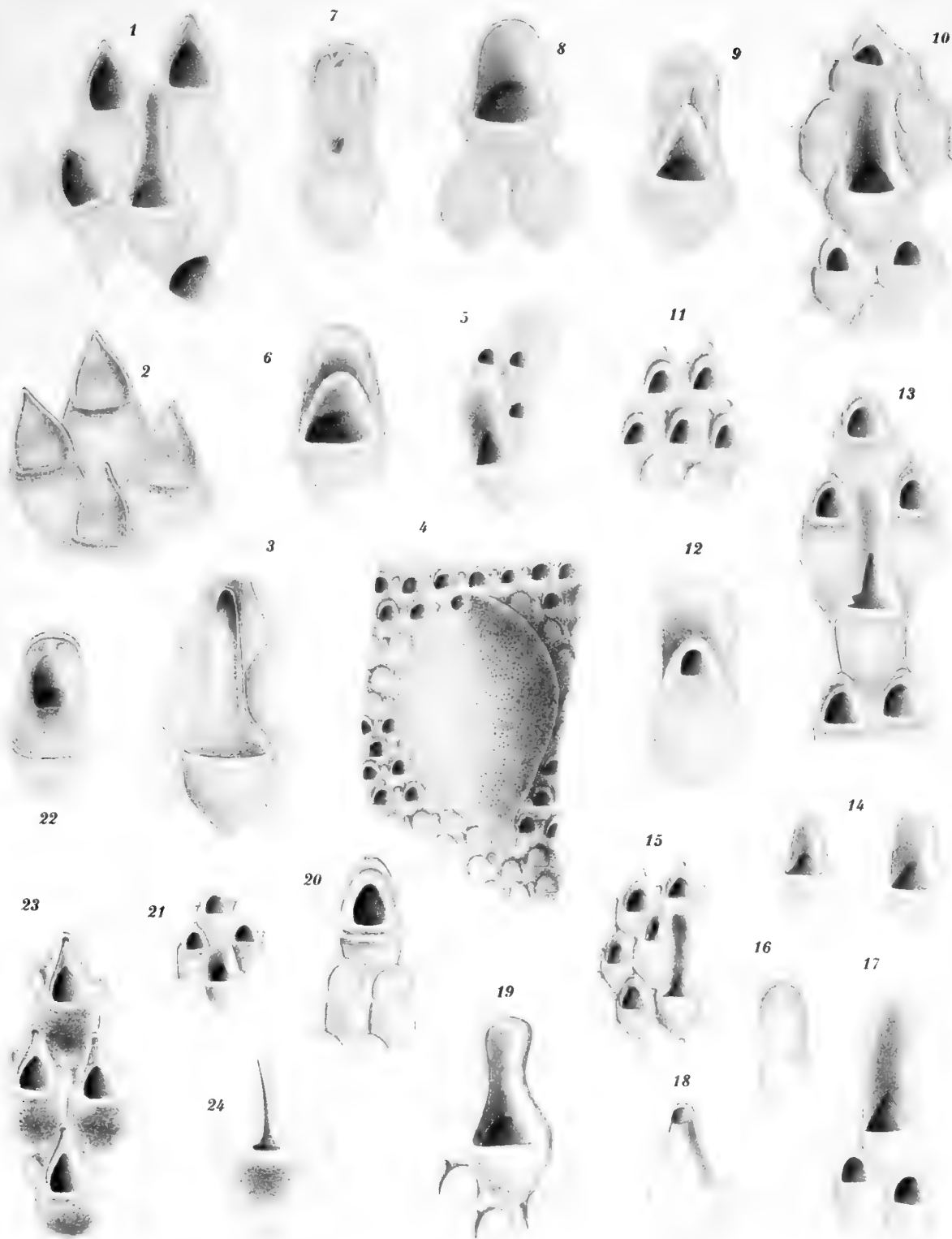


Plate III.

- Fig 1. *Meliceritites lamellosa* d'Orb. Three zooecia with opercula, and two heterozooecia in old zooecia, the aperture of which is not wholly taken up by the new zooids. The lateral margins of the heterozooecial apertures are not distinct. $\times 34$.
- 2. The same species. Two regenerated zooecia with opercula and a heterozooecium. $\times 34$.
 - 3. The same species. Four heterozooecia one of which takes up the whole aperture of an old zooecium. Two of them are provided with a mandible. $\times 34$.
 - 4. The same species. Two heterozooecia in old zooecia. The aperture of the distal one is partly closed. $\times 34$.
 - 5. The same species. Four heterozooecia one of which takes up the whole aperture of an old zooecium. The distal one on the left side has been regenerated. $\times 34$.
 - 6. A regenerated heterozooecium. $\times 34$.
 - 7. The same species. Four regenerated zooecia, two of which are provided with opercula and one with a closure-lamina. Three regenerated heterozooecia. $\times 34$.
 - 8. The same species. $\times 34$.
 - 9. The same species. An ooecium. $\times 20$.
 - 10. *Meliceritites micropora* d'Orb. Three regenerated zooecia and two zooecia with opercula. $\times 34$.
 - 11. *Meliceritites Steenstrupi* Pergens. The hinder surface with the heterozooecia. $\times 20$.
 - 12. The same species. Three heterozooecia. $\times 34$.
 - 13. The same species. Two zooecia with opercula. $\times 34$.
 - 14. *Meliceritites armata* n. sp. The hinder surface with kenozooecia and heterozooecia. $\times 20$.
 - 15. The same species. A zooecium with operculum. $\times 34$.
 - 16. The aperture of a heterozooecium. $\times 34$.
 - 17. *Meliceritites cenomana* d'Orb. A heterozooecium. $\times 34$.
 - 18. The same species. $\times 34$.
 - 19. *Meliceritites Canui* n. sp. The hinder surface with the heterozooecia. $\times 20$.
 - 20. The same species. Three heterozooecia. $\times 34$.
 - 21. The same species. Two zooecia with opercula. $\times 47$.
 - 22. A heterozooecium. $\times 47$.
 - 23. A heterozooecium the aperture of which is partially closed. $\times 47$.
 - 24. A heterozooecium with a very elongate aperture. $\times 47$.
 - 25. A heterozooecium the aperture of which is not far from being perfectly closed. $\times 47$.
 - 26. A zooecium with a closure-plate. $\times 47$.
 - 27. A heterozooecium which takes up the aperture of an old zooecium. $\times 47$.

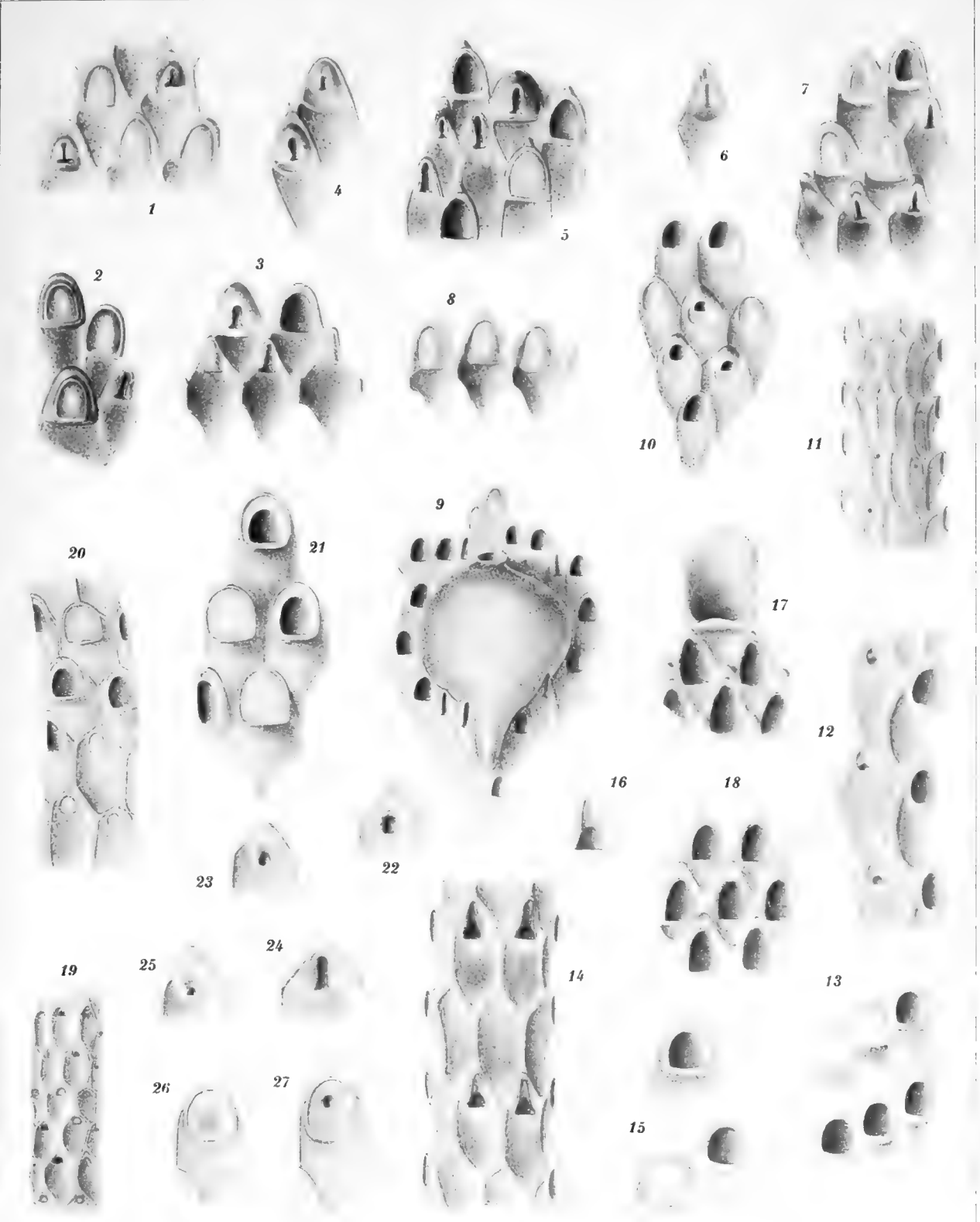


Plate IV.

- Fig. 1. *Meliceritites durobrivensis* Greg. Five zooecia with opercula and three heterozooecia two of which are provided with opercula. Chatham. $\times 34$.
- 2. The same species from Fécamp. Three heterozooecia. $\times 34$.
 - 3. The same species. Two large heterozooecia and three zooecia which have been regenerated by means of heterozooecia. Luton. $\times 34$.
 - 4. The same species. A regenerated zooecium with operculum. Chatham. $\times 34$.
 - 5. The same species. Chatham. $\times 34$.
 - 6. The same species. A zooecium the aperture of which is almost perfectly closed by a concave lamina. There is only left a small opening in the distal half. Chatham. $\times 34$.
 - 7. *Meliceritites durobrivensis*, var. *parviarmata*. There is seen a number of small heterozooecia two of which in old zooecia, one zooecium with operculum and another with a concave closure-lamina. Gillingham. $\times 34$.
 - 8. *Meliceritites durobrivensis* Greg. There are seen four regenerated zooecia, the distal of which has been regenerated twice while the two at the left side have been regenerated several times. Chatham. $\times 34$.
 - 9. *Meliceritites undata* d'Orb. $\times 34$.
 - 10. The same species. Three zooecia have been regenerated by means of heterozooecia. $\times 34$.
 - 11. The same species. Two regenerated zooecia with opercula.
 - 12. The same species. Three heterozooecia one of which is intermediate between a heterozooecium and a zooecium. $\times 34$.
 - 13. The same species. Two heterozooecia. $\times 34$.
 - 14. The same species. A zooecium regenerated by means of a heterozooecium. $\times 34$.
 - 15. The same species. A zooecium with a concave closure-lamina which is not yet closed in the middle. $\times 34$.
 - 16. The same species. A regenerated zooecium with operculum. $\times 34$.
 - 17. The same species. $\times 34$.
 - 18. *Meliceritites Lorieri* d'Orb. Two zooecia with opercula. $\times 34$.
 - 19. *Meliceritites durobrivensis*, var. *parviarmata*. A portion of the incrusting base, consisting chiefly of heterozooecia. Chatham or Luton. $\times 20$.
 - 20. Two heterozooecia from the same colony. $\times 66$.
 - 21. The same form. A portion of a two-layered colony with numerous heterozooecia. Basin de Paris. $\times 34$.
 - 22. *Meliceritites pentagonum* n. sp. Three heterozooecia. Tullstorp. $\times 20$.
 - 23. *Meliceritites Dollfusi* Pergens. One of the two heterozooecia shows a calcareous mandible. Gillingham. $\times 20$.

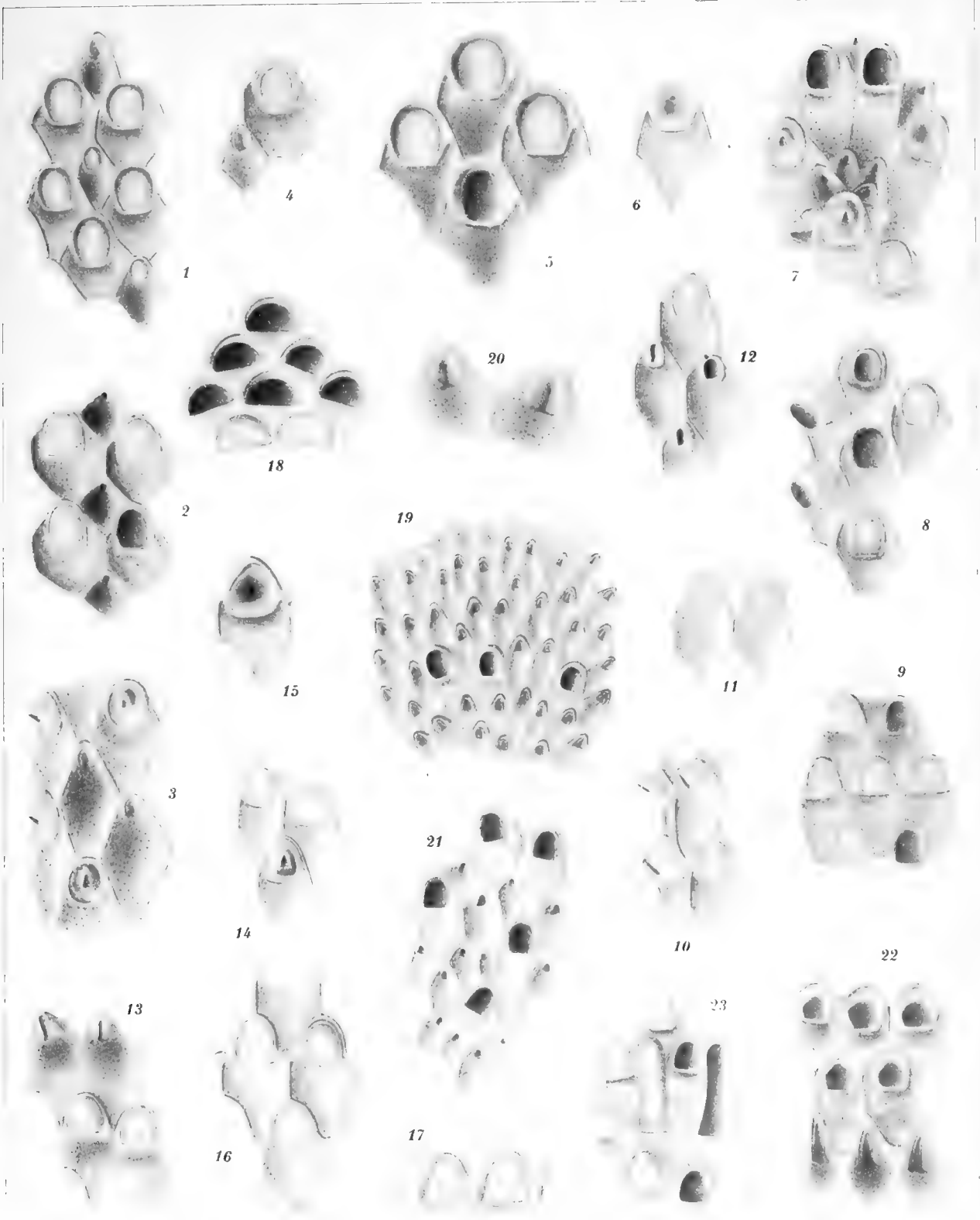


Plate V.

- Fig. 1. *Meliceritites dichotoma* d'Orb. Four zooecia with opercula and three heterozooecia the apertures of which have been covered by concave closure-plates. $\times 34$.
2. The same species. There is seen a regenerated zooecium and two zooecia with closure-plates. $\times 34$.
3. *Meliceritites hexagona* d'Orb. $\times 34$.
- 4. The same species. The two zooecia on the left-hand side have been regenerated twice, and the peculiar appearance of the unusually large zooecium to the right must no doubt be explained by a repeated regeneration together with the circumstance that the limits between the different peristomes have been effaced. $\times 34$.
- 5. The same species. A heterozooecium with mandible. $\times 34$.
- 6. *Meliceritites punctata* d'Orb. The zooecia with opercula. $\times 34$.
- 7. The same species. A heterozooecium. $\times 34$.
- 8. The same species. A heterozooecium with mandible. $\times 34$.
- 9. *Meliceritites Dollfusi* Pergens, var. *tubuliformis* n. A heterozooecium and a zooecium with operculum.
- 10. The same species. $\times 34$.
- 11. *Meliceritites plana* d'Orb. Two heterozooecia. France. $\times 20$.
- 12. The same species. Three zooecia with opercula and six with closure. St. Antoine du Rocher. $\times 34$.
- 13. *Meliceritites squamata* Marsson. One of the smaller heterozooecia. Möen (Denmark). $\times 34$.
- 14. The same species. A zooecium in an old heterozooecium. $\times 34$.
- 15. The same species. A new heterozooecium in an old one. $\times 34$.
- 16. The same species. A heterozooecium. $\times 34$.
- 17. The same species. A heterozooecium with a broader aperture and with mandible. $\times 34$.
- 18. *Meliceritites Roemeri* v. Hag. Tullstorp. $\times 20$.
- 19. The same species. Distally is seen a small heterozooecium in an old zooecium, to the left a new zooecium in an old one and to the right a concave closure-lamina. $\times 34$.
- 20. The same species. Three zooecia with a distinct oral ledge. $\times 34$.
- 21. A kenozooecium and a small heterozooecium. $\times 34$.
- 22. A small heterozooecium. $\times 34$.
- 23. Two small heterozooecia with a more elongate aperture. $\times 34$.
- 24. A small heterozooecium with a much larger aperture. $\times 34$.

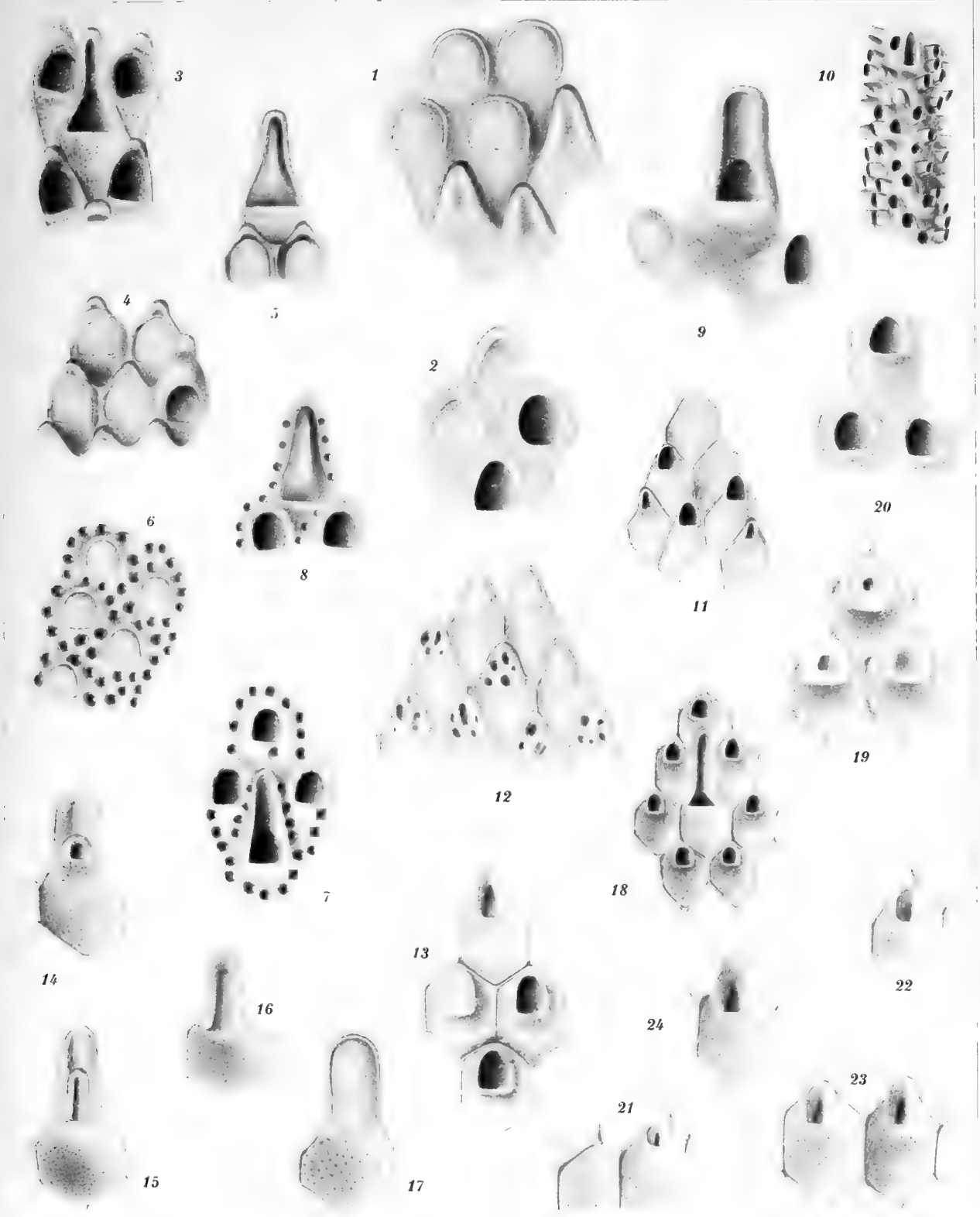


Plate VI.

- Fig. 1. *Meliceritites tuberculata* d'Orb. There are seen eight kenozoecia, two zooecia with opercula and two heterozoecia one of which has been regenerated. Basin de Paris. $\times 34$.
- 2. The same species. There is seen a heterozoecium with mandible. France. $\times 47$.
 - 3. The same species. Five zooecia and a kenozoecium surround an ancestrula of which only the aperture is seen. Basin de Paris. $\times 47$.
 - 4. *Meliceritites semiluna* d'Orb. All the zooecia are provided with a flat closure-plate, which rises from the free edge of the oral ledge. $\times 34$.
 - 5. The same species. Two zooecia with opercula. $\times 34$.
 - 6. The same species. Two heterozoecia. $\times 34$.
 - 7. *Meliceritites Filiozati* n. sp. There are seen the apertures of three ancestrulae, two kenozoecia and two heterozoecia. Fécamp. $\times 34$.
 - 8. Four zooecia and a kenozoecium surround the ancestrula of a future colony. Fécamp. $\times 34$.
 - 9. The same species. There are seen two heterozoecia and two kenozoecia. Fécamp. $\times 34$.
 - 10. The same species. One of the two zooecia is provided with a closure-plate which rises at a deeper level than the oral ledge. Fécamp. $\times 34$.
 - 11. *Meliceritites pyrenaica* d'Orb. All the apertures which are of very different form and size seem to have been regenerated, perhaps several times. Tours. $\times 20$.
 - 12. The same species. The suboral area is only represented by the pits. Villedieu. $\times 34$.
 - 13. The same species. Two zooecia with opercula. The beak and the proximal tooth are distinct. Villedieu. $\times 34$.
 - 14. The same species. The two strongly prominent zooecia seem to have been regenerated. Villedieu. $\times 20$.
 - 15. The same species. Three zooecia with opercula. The strongly developed suboral areas are convex owing to a rich deposition of calcareous matter, and the pits have almost vanished. Villedieu. $\times 34$.
 - 16. The same species. All the zooecia have no doubt been regenerated. Villedieu. $\times 20$.
 - 17. The same species. Zooecia from the distal end of a colony. The oral ledge and the proximal tooth are distinct, and only a single zooecium has been regenerated. Villedieu. $\times 20$.
 - 18. The same species. There are seen a heterozoecium and an operculum. Most zooecia have been regenerated. Villedieu. $\times 20$.
 - 19. The same species. The suboral area is short and broad and provided with two lateral pits. Villedieu. $\times 34$.
 - 20. The same species. The zooecia which have no doubt all been regenerated are placed in all directions. Villedieu. $\times 20$.
 - 21. The same species. Six zooecia surrounding a heterozoecium. They seem all to have been regenerated. Villedieu. $\times 20$.

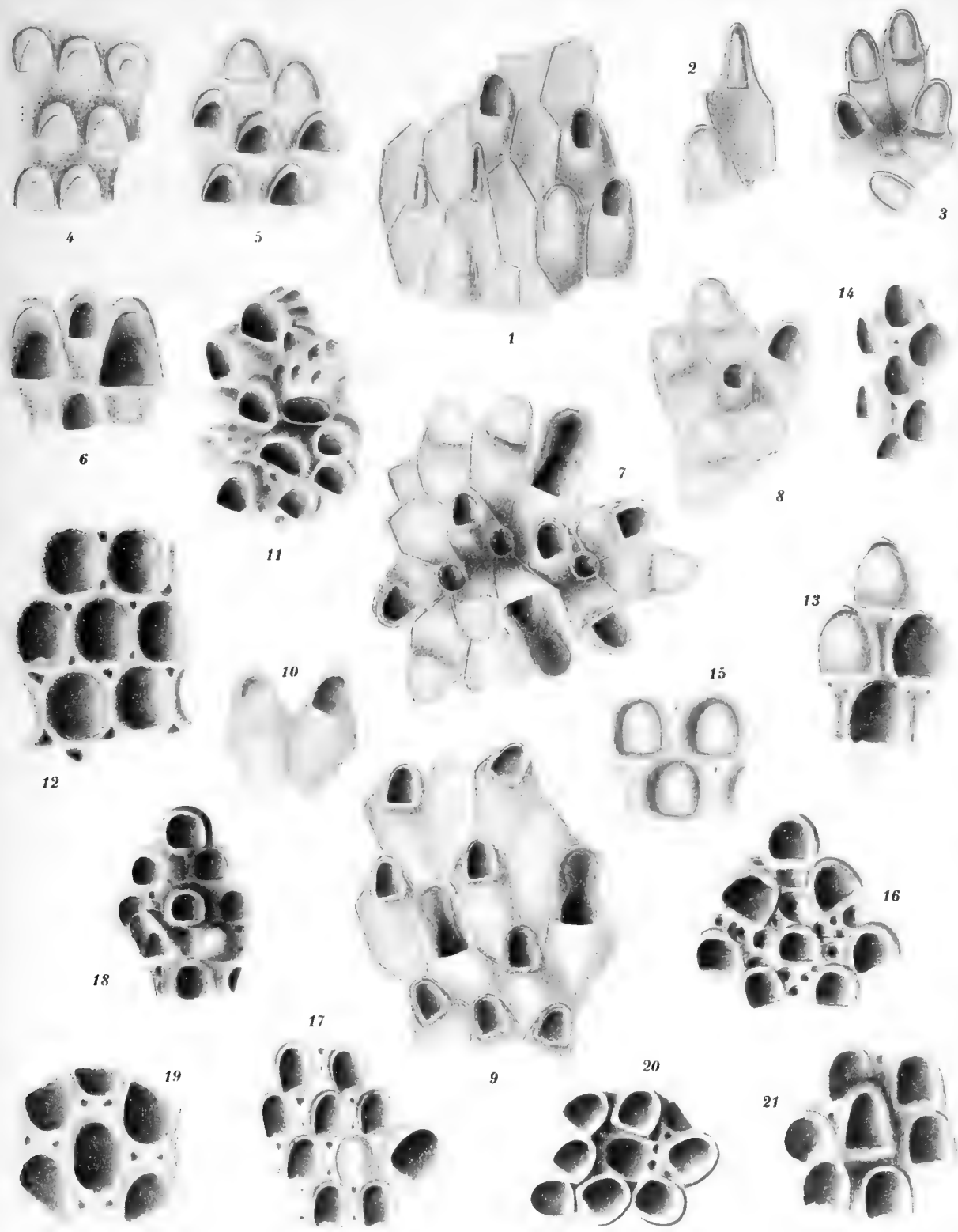
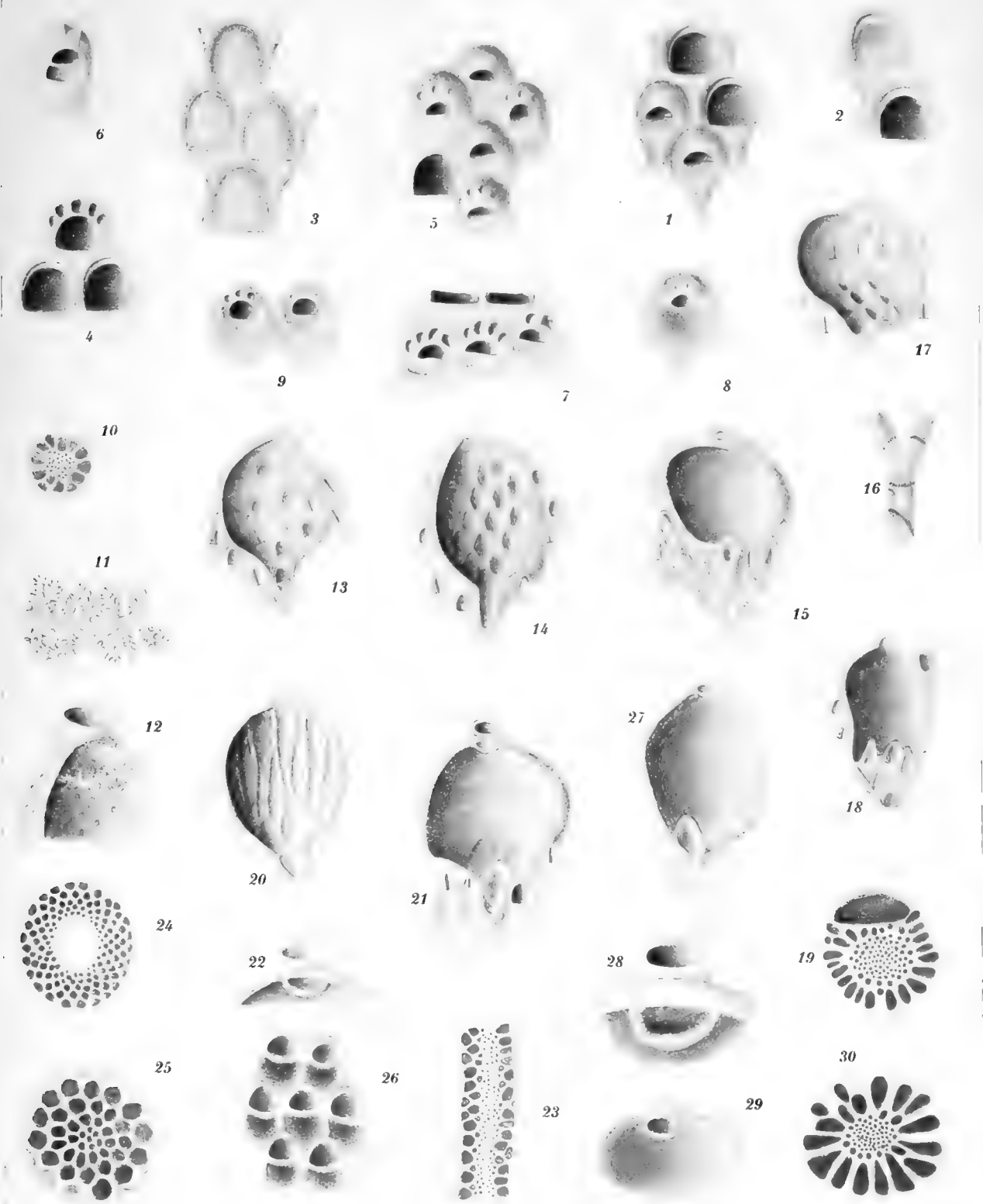


Plate VII.

- Fig. 1. *Meliceritites palpebrosa* nov. nom. Two zooecia with a convex closure-plate. $\times 34$.
- 2. The same species. A zooecium with a flat closure-plate. $\times 34$.
- 3. The same species. Four zooecia with opercula. $\times 34$.
- 4. The same species. A zooecium with a beginning convex closure-plate. $\times 34$.
- 5. The same species. Five zooecia with a convex closure-plate. The proximal part of the closure has in three of the zooecia been directed obliquely inwards. $\times 34$.
- 6. A zooecium with a distal convex and a proximal concave closure-plate. $\times 34$.
- 7. Three zooecia with a beginning distal and a well-developed proximal closure-plate. The proximal margin and a portion of the lateral margins of the aperture are still seen. $\times 34$.
- 8. A zooecium with a convex closure-plate. $\times 34$.
- 9. Two zooecia in which both the distal and the proximal part of the closure are well-developed. $\times 34$.
- 10. *Meliceritites durobrivensis* Greg. A transverse section through a branch. Chatham. $\times 34$.
- 11. A portion of the atrial ring in the ooecium of *Crisia eburnea*. Denmark. $\times 175$.
- 12. The distal part of the ooecium of *Crisia eburnea* after the removal of the frontal wall. The atrial ring is seen. $\times 66$.
- 13. An opened ooecium of *Mel. magnifica* after the adzooecial wall has been dissolved. The zooids covered by the ooecium are in this case completely developed. $\times 20$.
- 14. An opened ooecium of *Mel. magnifica* after the adzooecial wall has been dissolved. The zooids covered by the ooecium have no calcified frontal wall. $\times 20$.
- 15. An opened ooecium of *Mel. magnifica*. $\times 20$.
- 16. A fragment of *Mel. magnifica* showing three different superficial layers. $\times 1$.
- 17. An opened ooecium of *Mel. magnifica*, which belongs to the growing end of a superficial layer and the distal half of which covers an elder portion of the colony. The adzooecial wall has been dissolved. $\times 20$.
- 18. An open ooecium of *Mel. magnifica* in the proximal part of which is seen a zooecium and three heterozooecia besides the aperture of the gonozooecium. It is likely to suppose that the former have originally been covered by a bulging part of the adzooecial wall, which has later been destroyed. $\times 20$.
- 19. *Mel. magnifica*. A transverse section of a branch with an ooecium. $\times 14$.
- 20. An open ooecium of *Mel. lamellosa* d'Orb. The adzooecial wall together with a part of the underlying zooecial tubes have been dissolved. $\times 20$.
- 21. An opened ooecium of *Mel. lamellosa*. The proximal half of the atrial ring is seen. $\times 20$.
- 22. *Mel. lamellosa*. The distal end of an opened ooecium with the proximal half of the atrial ring. $\times 34$.
- 23. *Mel. lamellosa*. A transverse section of a colony. $\times 20$.
- 24. *Spiropora micropora* d'Orb. A transverse section. $\times 20$.
- 25. *Entalophora Roemeri* n. sp. A transverse section. $\times 14$.
- 26. Zooecia of *Ent. Roemeri*. $\times 20$.
- 27. An opened ooecium of *Mel. angulosa* d'Orb. $\times 20$.
- 28. *Mel. angulosa* d'Orb. The distal end of an opened ooecium with the proximal half of the atrial ring. $\times 66$.
- 29. The distal part of an opened ooecium of *Mel. angulosa* d'Orb. seen from the proximal end. The whole atrial ring is seen. $\times 34$.
- 30. *Mel. pyrenaica* d'Orb. A transverse section. $\times 11$.



A MONOGRAPH
OF
THE GENUS DRYOPTERIS

PART I
THE TROPICAL AMERICAN PINNATIFID-BIPINNATIFID SPECIES

BY
CARL CHRISTENSEN

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURV. OG MATHEMATISK AFD. X. 2



KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI

1912

Introduction.

The present work is the first part of a monographical review of the genus of ferns, *Dryopteris*, and it deals with the tropical American species having the lamina from subentire to bipinnatifidly cut. This delimitation is, of course, artificial, as the degree of cutting is of no greater value as systematic character, but it is chosen from practical reasons.

In earlier papers I have published a review of the species belonging to the two subgenera *Lastrea* and *Stigmatopteris*¹⁾. Since the publication of these papers I have received for determination or examination a large number of specimens of species belonging to the two subgenera mentioned; several of these species were previously unknown to me, and some others were found to be undescribed. I have now, therefore, not a few additions to my earlier papers, and as my former keys consequently now are uncomplete, I give in the present work new keys to all known species of the two subgenera. In this work 280 species are dealt with; about 100 of these were treated in detail in my earlier papers.

The systematic grouping of the 280 species into 10 subgenera is entirely new and it is, as to several points, very different from all older classifications. I dare maintain, however, that my classification is the most natural that has been proposed. It is based on a minute examination of not lesser than about 10,000 specimens belonging to about 500 species. Further, the study of that enormous material has enabled me to unravel the real relationship of several critical forms and to get an idea of the value as species of nearly all described "species". Not a few of these were described from very imperfect specimens, and many of them were described in such a way, that it is impossible from the descriptions alone to see, what the form in question may be. If I should, therefore, make myself any hope of clearing the synonymics of several species, it would be necessary to have for examination type-specimens, or, at least authentical specimens of as far as possible all species described. By the kindness of the curators of several collections I have succeeded in obtaining for study original specimens of all described species some few excepted. These original specimens are to be found in various museums and private collections, as follows:

- 1) 1. Revision of the American Species of *Dryopteris* of the group of *D. opposita*. — Kgl. Danske Vid. Selsk. Skr. VII, 4, 1907. A supplement hereto is
2. The American Ferns of the group of *Dryopteris opposita* contained in the U. S. National Museum. — Smithsonian Miscell. Collections (quarterly issue) 52: 365—396. 1909.
3. On *Stigmatopteris*, a new genus of ferns, with a review of its species. — Bot. Tidsskrift 29: 291 - 304. 1909.

In Muséum d'Histoire naturelle, Paris, are to be found the type-specimens of species described by LAMARCK, POIRET, DESVAUX, FOURNIER and some of those described by FÉE. I paid a visit to the museum in the summer 1909 and found the majority of the type-specimens mentioned. The original specimens of the species described by FÉE in his 5.—10. mémoire sur la famille des fougères were not found there, and I dare not know, where they are.

Prince ROLAND BONAPARTE's comprehensive herbarium, no doubt the largest private collection in existence, contains numerous authentic specimens; especially are SPRUCE's South-American collections richly represented. The type-specimens of the species recently described by Dr. ROSENSTOCK from SPRUCE's collections are in Herb. BONAPARTE, and I have had them for study.

In February 1909 I was at Stockholm, where I found in »Riksmuseets Afdelning för Fossila Växter och Archegoniater« the original specimens of species described by O. SWARTZ a hundred years ago¹⁾. Authentic specimens of not a few of the Swartzian species are to be found in the Botanical Museum of Copenhagen.

On loan I have received from

Kgl. Botanisches Museum, Dahlem bei Berlin, the type-specimens of species described by WILLDENOW, LINK, KLOTZSCH, METTENIUS, KUHN and HIERONYMUS.

The University Museum of Prague, the genera *Nephrodium* and *Lastrea* from PRESL's herbarium, which besides the original specimens of species described by PRESL also contains several authentic specimens of species described by KUNZE.

The University Museum, Vienna, some original specimens of species described by CHRIST.

Royal Botanic Gardens, Kew, all type-specimens of species described by HOOKER and BAKER, besides several authentic specimens of species described by SODIRO and JENMAN.

Botanisk Museum, Copenhagen, LIEBMANN's Mexican collections.

New York Botanical Garden, some original specimens of JENMAN's herbarium, kindly sent to me by Miss MARGARET SLOSSON.

U. S. National Museum, Washington, type-specimens of species described by BRACKENRIDGE, DAVENPORT, DONNELL SMITH, MAXON and JENMAN.

Finally I have received from Dr. H. CHRIST, Basle, and Dr. E. ROSENSTOCK, Gotha, the type-specimens of the species described by these two celebrated pteridologists. From Dr. CHRIST I have received also several authentic specimens of species described by SODIRO.

As it will be seen from the enumeration above there remain only some few type-specimens, which I have not seen. The "species", which I have not seen, are enumerated partly under each subgenus, partly at the end of the work.

¹⁾ See my paper: Über einige Farne in O. Swartz's Herbarium. — Arkiv für Botanik 9¹¹. 1910.

Besides the type-specimens I have had for examination the whole material of the species dealt with, that is contained in the museums of Copenhagen, Stockholm, Berlin, Washington, Herb. PRESL, and the large collections in Herb. ROSENSTOCK and Herb. CHRIST. Dr. CHRIST's very valuable collection of ferns is now incorporated in Herb. ROLAND BONAPARTE, Paris. Smaller collections I have received from Prince BONAPARTE, the museums of Lund, Bruxelles, München and Vienna.

To the curators of the museums enumerated above I owe my most sincere thanks for the kindness they have shown to me by lending me such comprehensive collections for study. My special thanks I must convey to His Highness, Prince ROLAND BONAPARTE, for the great aid I received during my sojourn in Paris, to Dr. H. CHRIST and Dr. E. ROSENSTOCK for numerous courtesies, to Miss MARGARET SLOSSON, New York, and to Mr. W. R. MAXON, Washington, for his most valuable assistance by sending to me the very comprehensive collections from Central America and the West Indies contained in the U. S. National Herbarium. These collections, consisting nearly exclusively of specimens collected during the last twenty years, f. inst. the large collection gathered in Central America by J. DONNELL SMITH and his collectors, are very rich in beautiful and complete specimens.

Finally I beg Dr. C. H. OSTENFELD, curator of the Botanical Museum, Copenhagen, accept my best thanks for his never failing benevolence, without which I scarcely should have got such a large material for examination.

Under each species I quote the specimens seen by me, indicating in a parenthesis where the specimen is to be found. The letters in parenthesis mean:

- B* = Herbarium Berolinense, Berlin.
- RB* = Herb. Roland Bonaparte, Paris.
- C* = Herb. H. Christ, now in Herb. Bonaparte.
- CC* = Herb. Carl Christensen, Copenhagen.
- H* = Herb. Hauniense, Copenhagen.
- Kew* = Royal Botanic Gardens, Kew.
- L* = Herb. Lundense, Lund.
- R* = Herb. E. Rosenstock, Gotha.
- Rg* = Herb. Regnellianum, Stockholm.
- S* = Herb. Holmiense, Stockholm.
- W* = U. S. National Herbarium, Washington.

The 46 figures, illustrating 88 species, are about all illustrations of type-specimens and show, as a rule, a pinna reduced to $\frac{1}{5}$ of its natural size, and two or three segments enlarged $1\frac{1}{2}$.

General Remarks.

The 280 species, dealt with in this work, are grouped into ten subgenera. In a recent preliminary paper¹⁾ I have characterized the subgenera and pointed out the principles, upon which my classification is based. Referring to that paper I shall here remark only, that the kind of the trichomes, the most important character, is not absolutely constant within each subgenus. That character is, like all other characters, not sufficient for basing a natural classification upon it alone, but it is, no doubt, the best and most constant character, by which groups of related species can be distinguished from each other. Against my classification that objection may be made that I, when using a single character as distinguishing mark, cannot omit errors similar to those so evident in all earlier classifications based upon a single character. To this I want to say that my classification is not based upon a single character, the structure of the trichomes. By examining again and again and comparing the thousands of specimens I rather quickly succeeded in being able to group the different forms thus, that the species referred to each group were no doubt intimately related. The affinity is not shown by a single common character, but by a certain conformity in the structure, colour and general habit of the related species. That conformity is easily seen by the trained eye, but it is very difficult to describe. Having sorted my material after that method I then found, that the structure of the trichomes of species referred to a certain group was remarkably alike and on further examination I found, that no other character is so constant as that mentioned. The find was of much value to me, for now it became an easy work to characterize my new subgenera in such a manner, that each pteridologist can refer with approximate accuracy a given form to its right systematic position. As a matter of fact I shall point out that all the 280 species, four or five perhaps excepted, could be determined to subgenus from an examination of the scales and hairs alone.

Qualitatively the ten adopted subgenera are not of exactly the same value. If

¹⁾ On a natural classification of the species of Dryopteris. Biologiske Arbejder, tilegnede Eug. Warming 3. Nov. 1911, pp. 73-85.

Dryopteris were to be divided into smaller genera, a very natural treatment indeed, the ten subgenera could not all be considered good, natural genera. Such are, in my opinion:

1. *Eudryopteris*.
2. *Stigmatopteris*.
3. *Ctenitis*.

These three are very different from the following and it is in reality highly unnatural to unite them all under a single genus. Intermediate forms with decomposed lamina seem to exist between *Stigmatopteris* and *Ctenitis*, but the two groups are as a whole very well defined.

4. *Lastrea*.

Under this fall as specialized groups *Glaphyopteris* and *Steiropteris*; the latter approaches:

5. *Cyclosorus*.

Leptogramma can scarcely be separated from *Cyclosorus* as genus.

6. *Goniopteris*.

A most natural genus, probably also including *Meniscium*.

In the treatment of the single species I have followed the principles, which I have explained in my paper on the group of *D. opposita*. Instead of giving detailed descriptions of all species I have chosen to describe certain central species under each narrower group, while I for all other species have confined myself to point out those characters, by which they differ from their nearest relatives. If the original diagnosis of a species does not mention essential characters, what very often is the case, I add the necessary notes.

My keys are elaborated so detailed as possible. Several species being exceedingly variable I have considered all forms known to me, and, therefore, you will find not rarely the same species occurring twice or even several times in the key.

Finally I shall make a few remarks on the geographical distribution of the species. As previously pointed out by me there are only some few species, which are common to Andes-West-Indian islands and South Brazil. Out of the 280 species the following 14 are common to the two regions:

Eudryopteris: *D. paleacea*, *D. patula*.

Stigmatopteris: *D. rotundata*.

Ctenitis: *D. submarginalis*.

Lastrea: *D. opposita*, *D. oligocarpa*, *D. pachyrachis*, *D. cheilanthoides*.

Glaphyopteris: *D. decussata*.

Cyclosorus: *D. patens*, *D. oligophylla*, *D. mollis*, *D. gongylodes*.

Goniopteris: *D. vivipara*.

The forms of these 14 species occurring in Andes-West-Indian islands and South Brazil respectively are, however, very rarely quite uniform, but, on the other hand, not so different that they can be separated from each other as species. Thus a pronounced difference between the fern-flora of the two regions is clearly seen, but there is also a distinct resemblance, indicating that the whole tropical American flora was in earlier periods not so specialized as in recent times. This resemblance is shown mainly thereby that each of the two regions is inhabited by a long series of species, which are, in the other region, superseded by other, but closely related species. This circumstance can mean that the floristic separation of the two regions took place so long ago, that the species, which were originally common to both regions, have had time to be segregated into several daughter-species, but, on the other hand, the segregation is till now not so far proceeded that separate genera or groups of specially characterized species could have been developed. In every case, such a specialization is in first beginning.

277 species out of the 280 are found in America only. Two, *D. mollis* and *D. gongylodes*, are cosmopolitic within the tropics and subtropics, and one, *D. eriocaulis* from Brazil, is not specifically distinct from the West-African *D. cirrhosa*. It is possible that some others of the South-Brazilian species occur also in tropical West-Africa.

Key to the Subgenera.

1. Lamina without true hairs consisting of a single row of one or some few cells; rhizome, stipe and rachises more or less clothed with scales, which can be very narrow, hairlike. Veins free, at least not regularly goniopteroid or meniscioid. Aërophore none.
2. Costulæ II—III or IV run out from the costa under a very acute angle. Sori with large, reniform, generally persistent indusia. Lamina not pellucido-punctate but often glandular beneath. Most species bipinnate-decompound with furcate veins.

Subgenus 1. *Endryopteris* (Species 1—11).

2. Costulæ run out from the costæ under an open angle. Sori exindusiate, or (in the section *Peltochlaena*) covered by deciduous, large, peltate indusia. Lamina pellucido-punctate by immersed glands. Most species pinnate-bipinnatifid with simple veins, which do not reach the margin. Apex of pinnæ sharply serrate to the very point.

Subgenus 2. *Stigmatopteris* (Species 12—28).

1. Lamina more or less hairy by true hairs of different structure. Costulæ run out from the costa under an open angle. Veins simple, rarely furcate, free or anastomosing (goniopteroid or meniscioid veins).
2. Hairs articulated, cylindrical, rufous, consisting of 2—4 short cells (unicellular hairs are found in *D. leptosora* and *D. platyloba* only); scales often many, never pubescent, their margins generally more or less dentate or fimbriate (not ciliated by hairs). Veins free, simple or forked, the basal ones not truly connivent to sinus, i. e. their apices do not meet at the sinus. Aërophore none; glandular hairs common. Lamina bipinnatifid-decompound, rarely reduced downwards.

Subgenus 3. *Ctenitis* (Species 29—53).

2. Hairs unicellular or pluricellular (in the latter case they are long, soft and thin, subulate), simple or branched. Scales entire or subentire, seldom many,

often ciliated by simple or branched hairs or pubescent throughout. Lamina pinnatifid-pinnate-bipinnatifid, rarely bipinnate.

3. Veins free, the basal ones run out to the margin above sinus, rarely furcate. Lamina bipinnatifid or bipinnate, often much reduced toward the base, the lower pinnæ being auriculiform or reduced to mere warts along the stipe. Aërophores frequent. Hairs nearly always simple.
4. No keel below the sinus. Sessile glands common.
5. Without aërophores at the base of the secondary veins (costulæ) beneath. Lamina nearly always reduced downwards. Veins not very close. Indusium often present but generally small.

Subgenus 4. *Lastrea* (Species 54—170).

5. Large aërophores at the base of the pinnæ beneath and smaller ones at the base of the secondary veins. Veins very numerous and close, simple. Indusium not seen.

Subgenus 5. *Glaphyopteris* (Species 171—176).

4. A carinate fold (keel) below the sinus extending toward costa, parallel to the secondary veins. Sessile glands none.

Subgenus 6. *Steiropteris* (Species 177—189).

3. Veins free or anastomosing, nearly always simple; the lower basal ones either run to the sinus, more or less connivent, or they are truly united in the leaf-tissue and send a common-branch to the sinus; in the two last subgenera often several pairs of veins are goniopteroid or all meniscioid. Lamina rarely reduced below.

4. Sori round or linear, not confluent. At least the uppermost veins not meniscioid.

5. Hairs simple, at least not branched with 2--6 branches on a short stalk, the scales not furnished with branched hairs.

6. Sori round, as a rule with reniform, persistent, setose indusia. Sporangia glabrous.

7. Lower basal veins run to the sinus, below which is a cartilaginous membrane, that in dried specimens becomes folded and forms a keel running from the sinus toward the costa. All veins free. Aërophores at the base of the pinna frequent; glands none.

Subgenus 6. *Steiropteris* (Species 177—189).

7. Lower 1—4 pairs of veins run to sinus or to a membrane, which scarcely forms a keel but is often protruded beyond the sinus as an apophysis, or the basal pair of veins are united in the leaf-tissue and send an excurrent veinlet to the sinus. Aërophore none; under-surface often glandular.

Subgenus 7. *Cyclosorus* (Species 190—202).

6. Sori oblong or linear, exindusiate; sporangia setose.

Subgenus 8. *Leptogramma* (Species 203—205).

5. Scales always with few or many branched hairs; branched, short-stalked hairs are found also on the rachis and, in some species, also on ribs and leaf-tissue. Veins free or anastomosing, goniopteroid or meniscioid. Lamina often proliferous.

Subgenus 9. **Goniopteris** (Species 206—267).

4. Sori confluent, exindusiate, all veins meniscioid.

Subgenus 10. **Meniscium** (Species 268—280).

Subgenus 1. *Eudryopteris* C. Chr., Biolog. Arbejder tilegnede Eug. Warming.
p. 76. 1911.

The typical species of this subgenus is our common *D. filix mas*, and most species of *Dryopteris* from the northern temperate region also belong hereto. In tropical America the subgenus is fairly well represented in Mexico, where a series of decompound forms occur, while South America is poor in species. As I have specimens of all but one known from tropical America, I give here a short review of all species of this subgenus, although I intended to include in the present work the pinnatifid-bipinnatifid species only¹).

Eudryopteris is a very natural group, or, I firmly believe, a distinct, well delimited genus, *Dryopteris* sens. strict. It is well characterized by venation, structure of scales and the total absence of common hairs. The venation is the best character. The veins are free and generally forked; costules (of II.—III. or IV. order) run out from the costa under very acute angles, at first nearly parallel to the costa. By this character the subgenus agrees with *Cystopteris* and I shall in connection herewith call attention to the fact, that certain Mexican forms, which no doubt belong to *Dryopteris*, not rarely have perfectly cystopteroid indusia. Also in pubescence *Eudryopteris* agrees very closely with *Cystopteris*. The scales are thin, entire or fimbriate, consisting of long, narrow often very irregular cells with small lumina and flexuose cell-walls, their margins often glandulose (*paleae cystopteroidae*). The scales of the costæ and veins, if present, are generally very narrow and hairlike, but they consist of 2—3 rows of cells and are therefore scales, not hairs. The leaf of several species is glandulose by short-stalked, capitate glands; the hairs of *D. Karwinskyana* are a peculiar kind of such glands. All species have a short oblique rhizome, which like the fasciculated stipites below is clothed with a dense mass of large, thin, mostly ovate scales. It will be seen from the above that *Eudryopteris* in most characters agrees with *Cystopteris*; the main difference between the two is found in the position and shape of the indusium, but even here we find intermediate forms, as already mentioned above. To me it is probable that *Eudryopteris* and *Cystopteris* are closely allied to each other and that it is unnatural to place them in two different tribes.

¹) A short extract of the review I published in American Fern Journal. 1: 93—97. 1911.

Eudryopteris is not nearly related to the other subgenera of *Dryopteris*. In the lack of simple hairs and in the structure of the scales it agrees with *Stigmatopteris*, but the venation and indusia are very different. Most species of *Eudryopteris* have large, persistent, reniform, often glandulose indusia, while *Stigmatopteris* includes a larger number of exindusiate species and some few species having large, circular, peltate indusia. Some species of *Eudryopteris* resemble closely certain species of *Ctenitis*, but they differ always by venation and lack of articulated, reddish hairs on the costæ above.

Eudryopteris includes, as delimited here, the genus *Dichasium* of A. BRAUN and FÉE and at least partly *Hypodematium* Kunze. Most of the species are large, with a bipinnatifid-decompound, lanceolate or deltoid lamina. In *D. Saffordii* the lamina is narrowed downwards about as in a species of § *Lastrea*. Commonly the leaf is fresh-green above and pale beneath, thick of texture and not rarely coriaceous; still thinly herbaceous forms are also to be found, f. inst. some of the *Cystopteris*-like small Mexican forms.

Key.

1. Lamina bipinnatifid; segments entire or toothed.
 2. Small. Lamina deltoid, thin. Indusium grey. 4. *D. mexicana* (Pr.) C. Chr.
 2. Larger. Lamina lanceolate.
 3. Lamina coriaceous, glandular throughout.
 1. *D. Saffordii*, C. Chr.
 3. Lamina membranous-chartaceous, eglandulose.
 4. Segments rectangular with parallel, entire edges and truncate toothed apex, the upper basal one rarely enlarged and free. Rachis densely chaffy by long glossy scales.
 3. *D. paleacea* (Sw.) C. Chr.
 4. Segments attenuate, toothed throughout, the upper basal one generally enlarged and free. Rachis less scaly.
 2. *D. filix mas* (L.) Schott.
 1. Lamina bipinnate-quadripinnatifid.
 2. Indusium flat or absent.
 3. Small species; lamina 10—30 cm. long, pinnæ rarely more than 8 cm. long often 3—5 cm. only. Both surfaces more or less glandular.
 4. Lamina deltoid or ovate-deltoid (the basal pinnæ not conspicuously shorter).
 5. Indusium large, reniform, grey. Surfaces finely glandulose. Most upper pinnæ decurrent; secondary segments or pinnules broad, ovate or oblong, subentire or toothed, the teeth close, rather obtuse 4. *D. mexicana* (Pr.) C. Chr.
 5. Indusium small, brown or absent. Both surfaces densely glandulose. Pinnæ not decurrent; secondary segments or pinnules narrow, linear, sharply and remotely toothed.
 5. *D. glandulifera* (Liebm.) C. Chr.

4. Lamina lanceolate (basal pinnæ shorter).
5. Sori not close to the secondary veins; stipe reddish; segments unequal-sided, more deeply cut on the anterior side. Underside paler..... 7. *D. indecora* (Liebm.) C. Chr.
5. Sori in two rows close to the secondary vein; stipe stramineous; segments equal-sided, deeply cut on both sides. Both surfaces concolorous 6. *D. cinnamomea* (Cav.) C. Chr.
3. Large species. Lamina often 50 cm. or more long, pinnæ 10—25 cm.
4. Indusium persistent 8. *D. patula* (Sw.) Und.
4. Indusium absent 9. *D. ulvensis*, Hieron.
2. Indusium very large, coriaceous, hemispherical, perfectly concealing the sporangia.
3. Surfaces not mealy-glandular. Leaf with age coriaceous, the segments broad 10. *D. Maxoni*, Und. et C. Chr.
3. Both surfaces densely mealy-glandular. Leaf thinner, finely cut. 11. *D. Karwinskyana* (Mett.) O. Ktze.

1. *Dryopteris Saffordii* C. Chr. Amer. Fern Journal 1: 94, 1911.

Type from Peru, mountains back of Lima, Arroyo Railway, leg. WILLIAM E. SAFFORD III¹⁸⁹², nr. 994 (W).

Eudryopteris rhizomate ignoto. Stipitibus brevibus, 7—8 cm. longis, stramineis, squamis ovatis, pallide-luteis, tenuibus sparse instructis. Lamina lanceolata, 30 cm. longa, medio 10 cm. lata, versus basin attenuata, papyracea vel coriacea, infra pallida, bipinnatifida, rachi straminea superne sulcata minute glandulosa et sparse squamosa. Pinnis oppositis, sessilibus, attenuatis, acutis, mediis 5 cm. longis, 1—1½ cm. latis, inferioribus sensim reductis, magis remotis, infimis 2 cm. longis, utrinque minute glandulosis, ad alam 1 mm latam pinnatifidis. Laciniis obliquis, triangularibus, acutis, leviter serratis, basali acroscopica longiore et latiore, subpinnatifida. Venis immersis, indistinctis, saepe furcatis, 3—4 jugis. Soris majusculis, medialibus; indusiis reniformibus, persistentibus, rufis, minute glandulosis.

A near relative of *D. filix mas*, different by the less cut lamina, which is nearly coriaceous and densely viscid throughout by minute, glossy glands.

2. *Dryopteris filix mas* (L.) Schott.

Our common European Male Fern is widely distributed in North America, from New Foundland to Greenland and Alaska, south to Michigan and South Dakota and along the Rocky Mountains and the Pacific coast to Arizona and Mexico. It varies in North America quite as much as in Europe, and some of the European forms are also to be found in America. I shall not here try to name and describe the different forms, but only mention that some of the forms of the Rocky Mts. differ not a little from the type. Thus a peculiarly looking variety is found in Washington (SUKSDORF nr. 1230, W) and a large-growing tripinnatifid variety, which

closely resembles the var. *affine* (Fisch. and Mey.), occurs in the Rocky Mts. (Montana, Idaho), where rather typical forms also are met with (Utah, New Mexico, Arizona, California).

In Mexico, where the following species is frequent, true *D. filix mas* is rare; the specimens referred here belong to a form, which approaches the following species, but it does not differ from *D. filix mas* in the main characters. This Mexican form varies considerably in the degree of cutting, from merely bipinnatifid to deeply tripinnatifid. Very large specimens collected by SCHAFFNER (B, without exact locality) are not unlike the subspecies *elongata* (Sw.) and belong perhaps to a distinct variety.

State of Mexico: Nevada de Toluca, J. N. ROSE and PAINTER nr. 7944 (W).

State of Oaxaca: Sierra de San Felipe, CHARLES L. SMITH nr. 2076 (W).

State of Puebla: Ixtaccihuatl, 2610 m., F. NICOLAS nr. 5550 (RB).

3. *Dryopteris paleacea* (Sw.) C. Chr. Amer. Fern Journal 1: 94, 1911.

Syn.: *Aspidium paleaceum* Sw. Syn. 52, 1806; Fourn. Mex. pl. 1: 92, 1872.

Aspidium parallelogrammum Kze. Linnaea 13: 146, 1839.

Dichasium parallelogrammum Fée, Gen. 302 tab. 23 B fig. 1. 1850—52.

Aspidium resp. *Nephrodium filix mas* var. *parallelogrammum* resp. *paleaceum* auctt. plur.

Aspidium crinitum Mart. et Gal. Mém. Ac. Brux. 15: 66 tab. 17, fig. 2, 1842.

Aspidium Pseudo-Filix-mas Fée, 8 mém. 103, 1857.

Aspidium chrysocarpon Feé, 8 mém. 103, 1857.

Type from PERU (LAGASCA). Not seen.

It is beyond question that *A. paleaceum* Sw. is identical with *A. parallelogrammum* Kze. from Mexico, leg. KARWINSKY (B!). The specific name has been attributed to DON, who under that name described a similar form from Himalaya (Prod. Fl. Nepal. 4, 1825), which is *A. patentissimum* Wall.

In treating this widely distributed American fern as a species, which by most authors is referred to *D. filix mas* as a variety, I have several reasons for doing so. It is very often identified with Central-Asiatic forms of *D. filix mas*, especially with the varieties *patentissima* and *fibrillosa* Clarke; I have, however, never seen Asiatic forms, which entirely agree with the American one. *D. paleacea* is a rather uniform species, which constantly differs from *D. filix mas* by the following characters: 1) Stipe and rachis very densely clothed with 1—2 cm. long, narrow, blackish or reddish, glossy, divaricating scales, 2) lamina always bipinnatifid, never bipinnate, 3) pinnæ not widened at base, long acuminate and sharply serrated to the very apex, the lower ones, which are somewhat reduced, not subdeltoid, 4) segments with parallel, entire or very faintly toothed edges, the apex truncate with 3—5 short teeth, the basal ones not enlarged and lobed (still often with an interne auricle), rarely free, 5) texture chartaceous or coriaceous. — The indusium is large and often biscuteloid (as FÉE termed it), which is especially the case in the an-

dine specimens. It is not fixed exactly in the centre; the sinus extends over the middle and reaches nearly a shallow sinus on the opposite side of the indusium, thus the indusium appears to be cleft into two halves. FÉE's figure (Gen. Fil. tab. 23 B) gives a good illustration of an extreme form. A. BRAUN founded a new section *Dichasium* on this character (Flora 1841: 710), and FÉE considered it being of generic value. I can not at all agree in this; in young specimens the indusium are of the common *filix-mas* type, but growing older it is cleft, before falling, in the said manner. Still it seems to be a good specific character.

D. paleacea varies in size and colour of the scales but is otherwise fairly constant. It is apparently not rare in the Andes from Mexico to Peru and Bolivia; it is found at high mountains of Jamaica and Haïti, and again in Southern Brazil. The Hawaiian plant referred hereto, *Lastrea truncata* Brack., is scarcely identical.

I have seen the following specimens:

San Domingo: infra Valle nuevo, 1900 m., EGGERS nr. 2306 (B).

Jamaica: Summit of Blue Mountain Peak, MAXON nr. 1411 (W = UNDERWOOD nr. 2541); W. HARRIS nr. 7486 (W); HART nr. 670 (W).

Mexico: Valle de Mexico, SCHAFFNER nr. 85 (B), HARTWEG nr. 570 (B), UHDE nr. 180 et 236 (B) — Forêt de San Nicolas, BOURGEAU nr. 1041 (B, H, W) — Pico de Orizaba, LIEBMANN nr. 2393 (H) — Oaxaca, GALEOTTI nr. 6348 (B = *A. crinitum* M. et G.) — EHRENBURG (B = *A. parallelogrammum* Kze. — Hidalgo, Trinidad, PRINGLE nr. 8750 (B, H, W) — Ixtaccihuatl, C. A. PURPUS nr. 10 (W) — Oaxaca, Cerro de San Felipe, C. GONZATTI and V. GONZALEZ nr. 530 (W) — Chiapas, San Cristobal, COLLINS and DOYLE nr. 136 (W).

Guatemala: Volcan de Agua, 33—3600 nr., MAXON and HAY nr. 3712 (W) — Depart. Quiché, San Miguel Uspantán, 6000', HEYDE et LUX, ed. DONN. SMITH nr. 3244 (W).

Costa Rica: Forêts de Barba, PITTIER nr. 1933 (B, W) — Cerro de Las Vueltas, PITTIER nr. 10604 (W) — El Paramo, PITTIER nr. 10621 (W) — forêts de Copey, 2670 m, TONDUZ nr. 11800 (W) — J. J. COOPER (W).

Panama, MAXON nr. 5275 (W).

Colombia, HARTWEG nr. 1512 (B) — Boqueron-Bogotá, STÜBEL nr. 457 (B) — Bogotá, KARSTEN (B).

Venezuela, prov. de Pamplona, FUNCK et SCHLIM nr. 1372 (RB, L).

Peru, DOMBEY (B), RUIZ (B, W), Agapata, LECHLER nr. 2020 (B).

Bolivia, MANDON nr. 71 (B) — Tunari Mts., O. KUNTZE (B) — M. BANG nr. 1784 (B, W).

Argentina: Sierra de Tucuman, P. G. LORENTZ (B).

Brazil: Rio, GLAZIOU nr. 4432, 7333 (B, H) — E. BRUNET nr. 53 (B) — Serra de Itatiaia, P. DUSÉN nr. 428 (Rg, W) — Theresopolis, TH. DE MOURA nr. 60 (B) — Minas Geraes, Caldas, Regnell nr. III 1451 b (B, Rg, W); MOSÉN nr. 2180 (H, Rg).

4. *Dryopteris mexicana* (Pr.) C. Chr. comb. nov. — Fig. 1.

Syn.: *Nephrodium mexicanum* Pr. Rel. Haenk. 1: 38, 1825 (non auctt.).

Aspidium flaccidum Fourn. Bull. Soc. Fr. 27: 328, 1880.

Nephrodium Fournieri Bak. Ann. Bot. 5: 317, 1891.

Dryopteris Fournieri C. Chr. Ind. 266, 1905.

The type-specimen of this species was collected in Mexico by HAENKE. I have seen in Herb. PRESL a single leaf labelled: "*Nephrodium mexicanum* Presl, rel. haenk. I: In Mexico (Hænke)", which I regard as typical. It does not agree with the forms generally referred to *D. patula* v. *mexicana*, but it is not essentially

from *A. flaccidum* Fourn., based on SCHAFFNER nr. 85, 1877, San Luis Potosi (herb. Paris!). PRINGLE nr. 11773 from Eslaba, Federal District, is probably the same. The species is perhaps not different from *D. cinnamomea*, with which it agrees by its small size, thin texture and large, grey indusia; it differs mainly by its deltoid lamina, lanceolate scales of the stipe and the less cut lamina; most pinnæ are decurrent in a narrow wing to the rachis, and only the basal pinnules of the lower pinnæ are free.

5. *Dryopteris glandulifera* (Liebm.)

C. Chr. Ind. 267. 1905.

Syn. *Polypodium glanduliferum* Liebm.

Vid. Selsk. Skr. V. 1: 206. 1849.

Mexico, inter Comaltepec et Trapiche de la Concepcion, Dep. Oajaca, leg. LIEBMANN nr. 2395 (H, type). — Morelos, Cuernavaca, H. Ross nr. 287 (CC).

Perhaps a small form of the large *D. patula* var. *Rossii*. In the specimens leg. Ross indusia are clearly seen. The species resembles very much *D. hirta* and allied species of *Ctenitis*, but the venation and pubescence is that of *Eudryopteris*.

6. *Dryopteris cinnamomea* (Cav.) C. Chr. Amer. Fern Journal 1: 95. 1911.

Syn. *Tectaria cinamomea* Cav. Descr. pl. 252. 1802.

Aspidium athyrioides Mart. et Gal. Mém. Ac. Brux. 15: 67 tab. 18. 1842.

Dryopteris athyrioides O. Ktze. Rev. 2: 811. 1891; C. Chr. Ind. 253.

Athyrium sphaerocarpon Fée, Gen. 186. 1850—52; Fourn. Mex. pl. 1: 101.

Nephrodium sphaerocarpum Hk. spec. 4: 139. 1862; Hk. Bak. Syn. 277.

Aspidium agatolepis Fée, 8. mém. 106. 1857.

Aspidium mexicanum Kze. Linn. 13: 147 (ex descr.).

?*Polystichum cystopteroides* Nees, Linn. 19: 685.

Type from Mexico: Chalma, leg. Luis Née (fragment in Herb. Sw. S.!).

In Ark. för Bot. 9¹¹: 43 I have said that *T. cinnamomea* Cav. probably is the same as *Asp. athyrioides* M. et G.; now I have no doubt that the two names are synonyms. I add here the original description of *T. cinamomea* Cav.

Tectaria cinamomea foliis tripinnatis glabris, foliolis linearibus acutis: fructificationibus solitariis. †

La planta es lampiña; los peciolos delgados, amarillentos; las hojas de color de canela tres veces pinadas, de un pie de largo con mas de medio de ancho, sin contar los peciolos; las hojuelas son estrechas, de dos á quatro líneas de largo,



Fig. 1. *D. mexicana* (Pr.) C. Chr. Part of the type-specimen of *A. flaccidum* Fourn. $\times \frac{1}{3}$, and pinna $\times 1\frac{1}{2}$.

terminadas en una puntita. Las fructificaciones solitarias en dos líneas inmediatas al nervio principal de las pinulas secundarias. Los puntos son redondos como igualmente sus tegumentos. Don Luis Née la encontró junto á Chalma, pueblo de la Nueva-España.

The best character of the species, the position of the sori, was clearly described by CAVANILLES. FÉE renamed the species of MARTENS and GALEOTTI, referring it to *Athyrium*, because the indusium is sometimes subhippocrepiform. *A. agatolepis* is the same according to specimens so named by FOURNIER (BOURGEAU nr. 1164). *D. cinnamomea* is exclusively a Mexican species and is very variable. The typical form can shortly be described thus: Rhizome oblique, like stipe densely clothed with large (up to 2 cm. long), ovate-acuminate, concolorous, red-yellow, thin and glossy, entire scales. Stipe stramineous, 10—15 cm. long. Lamina lanceolate or subdeltoid, firmly herbaceous or membranous, without hairs but more or less finely glandular on both sides by minute, glossy, whitish glands, especially on the costæ beneath, fully tripinnate, about 20 cm. long by 10 cm. broad; the pale rachis and the green costæ beneath with some few small scales. Pinnæ opposite, the lower somewhat shortened, the longest 5—8 cm. long, 2—3 cm. broad, deltoid-oblong, mostly equal-sided or somewhat reduced on the lower side at base. Pinnules oblong or subdeltoid, the lower ones free, the upper decurrent; free pinnules equal-sided, fully pinnate at base; segments III entire or toothed, acute, ascending. Veins indistinct, forked in the segments. Sori nearly always solitary in the segment close to the secondary midrib, furnished with a large, flat, greyish, finely glandulose indusium, which is as a rule truly reniform but sometimes subhippocrepiform. In mature leaves the indusium appears often to be lateral (*cystopteroid*), because the sporangia come out at one side only, viz. that opposite the sinus.

This typical form seems to be well-marked and not easily to confound with the much larger and less finely cut *D. patula mexicana*; the best characters are the position of the sori and shape of indusium, the equal-sided free, lower pinnules and the finely glandular surfaces. — The following specimens seen are typical:

Mexico: State of Mexico, near Toluca, J. N. ROSE and PAINTER nr. 6807 (W) — Durango, Tejamén, EDW. PALMER nr. 506 (W) — Hidalgo, Sierra de Pachuca, PRINGLE nr. 13815 (H, W) and J. N. ROSE and PAINTER m. 6745 (W) — Morelos, Sierra de Tepoxlan, ROSE and PAINTER nr. 7257 (W) — Michoacan, hills of Patzuaró, PRINGLE nr. 3362 (W) — Chihuahua, Arroyo Ancho, Sierra Madre, PRINGLE nr. 1445 (W), 1712 (S) — Barranca de Santa Fée, BOURGEAU nr. 1164 (H) — Montezuma près Cuantepec, BOURGEAU nr. 1165 (H).

7. *Dryopteris indecora* (Liebm.) C. Chr. Ind. 272. 1905.

Syn. *Lastrea indecora* Liebm. Vid. Selsk. Skr. V. 1: 272. 1849.

Aspidium indecorum Fourn. Mex. pl. 1: 97. 1872.

Aspidium inquinans Fée, 8 mém. 106. 1857 (teste FOURNIER).

Mexico, Yavesia, Dep. Oajaca, 7—7500 ft., leg. LIEBMANN nr. 2417 (H!)

A very doubtful species based on three imperfect leaves. It may be a form of *D. cinnamomea*. HIERONYMUS describes (Hedwigia 46: 346. 1907) a var. *obtus*a from Colombia, STÜBEL nr. 110, which I have not seen.

8. *Dryopteris patula* (Sw.) Und. Our nat. Ferns ed. IV. 117. 1893.

Syn. *Aspidium patulum* Sw. Vet. Akad. Handl. 1817: 64.

Lastrea polystichoides Pr. Epim. 38. 1849!

Aspidium imbecile Kze.; Ettingsh. Farnkr. tab. 107, fig. 10, tab. 109 f. 1. 1865!

Type from Brazil: Minas Geraes, leg. FREYREISS (S!).

To this species I refer a long series of forms, several of which are no doubt good species. The Brazilian type is thin-leaved, finely glandular throughout, the under-surface not much paler than the upper one; pinnæ about equal-sided, the pinnules more deeply lobed on the anterior side, the segments broad, shallowly toothed; indusium rather small, reddish, densely glandulose.

Minas Geraes: Caldas, MOSÉN nr. 2183 (H) — Serra da Piedade, WARMING nr. 871 (H). — GLAZIOU nr. 15759 (H).

Best agreeing with the type is a plant from Jamaica, HART nr. 153 (W). It is smaller and densely glandular throughout.

In Mexico and the Andes southwards a number of forms occur, which are described under several names but which I have failed to distinguish with certainty. The nomenclature is very confuse. Most authors refer the different forms to a single species, *D. patula* or *Nephr. mexicanum*; as shown above I believe that PRESL's species is the same as *Asp. flaccidum* Fourn. and probably specifically different from *D. patula*. I shall mention some of the more characteristic forms.

1. Very near the Brazilian type, but larger; pinnæ up to 25 cm. long, like the pinnules unequal-sided at base; surfaces slightly glandulose, the indusia large, flat, often peltate, finely glandular. Here I refer:

Aspidium paupertinum Kze. Linn. 18: 345. 1844, and

Aspidium apertum Fée, 8 mém. 106. 1857;

Dryopteris aperta C. Chr. Ind. 252. 1905.

Mexico: Oajaca, inter Comaltepec et Trapiche de la Concepcion, LIEBMANN nr. 2404 (H) — Orizaba, Izhuatlantillo, BOURGEAU nr. 2360 (H).

Aspidium leptorachis Kze. Linn. 18: 346. 1844 is evidently a less cut form of the same. The short, obtuse pinnules are deeply serrated, scarcely pinnatifid. *Aspidium roseum* Fourn. Mex. pl. 1: 97. 1872 is probably also belonging here.

2. var. *chaerophylloides* (Moritz) Bak. Syn. 276.

Like the preceding form, but surfaces and indusia not glandulose. It varies in size and cutting.

Guatemala: Alta Vera Paz, Cubilquitz, v. TUECKHEIM ed. DONN. SMITH nr. 8643, 8645 (W) — Jalapa, Laguna de Ayarza, HEYDE et LUX ed. DONN. SMITH nr. 4081 (W) — Alta Vera Paz, Coban, v. TUECKHEIM, ed. DONN. SMITH nr. 703 (W); v. TUECKHEIM nr. II 1370 (W) — Dept. Santa Rosa, Sambrerito, HEYDE et LUX ed. DONN. SMITH nr. 6408 (W).

Nicaragua: Ile d'Omotépé, LÉVY nr. 207 (H).

Costa Rica: Bords du Rio Torres à San José, TONDUZ nr. 9791 (W) — Haie près San José, TONDUZ 9780 (W) — Haies à la Verbena près Alajuelita, TONDUZ nr. 8790 (W) — Alajuela, ALFARO ed. DONN. SMITH nr. 6031 (W) — Alto del Sacatal, PITTIER nr. 10547 (W) — San José, BIOLLEY nr. 102 (C, CC, H, W) — Tablazo, BIOLLEY nr. 74 (C, CC, W).

Colombia: Santa Marta, H. H. SMITH nr. 1038 (Rg).
Ecuador: Niebly, SODIRO (RB).

var. **Rossii**, n. var.

Under the name *D. Rossii* I have described a Mexican fern as a new species after specimens collected in Morelos: Cuernavaca, Santa Maria by H. Ross nr. 279 (Herb. Munich, CC); my description is till now not published. Comparing my specimen with several other specimens from Mexico I fear that my proposed new species cannot be distinguished specifically from the Mexican form of *D. patula*, *Aspid. apertum* Fée, although the typical form looks very distinct. It differs by the deeply cut lamina, which is distinctly glandular on both surfaces, by its very unequal-sided pinnæ and pinnules, and especially by its narrow, toothed segments; under surface pale or even glaucous; stipe and rachis reddish, clothed with narrow scales. Indusium small, reddish. — The lamina is generally deltoid in outline, firm. — This is nearly exactly a *D. glandulifera* on an enlarged scale, — the basal pinnæ are up to 15 cm. long — and it also comes near to certain forms of *D. cinnamomea*, from which it differs by its larger, deltoid lamina, pale-green underside, position of sori and unequal-sided pinnules.

Mexico: State of Mexico, Hac. de la Encarnacion, ROSE and PAINTER nr. 8477 (W); near Tultenango, ROSE and PAINTER nr. 7861 (W) — Puebla, Ixtacihuatl, 2160 m. F. NICOLAS nr. 5549 (RB) — Jalisco, near Guadalajara, EDW. PALMER nr. 288 (W), ROSE and PAINTER nr. 7319 (W); Rio Blanco, EDW. PALMER nr. 149 (W); between Colotlan and Bolanos, J. N. ROSE nr. 2837 (W) — Morelos, Cuernavaca, H. ROSS nr. 279 (CC) — Michoacan, Mt. Patamban, E. W. NELSON nr. 6590 (W); Patzcuaro, PRINGLE nr. 3362 (W); near Morelia in different localities, F. ARSÈNE (RB).

Arizona: Huachuca Mts., Conservatory Canyon, Lemmon (W) — (A small form near *D. glandulifera*).

9. *Dryopteris ulvensis* Hieron. Hedwigia 46: 346 tab. 7 fig. 18. 1907.

Type from Ecuador, in valle Pastaza, STÜBEL nr. 848 (B).

This I have not seen. It is certainly closely allied to *D. patula* but is described as exindusiate.

10. *Dryopteris Maxoni* Underw. et C. Chr. Amer. Fern Journal 1: 96. 1911.

Type from Mexico: Morelos canyons above Cuernavaca, 5500 ft., leg. PRINGLE nr. 6190 (W! also S). —

Other specimens seen: type locality, J. N. ROSE and PAINTER nr. 6920 (W); Tlatzalan bei Tepotzlan, Morelos, C. et E. SELER nr. 4518 (B) — Michoacan: Coru Station, PRINGLE nr. 8846 (S, W) — Jalisco: near Etzatlan, ROSE and PAINTER nr. 7597 (W) — Very numerous specimens were collected in different localities near Morelia by J. ARSÈNE (RB, CC).

Eudryopteris rhizomate crasso, obliquo, sparse squamoso. Stipitibus crassis, stramineis, 3—4 dcm. longis, teretibus, squamis integris, ovatis, siccis, pallidis (vel in centro atrofusis) sparse instructis. Lamina deltoidea vel deltoideo-ovata vel ovato-lanceolata, 4—6 dcm. longa, chartacea vel coriacea, pallide viridi, pilis omnino destituta, ad rachin costasque infra squamis pallidis, parvis onusta, bipinnata-

tripinnatifida. Pinnis suboppositis, parum erectis, basalibus subovatis vix abbreviatis, medialibus a basi ad apicem acutum attenuatis, omnibus breviter petiolatis equilateralibus, maximis ad 15 cm. longis, Pinnulis remotis, equilateralibus, inferioribus liberis basi contractis, medialibus decurrentibus superioribus confluentibus, fere ad costulam pinnatifidis. Laciniis oblongis integris vel ad apicem truncatum saepe dentatis. Venis furcatis. Soris 1—2 in lacinia; indusiis magnis, coriaceis, hemisphaericis, reniformibus, minute glandulosis.

A very distinct species, well-marked by the very large, coriaceous, hemispherical indusia, which perfectly conceal the sporangia. By this character it resembles *D. Karwinskyana*, which is quite different in habit and pubescence. — Very characteristic are the thick, straw-coloured, somewhat fleshy stem and rachis.

11. *Dryopteris Karwinskyana* (Mett.) O. Ktze. Rev. 2: 813. 1891;
C. Chr. Ind. 272.

Syn: *Aspidium Karwinskyanum* Mett. Aspid. nr. 141. 1858.

Lastrea mexicana Liebm. Vid. Selsk. Skr. V. 1: 272. 1849.

A most distinct species with a finely cut, thin lamina, which throughout is covered by unicellular, cylindrical, hairlike glands. The sori are hemispherical and conceal the sporangia.

Mexico: Comaltepec, Dept. Oajaca, LIEBMANN nr. 2396 (H).

Guatemala: Dept. Santa Rosa, Rio de los Esclavos, HEYDE et LUX ed. DONN. SMITH nr. 4426; Mataquesuintla, HEYDE et LUX ed. DONN. SMITH nr. 6405 (W).

Nicaragua: Ile d'Omotépé, LÉVY nr. 132 (H).

Unknown species of uncertain position.

Dryopteris Wolfii Hieron. Hedwigia 46: 344 tab. 7 fig. 17. 1907.

Type from Ecuador, STÜBEL nr. 770, 919 et 998 (B).

By the author compared to *D. marginalis*, but the description and the figure do not agree perfectly with the characters of *Eudryopteris*.

Aspidium Huberi Christ, Hedwigia 45: 192. 1906. -

Type from Amazonas, Alto-Purus, HUBER nr. 4514.

Subgenus 2. *Stigmatopteris* C. Chr. Bot. Tids. 29: 292. 1909 (as genus).

In 1909 (loc. cit.) I proposed to refer to a proper genus, *Stigmatopteris*, a number of tropical American ferns, which show several characters by which they differ from species of *Dryopteris*: the lack of simple hairs, the serrated apex of the pinnæ, the pellucido-punctate lamina and the peculiar venation, the veins

terminating in the leaf-tissue in a clavate apex, not reaching the margin and, in some species, irregularly anastomosing. In lack of simple hairs and in the structure of the scales *Stigmatopteris* resembles *Eudryopteris*, but the venation and the exindusiate sori are quite different. I am fully convinced that *Stigmatopteris* is a very natural genus (perhaps even not a member of the *Dryopterideae*), but as the same can be said of several other subgenera of *Dryopteris* dealt with in this paper it is best for the sake of uniformity of treatment to place *Stigmatopteris* here among the other subgenera.

Since the publication of my paper on *Stigmatopteris* I have seen some other species, which belong here. It is very interesting to note that some of these species are indusiate, but the indusia are not reniform as in *Dryopteris* but peltate, large and circular, fixed at the centre, as a rule coriaceous and glabrous, the edges often upcurved. Species with such indusia have commonly been referred to *Polystichum*, but there exists no species of true *Polystichum* having a similar habit. In general habit the species here in question do not differ from *Stigmatopteris rotundata* and allied species, and as to all other characters they fully agree, *inter alia* also in the variable venation and in the presence of immersed glands. FÉE has named provisionally a plant from Guiana *Peltochlaena nephrodiiformis*, which no doubt belongs here. I can not say definitely what his species may be, but I use here his name *Peltochlaena* for a section of *Stigmatopteris* including those species having peltate indusia. It must be remembered that the very large indusia soon fall, and a fertile leaf with all indusia fallen could as well be referred to *Eustigmatopteris*.

The genus (subgenus) thus can be divided into two groups:

1. *Eustigmatopteris*. Sori exindusiate. Leaves as a rule thin.
2. *Peltochlaena* (Fée). Sori furnished with large, circular, peltate indusia.

Leaves generally papyraceous to coriaceous.

In my former paper I have exclusively dealt with the species of the former group and I have there mentioned and illustrated 12 species. Since then I have examined the type-specimens of some species described by HOOKER and BAKER, and the number of species known to me is now 17, which are all enumerated below and arranged in a key. For descriptions, synonymy and distribution of the 12 species I refer to my former paper.

Key.

1. Sori exindusiate. Lamina of most species herbaceous or membranous. *Eustigmatopteris*.
2. Pinnæ nearly entire, serrated or lobed not more than $\frac{1}{2}$ of the way down to the midrib, seldom a little more.
3. Sori round, not confluent; veins free.
4. Only the uppermost pinnæ with a shortly decurrent base.

5. Pinnæ linear, slightly lobed; lobes broader than long, oblique, as a rule faintly crenulate or obtusely dentate. Veins 2—5.
6. Veins short. Pinnæ not or shortly auricled at the upper base.
 7. Pinnæ 25 cm. long by $2\frac{1}{2}$ cm. broad; veins 4—5-jugate, rather patent; sori medial 12. *S. rotundata* (Willd.) C. Chr.
 7. Pinnæ 12—18 cm. long by $1\frac{3}{4}$ —2 cm. broad; veins 2—4-jugate, more erect, often very short and distant; sori near the midrib 13. *S. Carrii* (Bak.) C. Chr.
 6. Veins longer, ascending. Pinnæ with a large, broad auricle.
 27. *D. sancti gabrieli* (Hk.) C. Chr.
5. Pinnæ linear or lanceolate, lobed $\frac{1}{3}$ — $\frac{1}{2}$ of the way down; lobes generally longer than broad. Veins 5—7 jugate.
6. Pinnæ lanceolate, 12—18 cm. long, 3 cm. broad, with close, entire, falcate lobes. Veins 6—7-jugate. Light-green 14. *S. tijuccana* (Raddi) C. Chr.
6. Pinnæ linear or lanceolate with more or less repand or dentate lobes. Colour dark-green.
 7. Pinnæ narrow-linear; lobes patent with open sinuses between. Veins about 5 to a side. Sori inframedial. Immersed glands very large. Brazil.
 15. *S. prionites* (Kze.) C. Chr.
 7. Pinnæ linear-lanceolate, broadest at the middle; lobes falcate, rather close. Andine species.
 8. Sori medial without paraphyses. Lobes falcate, more or less dentate 16. *S. nephrodioides* (Kl.) C. Chr.
 8. Sori inframedial with articulated paraphyses. Lobes narrow, falcate, repand 17. *S. Michaëlis* (Bak.) C. Chr.
4. Most pinnæ with a long decurrent base, those of the upper third of the leaf connected by a broad wing to the rachis.
5. Pinnæ deeply lobed with narrow, repand lobes.
 17. *S. Michaëlis* (Bak.) C. Chr.
5. Pinnæ subentire, serrate or slightly lobed.
 6. Pinnæ approximate, linear, serrate, about 15 cm. long, $2\frac{1}{2}$ cm. broad 18. *S. longicaudata* (Liebm.) C. Chr.
 6. Pinnæ distant, about 15 cm. long, 3—4 cm. broad, subentire or irregularly crenate or slightly lobed; veins frequently anastomosing 19. *S. alloëoptera* (Kze.) C. Chr.
3. Sori confluent; venation meniscioid 20. *S. opaca* (Bak.) C. Chr.
2. Pinnæ incised to a narrow wing to the costa, or lamina bipinnate-tripinnatifid. Veins often furcate.
3. Lamina bipinnatifid with toothed or lobed segments.
 4. Sori without indusium-like, inferior scale.
 5. Segments generally toothed throughout, separated by broad, open sinuses, somewhat contracted above the widened base, 5—7 mm. broad. Lower basal segment of most pinnæ considerably reduced. Brazil 21. *S. caudata* (Raddi) C. Chr.
 5. Segments close, up to 1 cm. broad, sharply toothed at their apex, the edges entire or faintly crenate. Lower basal segment scarcely abbreviated. Andes 24. *S. pellucido-punctata* C. Chr.
 4. Most sori with an inferior, indusium-like scale; Andine and West Indian species.

5. Lower basal segment of most pinnæ abbreviated. Segments patent or oblique, not contracted above their base; costæ and costulæ beneath rather scaly 22. *S. ichtiosma* (Sod.) C. Chr.
5. Basal segments both much enlarged; sterile leaf tripinnatifid, fertile one with narrow, crenate segments; costæ beneath sparsely scaly 23. *S. contracta* (Christ) C. Chr.
3. Lamina bipinnate with lobed pinnulæ 25. *S. prasina* (Bak.) C. Chr.
1. Sori furnished with large, circular, peltate indusia, which are deciduous and often not found **Peltochlaena.**
2. Veins anastomosing about as in *Goniopteris*. Pinnæ irregularly cut, subentire or deeply lobed 26. *D. varians* (Fée) O. Ktze.
2. Veins free. Pinnæ regularly serrulate or lobed.
3. Lamina membranous. Pinnæ 15—20 cm. long, broadly and shallowly serrate with a broad auricle at the upper base. 27. *D. sancti-gabrieli* (Hk.) C. Chr.
3. Lamina coriaceous. Pinnæ 10—12 cm. long, lobed about halfway down to the midrib, scarcely auricled 28. *D. subobliquata* (Hook.) O. Ktze.

Group 1. **Eustigmatopteris** (species 12—25).

12. **Stigmatopteris rotundata** (Willd.) C. Chr. Bot. Tids. 29: 297 fig. 2. 1909.
 Area: Lesser Antilles from Montserrat to Trinidad. South Brazil.
Nephrodium Imrayanum Hk. spec. fil. 4: 86. 1862 from Dominica, IMRAY (Kew!)
 is exactly this species. The specimen is absolutely exindusiate.

13. **St. Carrii** (Bak.) C. Chr. l. c. 298 fig. 3.

Area: Brazil.

The type-specimen from Rio, leg. CARR (Kew!) shows that I have rightly understood this species, which is perhaps not specifically different from *S. rotundata*.

Aspidium brachyneuron Fée, Cr. vasc. Brés. I: 133. 1869 from Bahia, BLANCHET, is probably this species.

14. **St. tijuccana** (Raddi) C. Chr. l. c. 298, fig. 4.

Area: Brazil, Rio-S. Paulo.

15. **St. prionites** (Kze.) C. Chr. l. c. 298, fig. 5, 6.

Area: Brazil. — *Phegopteris Ulei* Christ, Bull. L'Herb. Boiss. II. 2: 634. 1902; *Dryopteris Ulei* C. Chr. Ind. 299 from S. Catharina, ULE nr. 70, is probably a form of this species.

16. **St. nephrodioides** (Kl.) C. Chr. l. c. 299, fig. 8.

Area: Venezuela Costa Rica.

17. **St. Michaëlis** (Bak.) C. Chr. l. c. 300, fig. 9.

Area: Ecuador-Colombia.

Additional synonyms: *Polypodium sylvicolum* Bak. Journ. Bot. 1881: 205.
Dryopteris sylvicola C. Chr. Ind. 297, 1905.

The type-specimen of *P. sylvicolum* Bak. from Colombia, Prov. Antioquia, KALBREYER nr. 1807 (Kew!) is nearly exactly identical with the type from Ecuador. It is not so deeply cut, but the segments are similarly falcate and broadly crenate-repand, and the sporangia are intermixed with brown articulate paraphyses just as in the type.

18. **St. longicaudata** (Liebm.) C. Chr. l. c. 300, fig. 10.

Dryopteris longicaudata Maxon, Contr. U. S. Nat. Herb. 13: 18, 1909.

Area: Mexico along the Andes to Peru and Bolivia.

19. **St. alloëoptera** (Kze.) C. Chr. l. c. 300, fig. 11.

Area: Costa Rica—Peru.

Additional synonyms: *Polypodium oligophlebium* Bak. Syn. Fil. 506, 1874.
Dryopteris paucinervata C. Chr. Ind. 283, 1905.
Polypodium heterophlebium Bak. Journ. Bot. 1884: 363.
Dryopteris heterophlebia C. Chr. Ind. 270, 1905.
 ? *Polypodium coalescens* Bak. Journ. Bot. 1877: 164.
Dryopteris coalescens C. Chr. Ind. 258, 1905.

The specimens on which BAKER founded his two first named species are rather typical *S. alloëoptera*. *P. heterophlebium* from Costa Rica, leg. HARRISON (Kew!) has narrower pinnæ and approaches *S. longicaudata*, from which it differs by its shorter and broader pinnæ with many veins anastomosing. — *P. oligophlebium* from Peru, Tarapoto, Cerro de Guayrapurima, SPRUCE nr. 4653 (Kew!) is that form with few but very broad pinnæ, which I have figured. In general habit and cutting it is nearly identical with *S. opaca*. The long stipe is clothed with several large, brown scales.

P. coalescens Bak. was founded on a most wonderful mixture of different things collected by SODIRO in Ecuador (Kew!). The type-specimen (a single sheet) consists of three young sterile plants and one fertile pinna. The latter is no doubt a pinna of *S. alloëoptera*, and two of the young plants are probably belonging to the same species. The third plant is entirely different and does not belong to *Stigmatopteris*; what it may be, I dare not decide. I should not hesitate to make *P. coalescens* Bak. a synonym of *S. alloëoptera*, had SODIRO (Cr. vasc. quit. 302) not described the species more fully, and his description does not agree perfectly with the specimen seen; thus he describes the stipe and rachis as pulverulento-pubescent, both surfaces pubescent and lamina impari-pinnate with the terminal pinna stalked, broader and shorter than the 5—7 pairs of lateral ones. These characters do not correspond to any species of *Stigmatopteris*. On the other hand his description very well agrees with the fertile pinna seen in other characters: size and shape of the pinnæ, venation and position of the sori.

20. *Stigmatopteris opaca* (Bak.) C. Chr. comb. nov.

Syn. *Meniscium opacum* Baker, Journ. Bot. 1877: 166; Sod. Cr. vase. quit. 390.

Dryopteris Christii C. Chr. Index 257, 1905.

Type from Ecuador, leg. SODIRO (authentic specimens in C!).

Closely allied to *St. alloëoptera*, resembling it in size and its broad partly decurrent pinnæ, differing by its thick texture, meniscioid venation and confluent sori. By these characters it resembles very much species of *Meniscium*, to which genus or subgenus it was hitherto referred. A minute examination of a specimen will, however, clearly show great differences between it and species of *Meniscium*, and further that it is in nearly all characters a true *Stigmatopteris*. The lamina is distinctly pellucido-punctate, simple hairs are wanting but rachis and costæ beneath are sparsely clothed with small, reddish scales similar to those of other species of *Stigmatopteris*. The veins anastomose richly but not so regularly as in species of *Meniscium*, and the outer veinlets do not reach the margin but end in a clavate apex in the parenchyma; in *Meniscium* the outer veinlets always are united with the thickened margin. The sori are large and generally confluent as in *Meniscium*.

21. *St. caudata* (Raddi) C. Chr. l. c. 302 fig. 12.

Area: South Brazil.

22. *St. ichtiosma* (Sod.) C. Chr. l. c. 302 fig. 13.

Area: Ecuador-Colombia. Cuba, Jamaica.

Additional synonyms: *Polypodium dentatum* Bak. Ann. Bot. 5: 456, 1891.

Dryopteris longipetiolata C. Chr. Index 275, 1905.

BAKER's proposed new species, founded on a part of a leaf gathered in Ecuador by SODIRO (Kew!), is absolutely identical with SODIRO's species. SODIRO remarks (Cr. vase. quit. 642) that BAKER's species does not exist in his herbarium, and he believes that it may be his *Nephrodium crinitum* β *glaucescens*, l. c. 251. It must be granted that it is impossible from BAKER's description to see, what his species may be; he compares it with *P. decussatum* and *P. rude* (!), which species *St. ichtiosma* is not at all related to. *N. crinitum* Sod. with its variety are forms of *D. submarginalis*.

23. *St. contracta* (Christ) C. Chr. l. c. 304, fig. 14.

Area: Costa Rica.

24. *St. pellucido-punctata* C. Chr. l. c. 304, fig. 15.

Type from Peru, Tarapoto, Mt. Guayrapurima, SPRUCE nr. 4720 (Kew!) A very large species, but bipinnatifid only. Stipe 1 cm. thick, rather scaly; pinnæ up to 45 cm. long, 7 cm. broad.

25. **Stigmatopteris prasina** (Bak.) C. Chr. comb. nov.

Syn. *Polypodium punctatum* Spruce, Hk. spec. fil. 4: 262, 1862.

Polypodium prasinum Bak. Syn. Fil. 312, 1867; ?Sod. Cr. vasc. quit. 292.

Dryopteris prasina C. Chr. Index 285, 1905.

Type from Peru, Tarapoto, Mt. Guayrapurima, leg. SPRUCE nr. 4719 (Kew!, also RB). I consider SPRUCE nr. 4719 the type-number of this species, to which the first name of the plant, *Pol. punctatum* Spruce, was given by SPRUCE; his nr. 5714 from Chimborazo (Kew, RB) belong to *St. ichtiosma*. *S. prasina* is fully bipinnate; the lower pinnulæ are free but broadly adnate to costa, lobed nearly half of the way down to the midrib into square or rectangular, entire or faintly serrated lobes; veins pinnate in the lobes, 3–4 to a side, simple. Sori about medial, without an indusium-like scale as in *S. ichtiosma*. — Plant very large, up to 2 m. high; stipe at base with many squarrose scales, which are up to 3 cm. long. Rachis, costæ and costulæ beneath richly fibrillose by reddish scales. Pinnæ 35 cm. or more long, lanceolate, short-stalked, 6 cm. broad at the middle; pinnulæ about 4 cm. long, 1 cm. broad. Lamina thin, grass-green, paler beneath, very distinctly pellucidopunctate.

A specimen from Ecuador, Andes of Quito, SODIRO (B) may be specifically different. It is *Pol. prasinum* Sod. Cr. vasc. quit. 292. It is much larger than the Peruvian type and less scaly. The pinnæ are distinctly stalked, the pinnules sessile, 10 cm. long by 2 cm. broad, incised $\frac{3}{4}$ of the way down to the midrib into obtuse, serrulate segments. Veins in about 5 pairs to a segment, as a rule forked near the middle, the anterior branch very short and soriferous. In texture, colour and immersed glands the specimen agrees with *S. prasina*.

Group 2. **Peltochlaena**. (Species 26–28)¹.

26. **Dryopteris varians** (Fée) O. Ktze. Rev. 2: 814, 1891; C. Chr. Ind. 299.

Fig. 2 c.

Syn. *Nephrodium varians* Fée, 11 mém. 88 tab. 24 fig. 2, 1866.

Type from Trinidad, leg. GERMAIN (Herb. Mus. Paris!). — Bot. Gard. Herb. Trin. nr. 1225 (W); Aripo-Savannah „im schattigen Walde an Bäume kletternd“, OTHMER nr. 116 (Herb. Monac., C, CC). British Guiana, Essequibo, APPUN nr. 27 (B, RB).

This is a most distinct species. Leaves somewhat dimorphous, the sterile ones being considerably broader than the fertile ones, about 10 cm. long by 2 cm. wide, short-stalked, truncate at the upper base but scarcely auricled, cuneate at the lower one, the margins subentire or repand or broadly serrulate, always sharply serrate towards the apex. Fertile leaves with pinnæ scarcely more than 1 cm. broad, very variable in cutting, being from almost entire (the serrated apex ex-

¹) The species of this group I have not named *Stigmatopteris* but use the names of Index Filicum.

cepted) to deeply and irregularly pinnatifid. The variations are often to be found in the same leaf. — Lamina with a subdistinct terminal pinna, herbaceous-membranous, perfectly glabrous and naked, dark-green. Veins 3—4 to a side, anastomosing about as in certain species of *Goniopteris*. Sori small, furnished with a deciduous, membranous, glabrous, reddish, wrinkled, peltate indusium. — The Guiana-specimens differ from the type from Trinidad by more numerous pinnæ, which have a longer petiole and are more equally cuneate at the base on both sides.

D. varians is in some characters, f. inst. the wide-creeping rhizome and perhaps in the lack of immersed glands, rather different from the general type of *Stigmatopteris*. In the Trinidad-specimens I have not found immersed glands, while they are seen indistinctly in the specimens from Essequibo. As to other characters it does not differ materially from *Stigmatopteris*, and I think it best to place the species here. From all other groups of *Dryopteris* it is widely different.

27. *Dryopteris sancti-gabrielii* (Hook.) O. Ktze. Rev. 2: 813, 1891;
C. Chr. Index 290. — Fig. 2 a.

Syn. *Polypodium Sancti-Gabrielii* Hook. spec. fil. 4: 233, 1862.

Nephrodium Sancti-Gabrielii Bak. Flor. bras. 1²: 469, 1870.

Aspidium Imrayanum Fée, Cr. vasc. Brés. 1: 133, 1869.

Type from Amazonas, São Gabriel, SPRUCE nr. 2153 (Kew!).

Venezuela, Merida, ENGEL nr. 231 (B).

Trinidad, FENDLER nr. 97 (W).

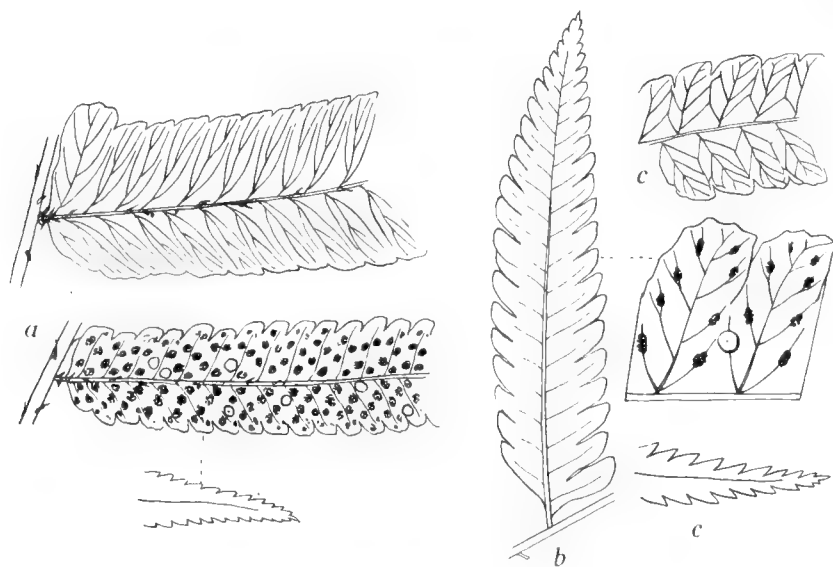


Fig. 2. a. *D. sancti-gabrielii* (Hook.) O. Ktze. Basal part of sterile and fertile pinna and apex of a pinna, $\times \frac{4}{5}$. — b. Pinna of *D. subobliquata* (Hook.) O. Ktze., $\times \frac{4}{5}$, and two segments $\times 1\frac{1}{2}$. — c. Middle part and apex of a pinna of *D. varians* (Fée) O. Ktze., showing venation, about nat. size.

This is a large species with subdimorphous leaves, the sterile ones being much larger than the fertile. Sterile lamina up to 50 cm. long, 40 cm. broad, truncate at base, upwards gradually narrowed to the pinnatifid apex. Pinnæ 15–20-jugate, short-stalked, 15–20 cm. long, 3 cm. broad, the lower base bluntly rounded, the upper one truncate and with a broad, obtuse auricle; margins broadly and shallowly serrulate, towards the submucronate apex more distinctly serrate. Texture membranous to firmly papyraceous, colour fresh-green, paler beneath. Stipe, rachis and costæ beneath sparsely clothed with reddish or blackish small scales. Veins in groups of 3–4, upcurved, not reaching the margin, free. — Fertile lamina similar, but the pinnæ only 10–12 cm. long by $1\frac{1}{2}$ – $1\frac{3}{4}$ cm. broad. Sori in the lower half of the vein, in most specimens apparently exindusiate, but in a single of FENDLER's Trinidad-specimens the sori are covered by large, circular, peltate, glabrous, coriaceous indusia; a single indusium is found in the type-specimen.

D. sancti-gabrielii can with certainty be referred to *Stigmatopteris*, differing from the general type by its peltate indusia.

28. *Dryopteris subobliquata* (Hook.) O. Ktze. Rev. 2: 813, 1891;

C. Chr. Index 296. — Fig. 2 b.

Syn. *Polypodium subobliquatum* Hk. spec. 4: 240, 1862.

Nephrodium subobliquatum Bak. Syn. 261, 1867.

Type from Surinam, HOSTMANN nr. 15 (Kew!).

Closely related to the preceding species, but easily recognizable by several characters. Leaf generally coriaceous, entirely glabrous, lamina lanceolate, about 40 cm. long with 10–15 pairs of rather remote, subopposite, short-stalked pinnæ, which are all alike, 12 cm. long by $2\frac{1}{4}$ –3 cm. broad, the lower base shortly cuneate, the upper one truncate, scarcely auricled, the margins lobed about half-way down to the midrib into oblong, obtuse, entire segments; apex of pinnæ mucronate. Veins simple, free, 3–4 to a side, short, not reaching the margin. Sori small, furnished with an early falling, large, coriaceous, glabrous, peltate indusium.

I call here this distinct species *D. subobliquata*, although I have no doubt that it is identical with *Aspidium guianense* Kl., Linnaea 20: 364, 1847; *Peltochlaena nephrodiiformis* Fée, Gen. 289 is probably the same. I have, however, not seen the original specimen of *A. guianense*, which commonly is referred as a subspecies to *Polystichum abbreviatum* (Schrad.), and I shall not here create a new combination of name, mainly because the position of the species in *Dryopteris* is very questionable. It is possible that it may be a free-veined form of *Polystichum abbreviatum*, still I think it can be regarded as a distinct species forming a connecting link between *D. sancti-gabrielii* and *Polyst. abbreviatum*. Its systematic position will naturally be in *Stigmatopteris*. Some of the specimens seen have distinct immersed glands.

Specimens seen:

French Guiana, LEPRIEUR nr. 42 (C, W); RICH (CC); — Demerara, JENMAN (B, C, W).

Colombia, Cauca, LEHMANN nr. 3802 (B).

Brazil, Pará, Aproya, J. HUBER nr. 753 (C). — Rio, GLAZIOU nr. 12373 (B, H).

Polystichum abbreviatum (Schrad.) Pr. (for synonymy see Index Fil. 575, the subspecies excluded) should probably also be placed in this group. It is very closely related to *D. subobliquata*, mainly different by the anastomosing veins. It is known from Brazil and also recorded from Guiana and Ecuador. Besides several specimens from Brazil I have seen one from Colombia, Córdoba, Dagua Valley, 30—100 m., H. PITTIER nr. 529 (W).

Subgenus 3. *Ctenitis* C. Chr.

Biolog. Arbejder tilegnede Eug. Warming, p. 77. 1911.

A natural subgenus including several species all having an erect or oblique rhizome, which like the bases of stipes is clothed at the apex with a dense mass of scales; these are as a rule very large, toothed at the margin or, more rarely, subentire, their apex very long and hairlike; in some species, f. inst. *D. strigilosa* Dav., the scales are long and narrow, dark-brown and rigid. — Lamina bipinnate-decompound, as a rule brownish green when dry, membranous or herbaceous, rarely coriaceous, not or a little narrowed downwards, more often deltoid and then the basiscop pinnulæ of the basal pinnæ are more or less enlarged. Rachis and costæ beneath always furnished with many or few scales, which are differently shaped in the different species (see fig. 3). Rarely they are quite entire, generally the margins are toothed, or, as in *D. deflexa*, long ciliated; always the apex is long and hairlike, and some few long ciliæ are to be found at the cordate base; in some species the base of the scale is bullate or subbullate. The cells are as a rule large and regular, most often rectangular, the inner cell-walls often very thickened and dark, while the outer ones are always thin and colourless (clathrate scales). The structure and shape of the scales, especially of those of the costæ beneath are important specific characters. — Besides the scales the species are furnished with short, articulate, reddish hairs, which in all species are to be found on the costæ above; they consist of 2—4 short, cylindrical cells separated by dark, thick walls (see fig. 3,1). If hairs occur elsewhere, on leaf tissue, veins or margins, they are generally of the same structure. Short, unicellular hairs I have met with only in two species, *D. platyloba* and *D. leptosora*. Some species are glanduloso-pubescent by hairs of a similar structure, but often shorter end ending at a gland. Sessile glands as in species of *Lastrea* scarcely occur. Aërophore none. — Veins simple or, in some species, forked, never anastomosing, the basal ones reaching the margin above sinus, or, in the less-divided species as *D. pedicellata*, running

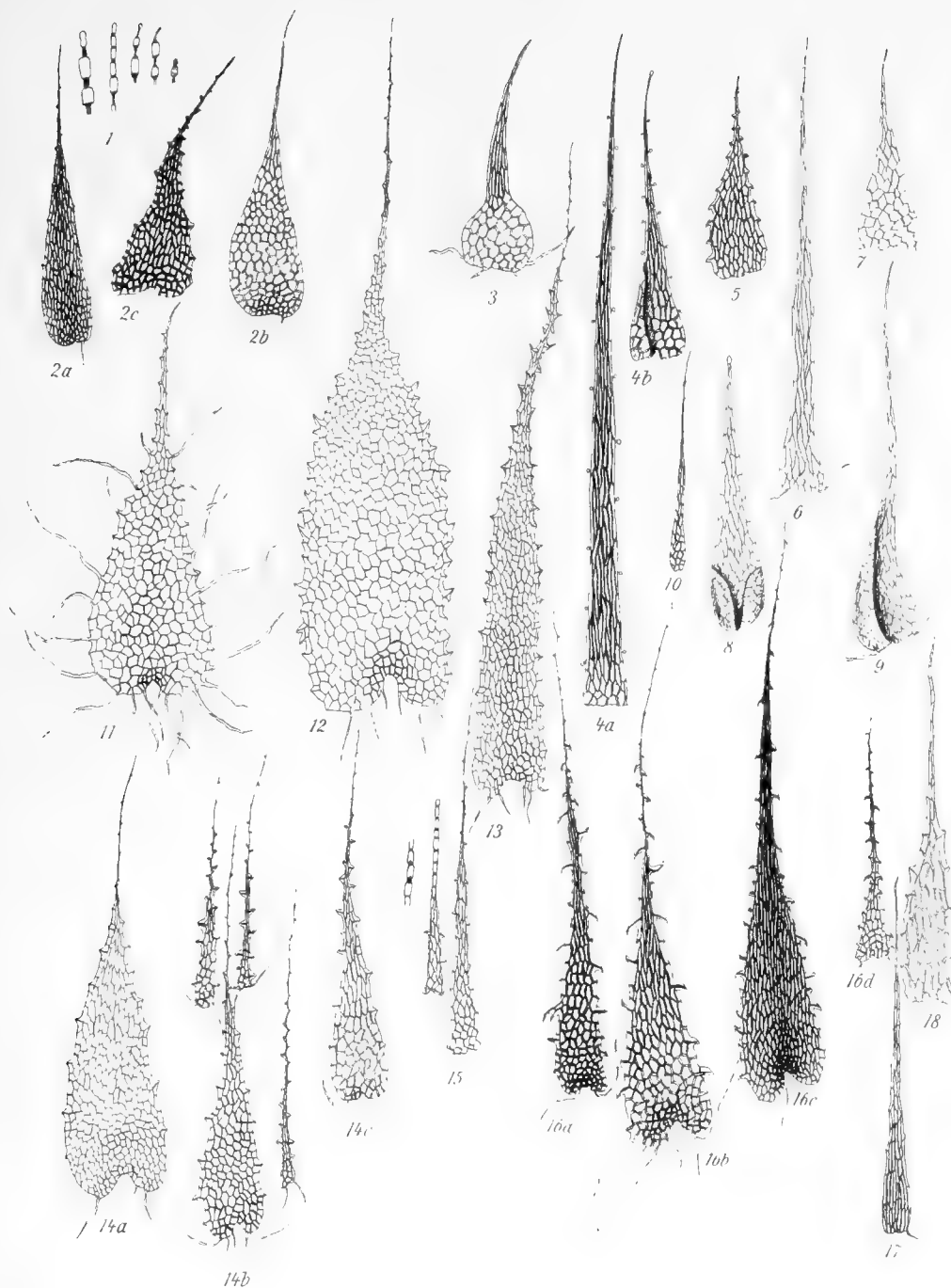


Fig. 3. Hairs and scales of different species of § *Clenitis*. The scales are all, 4a only excepted, from the costæ of the pinnae beneath. — 1. Hairs of the common type. 2. *D. falciculata*: a. type; b. *Pol. ciliatum* Pr.; c. var. *paranaensis*. 3. *D. nigrovenia*. 4. *D. strigilosa*, a. from stipe. 5. *D. pedicellata*. 6. *D. refulgens*. 7. *D. refulgens* var. *peruviana*. 8. *D. vellea*. 9. *D. cirrhosa* var. *eriocaulis*. 10. *D. alsophilacea*. 11. *D. deflexa*. 12. *D. fenestralis*. 13. *D. Anniesii*. 14. *D. submarginalis*: a. type (*D. Sellowii*), b. 4 scales of var. *tenuifolia*, c. var. *caripensis*. 15. *D. Karstenii*, 3 forms. 16. *D. etenitis*: a. type. b. c. *f. amaurolepis*. b. lateral scale and c. frontal scale from the same costa, d. *f. isabellina*. 17. *D. honesta*. 18. *D. yungensis*.

towards the sinus but not meeting there and thus not being truly connivent; costulæ run out from the costæ under open angles. Sori indusiate or not, the indusium rarely large and persistent. Sporangia glabrous.

Ctenitis is a very natural group, by the peculiar pubescence different from all other groups of the genus, and I have no doubt that the differences are of generic value. The species vary being from bipinnate to decompose, most of them with characteristic obtuse segments, rather small to large. All intermediates between bipinnate and large decompose species are found, such are f. inst. *D. strigilosa* and *D. Hemsleyana*. In America the bipinnate species are numerous in Southern Brazil, while only one, *D. vellea*, occurs in the West Indies, where several decompose species (*D. hirta*, *D. ampla*, *D. nemorosa* etc.) are to be found. In the Old World's tropics the subgenus is fairly well represented. Among them is the African *D. cirrhosa* (Schum.) O. Ktze., which I cannot distinguish specifically from the Brazilian *D. eriocaulis* (Fée) O. Ktze., a new example of the well-known relation between the floras of South Brazil and tropical West Africa.

Ctenitis is not nearly related to the other subgenera of *Dryopteris* and in my opinion it is even not congeneric with them. On the other hand it is intimately related to the genus *Psomiocarpa* Presl., recently restored by CHRIST¹), who referred to that genus two West Indian species, *Ps. aspidioides* (Griseb.) and *Ps. Maxoni* Christ. I know the first named of these species, which in habit, texture, colour and pubescence is perfectly agreeing with *Ctenitis*; by its contracted fertile frond it differs, however, so much that it naturally must be placed in a separate genus, which can be regarded as a derivate of *Ctenitis*. *Psomiocarpa* was previously united with *Polybotrya*, which genus is more closely related to *Polystichum*.

Ctenitis is further related to the genus *Aspidium* (*Tectaria*) at least to the majority of the species referred to that genus. The main difference is found in the venation, *Ctenitis* having free veins, *Aspidium* reticulated veins; in pubescence, texture and colour most species of the two genera fully agree. Species intermediate between the two genera exist, f. inst. *Dryopteris Brauniana* (Karst.) O. Ktze. from Colombia (KARSTEN; LINDIG nr. 261 (B); LEHMANN (RB)). It has free veins but is in habit rather a species of *Aspidium*, to which genus I refer it, wherefore it is not included in this paper.

In the following I deal with 24 species. It is, in this subgenus, impossible to draw a sharp line between the bipinnatifid and the more divided species. Some of the species vary considerably in cutting being bipinnatifid, bipinnate or tri-pinnatifid. I have chosen to include in this monograph those species only of which I have had type-specimens and which in the most developed form are scarcely more than bipinnate.

The 24 species can be divided into smaller groups, which are connected by intermediate characters:

¹ On *Psomiocarpa*, a neglected genus of ferns. Smithsonian Misc. Collections 56 nr. 23. 1911.

1) The group of *D. submarginalis* including the species 29—40, closely allied species with differently shaped but never bullate scales; stipe at base as a rule with a dense mass of large scales. Veins reaching the margin.

2) The groups of *D. Lindeni*, including the species 44—46, smaller Central-American species with very characteristic bullate scales; base of stipe scaly but not with a dense tuft of large scales; veins often furcate, not reaching the margin. The species 41—42, *D. nigrovenia* and *D. Tonduzii* are intermediate between the first and the second group.

3) The group of *D. platyloba*; hereto the species 48—52, characterized by broad, obtuse segments with normally furcate veins, which do not reach the margin. The lamina is often finely pellucido-punctate; by this character as well as by venation the species of this group resemble *Stigmatopteris*, from which they differ by the non-serrated apex of the pinnæ and by the presence of articulated hairs. In some of the species the articulated hairs are subulate, and very short, unicellular hairs are also found. To this group mainly decompose species belong; *D. refulgens* is an intermediate between the first and the third group.

Key.

1. Veins normally simple, still often forked in the basiscop enlarged segments of the lower pinnæ.
2. Segments normally entire or faintly crenate or serrate; the basal pair of pinnæ scarcely enlarged.
3. Pinnæ incised $\frac{1}{3}$ — $\frac{2}{3}$ of the way to the costa; the lower 2—5 veins run to the sinus.
4. Pinnæ incised $\frac{1}{3}$ — $\frac{1}{2}$ into broad, obtuse segments; the lower 4—5 veins (2 pairs) run to the sinus.
5. Pinnæ 10—15 cm long by $2\frac{1}{2}$ —3 cm broad, abruptly and shortly acuminate; scales few, dark, toothed. Brazilian species
 29. *D. pedicellata* (Christ) C. Chr.
5. Pinnæ 20 cm long by 4 cm broad, long acuminate; scales numerous, entire, reddish. Andes and Guiana... 30. *D. refulgens* (Kl.) C. Chr.
4. Pinnæ incised $\frac{1}{2}$ — $\frac{2}{3}$; 2 basal veins run to the sinus. Scales very few and small, dark. Lamina generally with a distinct terminal pinna.
 31. *D. alsophilacea* (Kze.) O. Ktze.
3. Pinnæ incised to a narrow wing to the costa or larger ones of some species fully pinnate in their basal part.
4. Scales of rachis dark and rigid, not thin, hairlike and not broad and reddish. Indusium persistent.

5. Scales of costæ entire or slightly toothed. Lamina often glanduloso-pubescent. Veins 5—10-jugate.
6. Scales ovate, not bullate; sori supramedial. Brazil and Guiana.
32. *D. falciculata* (Raddi) O. Ktze.
6. Scales of costæ from a broad, generally bullate base abruptly narrowed into a short, narrow apex. Central American and Andine species.
7. Both surfaces with scattered short, thick hairs, not glanduloso-pubescent. Larger species, generally dark-green.
41. *D. nigrovenia* (Christ) C. Chr.
7. Both surfaces (costa excepted) glabrous or glanduloso-pubescent.
8. Veins reaching the margin. Lamina thin, light-green.
9. Lamina glabrous; scales of stipe and rachis rather thin, from a broad base narrowed into a hair-like apex.
42. *D. Tonduzii* (Christ) C. Chr.
9. Lamina densely glanduloso-pubescent throughout; scales of stipe and rachis very narrow, black-brown.
43. *D. strigilosa* Dav.
8. Veins not reaching the margin. Scales bullate. Lamina firm, dark-green.
9. Pinnæ pinnatifid only; segments shallowly serrate.
44. *D. Lindeni* (Kuhn) O. Ktze.
9. Pinnæ pinnate in the lower part; segments or pinnules entire, ligulate or deeply lobed with 2—3 obtuse, oblique lobes 46. *D. lanceolata* (Bak.) O. Ktze.
5. Scales of costæ lanceolate with claw-shaped teeth. Veins 10—15-jugate. Lamina not glanduloso-pubescent, dark-green.
6. Sori medial or supramedial. Veins 10—11-jugate.
32. *D. falciculata* var. *paranaensis* C. Chr.
6. Sori inframedial. Veins 12—15-jugate 33. *D. ctenitis* (Link) O. Ktze.
4. Scales of rachis reddish or yellowish, thin, hairlike or broad. Lamina mostly light-green.
5. Scales of rachis not many, hairlike. Sori medial or supramedial.
6. Costæ beneath without articulated hairs. Veins 10—15-jugate. Indusium often absent. . . . 34. *D. submarginalis* (L. & F.) C. Chr.
6. Costæ beneath with articulated hairs among the scales. Veins 8-jugate. Indusium distinct . . . 35. *D. Karstenii* (A. Br.) C. Chr.
5. Rachis densely scaly by broad, reddish or dirty-brown scales. Sori medial or inframedial; indusium in most species persistent.
6. Scales toothed or fringed by long cilies, their base flat.
7. Scales with long cilies along the margins. Veins 6—8-jugate.
36. *D. deflexa* (Klf.) C. Chr.

7. Scales toothed, without long cilies. Veins 12--18-jugate.
 8. Scales of rachis adpressed. Sori near the midvein; indusium persistent 37. *D. Anniesii* Ros.
 8. Scales of rachis broad, loose. Sori medial without distinct indusium 38. *D. fenestralis* n. sp.
6. Scales entire with a pocket-shaped base.
 7. Veins 8--10-jugate. Upperside glabrous. West-Indian species. 39. *D. vellea* (Willd.) O. Ktze.
 7. Veins 10--12-jugate. Both surfaces hairy. Brazil. 40. *D. cirrhosa* var. *eriocaulis* (Fée)
2. Segments normally distinctly serrated or lobed, at least those of the basal pinnæ. Central American and Andine species.
3. Basal pinnæ not considerably enlarged. Indusium distinct.
 4. Veins reaching the margin.
 5. Surfaces with scattered, short, thick hairs, not glanduloso-pubescent. Larger species, generally dark-green 41. *D. nigrovenia* (Christ) C. Chr.
 5. Both surfaces (costæ excepted) glabrous or glanduloso-pubescent. Lamina thin, light-green.
 6. Surfaces not glanduloso-pubescent, scales of stipe and rachis rather thin, from a broad base narrowed into a hair-like apex, those of rachis bullate 42. *D. Tonduzii* (Christ) C. Chr.
 6. Surfaces throughout glanduloso-pubescent. Scales of stipe and rachis very narrow, black-brown 43. *D. strigilosa* Dav.
 4. Veins not reaching the margin. Scales of costæ bullate. Surfaces glabrous. Lamina firm, dark-green.
 5. Pinnæ pinnatifid only; segments serrated.
 6. Veins 2--3-jugate. Stipe and rachis slender. 44. *D. Lindenii* (Kuhn) O. Ktze.
 6. Veins 5-jugate. Stipe strong 45. *D. Salvini* (Bak.) O. Ktze.
 5. Pinnæ pinnate in the lower half; segments or pinnules ligulate. entire or deeply lobed with 2--3 obtuse lobes. 46. *D. lanceolata* (Bak.) C. Chr.
3. Basal pinnæ the largest, bipinnatifid; upper ones smaller, pinnatifid with entire segments.
 4. Small. Veins not reaching the margin. Sori very large, persistent. 46. *D. lanceolata* (Bak.) O. Ktze. var.
 4. Larger. Veins reaching the margin. Exindusiate. 47. *D. Hemsleyana* (Bak.) C. Chr.
1. Veins normally furcate.
2. Hairs of the common type, short, rarely unicellular. Veins generally not reaching the margin.
3. Segments narrow, uniformly serrate. Scales bullate. Indusium large.

4. Veins 2-3-jugate. Stipe and rachis slender. 44. *D. Lindenii* (Kuhn) O. Ktze.
4. Veins 5-jugate. Stipe strong 45. *D. Salvini* (Bak.) O. Ktze.
3. Segments broad, obtuse, entire or obtusely lobed. Scales not bullate. Exindusiate.
4. Costæ beneath without short, unicellular hairs.
5. Andine species.
6. Pinnæ scarcely incised to costa, the segments entire (the lower basal one excepted).
7. Sori placed at the base of the anterior branch of the forked vein. Scales narrow, dark 48. *D. honesta* (Kze.) C. Chr.
7. Sori above the middle of the anterior branch of the forked vein. Scales broader, yellowish. 49. *D. yungensis* Christ et Ros.
6. Lower pinnæ pinnate with lobed pinnulæ.
52. *D. biserialis* (Bak.) C. Chr.
5. Brazilian species *D. grandis* * *macroptera* (Klf.) C. Chr.
4. Costæ beneath hairy by short, unicellular hairs.
5. Lamina fresh-green. Basal segments reduced. 50. *D. leptosora* C. Chr.
5. Lamina brownish-green. Basal segments not reduced, the lower one of the upper pinnæ decurrent and adnate to rachis.
51. *D. platyloba* (Bak.) C. Chr.
2. Lamina very hairy by long, flexuose, soft, pluricellular hairs.
53. *D. hirsuto-setosa* Hieron.

29. *Dryopteris pedicellata* (Christ) C. Chr. comb. nov. — Fig. 4.

Syn. *Aspidium pedicellatum* Christ, Denkschr. Akad. Wien **79**: 14. (1906). 1907.
Dryopteris indecora Rosenst. Hedwigia **46**: 117. 1906 (non C. Chr. Ind.
 272. 1905).

Type from Brazil: São Paulo, prope Santos, leg. WETTSTEIN & SCHIFFNER.
 (Original in Herb. Mus. Wien! and C!).

A Brazilian representative of the Andine *D. refulgens* and together with that species different from all other species of the subgenus by the pinnæ being incised scarcely more than halfway to the costa. From *D. refulgens* it differs 1) by the dark-green leaf, which is 20—25 cm broad; pinnæ 2½—3 cm broad, 2) pinnæ from a truncate base oblong-linear with parallel edges suddenly narrowed into a short, acute apex; segments close, 4 mm broad, truncate and often emarginate at the apex; veins 7—9, the lower 2—4 running to the sinus, all reaching the margin, or often the lower-most pair do not reach the sinus but end in the leaf-tissue below the sinus, 3) rachis and costæ beneath sparingly scaly; scales small, dark reddish-brown, mostly ovate with a short apex and shortly toothed margins; cell-walls

dark and thick, the lumina small; those of the costæ beneath adpressed, not patent (fig. 3,5); 4) stipe at the base with some dark-brown, rather long scales. — Sori small, medial, exindusiate. — From *D. alsophilacea*, to which it is closely related, it differs by its more numerous and close pinnæ, its larger scales and by the short, pinnatifid apex of the lamina.

D. pedicellata is confined to Southern Brazil, where it seems to be a rather common species, I have seen the following additional specimens (for others see ROSENSTOCK Hedwigia 46: 117).

São Paulo: Santos, MOSÉN nr. 3089 (Rg.).

Sta. Catharina, Itapocú, SCHWACKE nr. 12935 (C). — São Francisco, ULE nr. 75 (RB).

Paraná, Serro do Mar, Ypiranga, DUSÉN nr. 6753 (S).

30. *Dryopteris refulgens* (Kl.) C. Chr. Ind. 288. 1905. — Fig. 5a.

Syn. *Polypodium refulgens* Kl. msc.; Hk. Bak. Syn. 307.

Phegopteris refulgens Mett. Ann. sc. nat. V. 2: 240. 1864.

Phegopteris tricholepis Fée, Cr. vasc. Brés. 1: 98 tab. 32, fig. 1. 1869.

Type from Guiana, SCHOMBURGK nr. 1128 et 1183 (B!).

A very distinct species, in cutting very near *D. pedicellata*, in size, colour and scales near *D. eriocaulis*. Stipe below with a dense mass of long, reddish scales. Lamina yellowish-green, firmly herbaceous or membranous, 50–60 cm long; pinnæ about 20 cm long by 4 cm broad, acuminate, incised about halfway to the costa into 7 mm broad, bluntly rounded or subacute, faintly toothed or entire segments. Rachis and costæ beneath scaly by red, narrow, entire scales with thin cell-walls, much like those of *D. eriocaulis* but their base flat; scales of costæ patent, mostly at right angles to the costa (fig. 3,6). Margins and veins beneath finely pubescent, the underside sometimes glandular by red, sessile glands. Veins 9–11-jugate, the lower 3–5 running to the sinus, the lowermost much ascending, the upper ones not reaching the margin. Sori medial or slightly inframedial, exindusiate.

Judging from the structure of the scales this very beautiful species is more related to *D. eriocaulis* than to *D. pedicellata*, and stands to that species as *D. pedicellata* to *D. ctenitis*.

Guiana: SCHOMBURGK nr. 1128 and 1183 (B). — Demerara, JENMAN (B).

Colombia: Magdalena, LINDIG nr. 382 (B).

Panamá: SCHOTT nr. 7 (B, W).

Brasília: Rio Negro, São Gabriel, SPRUCE nr. 2100 (B = *Ph. tricholepis* Fée).

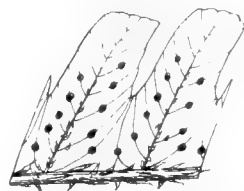
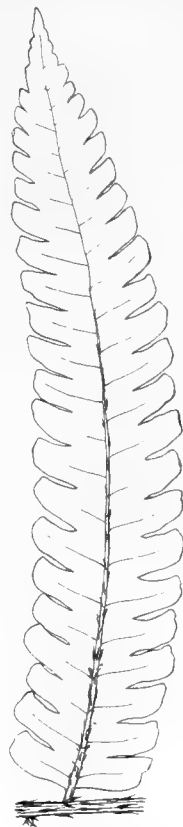


Fig. 4. *D. pedicellata* (Christ) C. Chr. Pinna $\times \frac{1}{2}$; segments $\times 1\frac{1}{2}$. (Orig.)

var. *peruviana* n. var. — Fig. 5b.

Syn: *Dryopteris refulgens* Hieron. Hedwigia. 46: 324. 1907.

Pinnæ incised scarcely more than one third with about 3 pairs of veins running to the sinus; lower veins irregularly flexuose and sometimes united. Scales few and small, ovate with a short apex. Otherwise typical.

Peru: Tarapoto, SPRUCE nr. 4657 (L, B), 4712 (W) — STÜBEL nr. 1097 :B.

31. *Dryopteris alsophilacea* (Kze.) O. Ktze. Rev. 2: 812. 1891. — Fig. 5c.

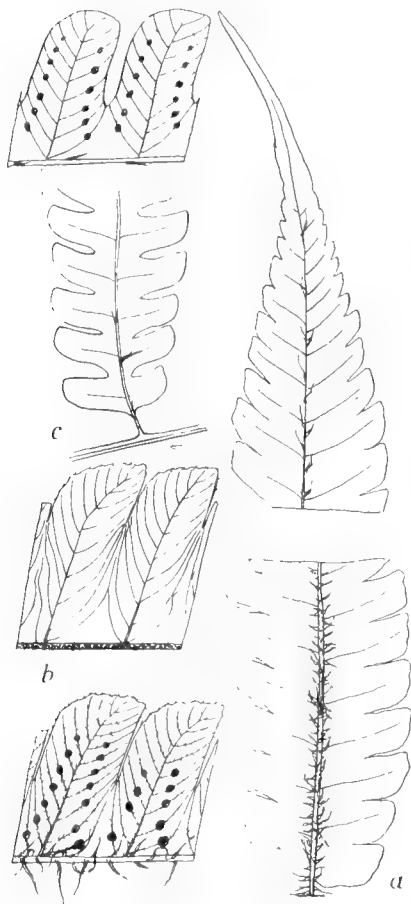


Fig. 5. a. Base and apex of *D. refulgens* (Kl. C. Chr. $\times \frac{1}{2}$, and two segments $\times 1\frac{1}{2}$ (orig.). b. Segments $\times 1\frac{1}{2}$ of *D. refulgens* var. *peruviana*. c. Base of pinna of *D. alsophilacea* Kze. O. Ktze. $\times \frac{4}{5}$ and segments $\times 1\frac{1}{2}$ (orig.)

Syn. *Aspidium alsophilaceum* Kze.; Mett. Aspid. nr. 218. 1858.

Nephrodium alsophilaceum Bak. Fl. bras. 1²: 474. 1870; Syn. Fil. 495.

Polypodium aspidioides Pr. Del. Prag. 1: 170. 1822.

Lastrea aspidioides Pr. Epim. 41. 1849!

Dryopteris tenuifolia C. Chr. Ind. 297. 1905, non Ktze.

Aspidium tijucense Fée, Cr. vasc. Br. 2: 72. tab. 102. 1872—73.

Dryopteris itatiaiensis C. Chr. Ind. 272. 1905.

Type from Rio leg. POHL (herb. PRESL!).

FÉE's plate cited above is a good figure of this species, which is intermediate between *D. pedicellata* and *D. falciculata* var. *paranaensis*; it is marked by its remote and long-stalked pinnæ, its distinct terminal pinna and its few and small scales. Pinnæ in 6—7 pairs, at distances of 5—6 cm, shortly acuminate, incised $\frac{2}{3}$ of the way to the costa into falcate, entire, obtuse segments; scales of the costæ beneath very few and small, narrow-linear, scarcely toothed, dark-coloured, (fig. 3,10). Surfaces glabrous; colour dark-green; texture membranous. Veins 8—10-jugate, the lower 2 running to the sinus. Sori medial with a subpersistent, reddish, glabrous indusium. Rhizome oblique with numerous red-brown scales at the top, but not with a dense mass of long scales.

A very constant species, which I had identified (following METTENIUS) with *Lastrea tenuifolia*

Presl and in Ind. Fil. therefore renamed *Dr. tenuifolia*. *L. tenuifolia* Pr. is however a form of *D. submarginalis*, and therefore I must fall back to the specific name *alsophilacea*. The species is confined to the mountains of South-eastern Brazil, and I name here some collector-numbers.

Rio: SCHOTTMÜLLER nr. 35 (B), MOSÉN nr. 99 (Rg, S), DUSÉN nr. 1611 (C, W), GLAZIOU nr. 5262 (B, H).

Rg = *A. tijuense* Fée).

S. Paulo: WACKET nr. 185 (R), RIO TIETÉ, GERDER nr. 87 (R), HANS nr. 127 (R).

Santa Catharina: Joinville, E. ULE nr. 42 (RB).

32. *Dryopteris falciculata* (Raddi) O. Ktze. Rev. 3²: 378. 1898; C. Chr. Ind. 264.

Syn. *Aspidium falciculatum* Raddi, Opusc. sci. Bol. 3: 289. 1819; Pl. Bras. 1: 31 tab. 47, 1825.

Polypodium ciliatum Presl, Del. Prag. 1: 169. 1822!

Aspidium chrysolobum Klf.; Link, Hort. Berol. 2: 117. 1833.

Dryopteris chrysoloba O. Ktze.; C. Chr. Ind. 257 (which see).

Polypodium distans Klf. Enum. 113. 1824!

Aspidium Schomburgkii Kl. Linnaea 20: 369. 1847!

Aspidium Schottianum Kze.! (Bak. Syn. 262. 1867).

Aspidium mucronatum Beyrich msc.!

Aspidium sericeum Fée, Cr. vasc. Brés. 1: 144 tab. 42, fig. 1. 1869!

Type from Rio, not seen, but numerous specimens from Rio agree perfectly with Raddi's figure, and I consider them typical. They can be described as follows:

Rhizome obliquely erect. Stipes fasciculated, up to 3—4 dcm long, slender, brownish stramineous, very shortly hairy and like the apex of the rhizome clothed with black-brown, rather thick and rigid, entire and long acuminate scales, which are 1 mm broad and 4—5 mm long; in the upper part of the stipe the scales become fewer and deciduous. Rachis like the stem, somewhat angular and bisulcate above. Lamina deltoid-lanceolate with the lowest pair of pinnae not or a little reflexed, towards the apex rather suddenly narrowed but without a distinct, terminal pinna, herbaceous, generally dark-green, 30—60 cm long, 20 cm broad. Larger pinnae shortly petiolulate, lanceolate, 10—13 cm long, 2—2½ cm broad, subopposite, ciliate, truncate at the base, acuminate, the basal pair often with the lower side enlarged. Costae above densely tomentose by crisped, pluricellular, reddish hairs, which in some forms also occur on the veins and leaf-tissue; costae beneath with some few small, entire or rarely slightly dentate, reddish-brown scales, which are ovate with a long hair-like apex and formed by almost isodiametric cells with rather thick walls (fig. 3²), surfaces glabrous or more or less glanduloso-pubescent. Basal segments sometimes quite free and very often much reduced. Larger segments a little falcate, as a rule faintly dentate, obtusely rounded at the apex or in some forms submucronate, 3—4 mm broad, connected by a very

narrow wing. Veins simple, 6—8 to a side, the basal ones much upcurved and reach the margin above sinus, the upper ones very oblique and run out in the teeth, about all soriferous. Sori middle-sized, yellowish brown, a little above the middle of the vein. Indusium rather large, yellowish, glanduloso-ciliate, persistent.

The form here described is very common in the mountains round Rio; the original specimens of *Pol. ciliatum* Pr. (hb. Presl!), *Aspid. chrysolobum* Klf. (B!) and *A. mucronatum* Beyrich (B!) differ not at all. I have seen numerous specimens, f. inst. GLAZIOU nr. 7949, MOSÉN nr. 98, 2696, 2697 etc., and they vary only a little, mainly in pubescence. Two extreme forms can be separated:

f. *glabrata* Hieron. msc.

Surfaces (costæ excepted) quite glabrous.

— GLAZIOU nr. 12288 (B, H) and others. — This form is probably *Nephrodium chrysolobum* Bak. Fl. Bras. and Syn. Fil.

f. *sericea* Fée = *Aspid. sericeum* Fée, *A. Schottianum* Kze.

Both surfaces throughout densely and shortly glanduloso-pubescent.

— GLAZIOU nr. 957, 1658 (H).

Between these two extremes one finds numerous intermediate forms, and I consider them all belonging to the typical form of the species. This seems to be rare in other parts of Brazil. I have seen the following specimens only:

Minas Geraes: Lagoa Santa, WARMING (H) — CLAUSSEN (B).

São Paulo: Santos, USTERI (C) — IGUAPE, WETTSTEIN u. SCHIFFNER (C) — ?ULBRICHT nr. 57b (R).

Parahyba: GÖLDI (C) = f. *glabrata*.

Rio Negro: Ega, MARTIUS (B) — Segments dentate, approaches *D. nigrovenia*.

British Guiana: SCHOMBURGK nr. 1167 (B = *A. Schomburgkii* Kl.; rather typical).

A specimen in B gathered and determined (?) by RADDI as *Polypodium falciculatum* Raddi, Opusc. sci. Bol. 3: 288. 1819; Pl. Bras. 1: 24 tab. 36 bis, belongs to the typical form of *D. falciculata*, but it does not agree perfectly with RADDI's description. What *P. falciculatum* Raddi may be is questionable; probably it is a form of the present species.

In the southern states of Brazil are to be found a series of forms, which do not differ essentially from typical *D. falciculata*, while they in some characters are so different, that it is possible that they belong to a distinct species, intermediate between *D. falciculata* and *D. ctenitis*. I refer them to *D. falciculata* as

var. **paranaensis** n. var.

Syn. *D. falciculata* Rosenstock, Hedwigia 46: 116. 1906, where Dr. ROSENSTOCK has given an excellent description of this variety.

Type specimen: Paran : Villa Nova, leg. ANNIES, Rosenstock, Fil. austr. bras. exsic. nr. 79 (R, W).

It differs from *D. falciculata* type by the following characters: 1) Stipe at base with numerous, long scales, almost as in *D. submarginalis*, 2) lamina larger

and broader: pinnæ up to 15 cm long by $3\frac{1}{2}$ —4 cm broad; segments 5 mm broad, costæ above and margins excepted quite glabrous, 3) stipe, rachis and costæ beneath rather densely scaly, the scales rigid, nearly black, more or less dentate with a long fibrillose apex, the cell-walls thick and dark (fig. 3^{2c}). 4) veins 10—11, more distinct than in the type.

From *D. ctenitis* it differs by fewer veins and medial or supramedial sori and by the shorter teeth of the scales.

Besides the specimens enumerated by ROSENSTOCK (l. c.) I have seen:

Santa Catharina: Itajahy, E. ULE nr. 190 (RB).

Paraná: Maréchal Mallett, P. DUSÉN nr. 3068 (C).

Minas Geraes: Serra do S. José d'El Rey, A. SILVEIRA nr. 358 (C).

Most specimens referred hereto with doubt by ROSENSTOCK belong to *D. submarginalis*.

33. *Dryopteris ctenitis* (Link) O. Ktze. Rev. 2: 812. 1891; C. Chr. Ind. 260.

Syn. *Aspidium ctenitis* Link, Hort. Berol. 2: 122. 1833!

Nephrodium ctenitis Bak. Syn. 265. 1867.

Lastrea distans Brack. U. S. Expl. Exp. 16: 192. 1854!

Aspidium squamigerum Mett. msc. (Bak. Syn. 265 nota)! (non Fée nec Mann).

Nephrodium squamigerum Rosenst. Hedwigia 43: 224. 1904.

Aspidium amaurolepis Fée, Cr. vasc. Brés. 1: 137 tab. 44 fig. 2. 1869!

Aspidium isabellinum Fée, Cr. vasc. Brés. 1: 137 tab. 45 fig. 2. 1869!

Dryopteris isabellina C. Chr. Ind. 272. 1905.

Nephrodium caripense β *squamigerum* Bak. Fl. Bras. 1²: 474. 1870.

LINK founded this species on plants cultivated in the Botanical Garden of Berlin and the type specimen (B!) belongs to a somewhat abnormal form, which after a detailed analysis of the original specimens of each was found to be identical with *A. squamigerum* Mett. and *A. amaurolepis* Fée. The species is well figured by FÉE on the plates quoted above. The rudimentary lower pinnæ of *A. isabellinum* (tab. 45 fig. 2) are not normal ones; in some specimens I have found similar abnormal pinnæ, which are not always the lower ones; in such pinnæ the lower segments only become fully developed, the costa suddenly terminating in a scaly bud.

D. ctenitis is intermediate between *D. falciculata* and *D. submarginalis* and it is very difficult to distinguish some of its forms from these species, still I think the following characters mark it sufficiently.

Leaf as dried dark-brown or red-brown, thin. Rachis clothed with dark and stiff, fibrillose scales with a long hair-like apex, shortly pubescent in the furrows above. Pinnæ rather distant, up to 20 cm long by 3—4 cm broad, the edges parallel from the base to above the middle. Costæ beneath more or less short-

hairy and more or less scaly by filiform scales, which are distinctly and sharply toothed by claw-shaped teeth; the cell-walls very dark and thick, the lumina small (fig. 3¹⁶). Under-surface often glandular. Segments patent or a little oblique, with the edges parallel, the apex truncate or bluntly rounded, entire or finely toothed, not mucronate. Veins 10—15, oblique. Sori inframedial, rarely almost medial, furnished with a persistent, reddish, glabrous or slightly ciliate and glandulose indusium.

From *D. falciculata* the present species differs by more veins in the patent, obtuse segments, inframedial sori and the different shape of the scales; from *D. submarginalis* it differs by the dark colour of the lamina and scales, obtuse segments, inframedial sori and persistent indusia.

The specimens examined can be referred to two forms:

1. forma *isabellina* (Fée).

Scales of rachis and costæ few and small, very like those of *D. submarginalis* f. *tenuifolia* but darker and more distinctly toothed (fig. 3^{16d}); rachis, costæ and both surfaces hairy by scattered, red, deciduous hairs. Pinnæ 2½—3 cm broad; segments rather oblique, finely toothed or often deeply serrate in the lower pinnæ. Veins 10—12, forked in the serrate segments; sori often nearly medial. — Stipe at base with a dense mass of soft, reddish scales.

This form, to which the original specimen of *A. ctenitis* Link seems to belong, is very closely related to *D. submarginalis* f. *tenuifolia*, from which it differs mainly by colour, structure of the scales and the persistent indusium.

Brazil: Rio, GLAZIOU nr. 2368 (C, H, Rg), 2371 (H), 2372 (H, Rg).

Minas Geraes: Lagoa Santa, WARMING (H) — Caldas, LINDBERG nr. 557 (B), MOSÉN nr. 2146 pt. (B, Rg).

São Paulo: Tieté, GERDER nr. 84a (R) — Toledo, SCHNAPP nr. 43 (C).

2) forma *amaurolepis* (Fée) — (*A. amaurolepis* Fée, *A. squamigerum* Mett., *Lastrea distans* Brack.)

Larger and more scaly than the preceding form, the scales larger and darker. Costæ beneath flat and sulcate, furnished with two kinds of scales: 1) lateral ones, which are red-brown and patent (fig. 3^{16b}), and 2) frontal ones, which are adpressed and almost black (fig. 3^{16c}). Both surfaces generally glabrous. Pinnæ 4 cm broad, the segments close, patent, obtuse; veins about 15; sori inframedial. — Stipe at base apparently without a dense mass of soft scales.

Between this excellent form and f. *isabellina* numerous intermediates are to be found, the two forms described being the two extremes of *D. ctenitis*.

Brazil: Rio, GLAZIOU nr. 1680, 2370 (H), MOSÉN nr. 100 (Rg, S), MARTIUS (Hb. Fl. Brazil nr. 324 (B), and others.

Minas Geraes: T. DE MOURA (B).

Bahia: DU PASQUIN 1851 (B). LUSCHNATH nr. 118 (B).

34. *Dryopteris submarginalis* (Langsd. et Fisch.) C. Chr. Ind. 296. 1905.

Syn. *Polypodium submarginale* Langsd. et Fisch. Ic. Fil. 12 tab. 13. 1810.

Aspidium caripense Mett. et auctt.

Nephrodium caripense Hk. Bak. Syn. 265 et auctt.

(For other synonyms see under the different forms).

Type from S. Brazil, Sta. Catharina.

As here understood this is the most variable species of the subgenus. Some of its forms are closely related to *D. falciculata* var. *paranaensis*, others to *D. ctenitis*; certain South Brazilian forms resemble very much *D. deflexa* and an andine form is not unlike *D. refulgens*. All these forms are, however, connected by all possible intermediates, and I have found it impossible to distinguish them specifically, while they all differ, as a rule, from the related species by some few characters, which are common even to the most different forms. The species is distributed from Mexico to Peru and from N. Brazil to Uruguay, and I first succeeded in recognizing the special characters of the different forms by sorting the more than 200 specimens examined by me after locality. Then it was found that almost all specimens from each of these three regions 1) Brazil from São Paulo southwards to Argentina and Uruguay, 2) Brazil from São Paulo northwards, and 3) Andes from Mexico to Peru constantly show some peculiarities, especially in the structure of the scales, by which they differ from specimens from another region but resembling them in size, habit, colour and other characters.

D. submarginalis may be distinguished from allied species by the following characters:

Stipe at base with a very dense mass of 3—4 cm long, thin, red-yellow scales. Leaf light-green, firmly herbaceous, ciliate, lower pinnæ generally reflexed; segments oblique, rarely falcate, entire or often faintly toothed, obtuse or mucronate. Rachis and costæ beneath more or less scaly, the scales red-yellow or red-brown with a long hair-like point, their margins toothed by short, mostly not curved teeth, the cell-walls clear. Costæ beneath without hairs. Veins simple, 8—15, the basal ones running out near the sinus. Sori medial or slightly supramedial, reddish-yellow. Indusium small, reddish, glabrous, generally absent. — As a rule the surfaces are glabrous; in some specimens scattered hairs can be found mainly on the veins above, in others the under-surface is slightly glandular.

In the following I shall try to explain the recognized forms. As the species was founded on a Brazilian form I begin with this, although it is not the most developed form of the species.

A. South-Eastern forms

1. forma *vera* (Langsd. et Fisch. tab. 13).

Syn. *Dryopteris Sellowii* Hieron. Hedwigia 46: 324 tab. 3 fig. 1. 1907!

Dryopteris soriloba Christ. in Fedde, Repertor. 6: 350. 1909!

? *Dryopteris collina* Christ., Bull. L'Herb. Boiss. II. 7: 922. 1907.

Lamina 4—7 dm long; pinnæ rather distant, 10—12 cm long, 2—2½ cm broad; segments oblique or often subfalcate, faintly toothed in the outer half and generally with a mucronate apex. Veins 10—23. Sori supramedial, very often confined to the outer half of the segment (*D. soriloba* Christ). Rachis and costæ more or less scaly; scales of costæ ovate with a long, hair-like point, clear and thin, their cells large with thin walls, the edges faintly and distantly toothed (fig. 3^{14a}).

This form comes near to *D. deflexa* in size, habit and colour of scales but is different by the subentire scales. It is confined to the Southern Brazilian States and the neighbour-republics; agreeing best with the original description and figure are the specimens from Rio Grande do Sul and Misiones; some of the specimens from Paraná and Sta. Catharina are intermediate between this and the following form. — *D. Sellowii* Hieron. is exactly this form.

Argentina: Tucuman, SIAMBON, HIERONYMUS u. LORENTZ nr. 795 pt. (B) — Misiones, Campament Bosetti, NIEDERLEIN (B); Soreto, NIEDERLEIN nr. 253 (B); Puerto Tamaren, NIEDERLEIN (B); Bonpland, EKMAN nr. 1, 2, 3 (Rg).

Uruguay: Coronilla, BERRO nr. 2382 (C, CC).

Paraguay: Colonia Elisa prope San Antonio, LINDMAN (Rg) — Cordillere de Mbatobi, Paraguari, BALANSA nr. 2917 (B, H) — Sierra de Amambay, HASSLER nr. 10454 (C, RB = *D. soriloba* Christ! and perhaps the same as *D. collina* Christ).

Brasilia: Rio Grande do Sul: Serra dos Tapes, Cascata, LINDMAN nr. A. 905 (Rg, W) — Excolonia Santo Angelo, LINDMAN nr. A. 959 (Rg) — Neu-Württemberg, Estancia L. Gomez, A. BORN-MÜLLER nr. 273 (R) — S. Leopoldo, Hamburger Berg, STÜBEL nr. 1173 (B = *D. Sellowii*). — Sta. Catharina: Blumenau, HAERCHEN nr. 49¹, 50 (R) — GOEDEN nr. 45 (R). — Paraná: S. Matheus, GÄNSLY nr. 8 u. 35 (R) — Villa Nova, ANNIES nr. 42 (R), — and others.

Intermediate forms between forma 1 and 2.

Sta Catharina: Blumenau, HAERCHEN, Ros. Fil. exsicc. nr. 153 (B, R, Rg, W).

São Paulo: Toledo, STIER nr. 32 (R), SCHNAPP nr. 30 (R), ULBRICHT nr. 55 (R) — Campinas, ULBRICHT nr. 59 (R).

2. forma *tenuifolia* (Presl).

Syn. *Lastrea tenuifolia* Pr. Epim. 37. 1849!

Aspidium caripense f. *macroloba* A. Br. Ind. sem. ht. Berol. app. 1857: 2!

Phegopteris Oreopteridastrum Fée, Cr. vasc. Br. 1: 97. 1869 (GLAZIOU 963).

Aspidium Sancti Pauli Christ, Denkschr. Akad. Wien 79: 15. 1907!

Larger than the preceding form, leaf one meter or more long, pinnæ 20 cm long by 3—3½ cm broad, often contiguous and erect, so that the leaf becomes a very compact habit. Segments slightly oblique or patent, seldom subfalcate, generally bluntly rounded at the apex and faintly toothed, sometimes mucronate. Scales as a rule fewer than in the typical form, those of the costa beneath generally hair-like consisting of a few rows of cells (fig. 3^{14b}). — Veins 12—15-jugate. Sori often medial, exindusiate (always?).

This is the common form of the mountains of the states Rio de Janeiro and Minas Geraes, where it was collected by almost all collectors. The original specimen of *Lastrea tenuifolia* Presl was collected near Rio by POHL (Herb. PRESL!);

in my Index it was identified with *D. alsophilacea* because METTENIUS referred PRESL's species to *A. alsophilaceum*. — I have examined a series of specimens and enumerate here the more important collector-numbers.

São Paulo: Villa Nova, ANNIES nr. 24 (R) — Rio Tieté, GERDER nr. 84 (R) — Campinas, HEINER nr. 516 (Rg — Serra de Caracal, MOSÉN nr. 4620 (Rg) — WETTSTEIN u. SCHIFFNER 1901 (= *A. Sancti Pauli* Christ, C, Hb. Wien).

Rio de Janeiro: GLAZIOU nr. 963, 15767 (H).

Minas Geraes: Caldas, LINDBERG nr. 559 b (B), REGNELL nr. 475 a (Rg, W); MOSÉN nr. 2146 pt. 2147, 2148, 2149, 2150, 4619 (Rg) — Corrego d'Olho d'Agua bei Antonio Pereira, SCHWACKE nr. 15052 (C) — Ouro Preto, SCHWACKE nr. 11291 (C); M. GOMES nr. 3113 (C).

Matto Grosso: H. SMITH (C).

Ceará: Serra de Baturité, HUBER nr. G. 137 (C).

B. Andine forms.

3. forma *caripensis* (Willd.).

Syn. *Polypodium caripense* H. B. Willd. sp. 5: 202. 1810!

Aspidium caripense f. *brachyloba* A. Br. Ind. sem. ht. Berol. app. 1857: 2!

Nearly identical with f. *tenuifolia*, but generally with smaller pinnae (15 cm × 2½ cm) and fewer veins (10—12), which are not so close as in the Brazilian forms. The scales of the costae beneath are never hair-like but rather uniform (fig. 3^{14c}). Underside often glandulose.

Mexico: Orizaba, WEBER (B), MÜLLER (B) — Mirador, LIEBMANN (H) — Sierra San Pedro, Nolasco, Talea etc. C. JÜRGENSEN nr. 228 (RB).

Guatemala: Coban, Alta Verapaz, v. TUERCKHEIM nr. II. 1204 (W).

Nicaragua: Cañada Yastica, Dep. Matagalpa, E. ROTHSCHUH nr. 237 (B).

Costa Rica: Turrialba, DONN. SMITH nr. 5095 (C, W); Navarro, WERCKLÉ nr. 16780 (RB, CC); Juan Viñas, Reventazon Valley, 1000 m, COOK and DOYLE nr. 185 (W); J. J. COOPER nr. 372. ed. DONN. SMITH sub nr. 6018 b (W).

Colombia: Caripe, HUMBOLDT (B, herb. Willd. nr. 19700); LINDEN nr. 1010 (RB).

Venezuela: Tovar, MORITZ nr. 38, 39, 207, 208 (B), 106 (C, B, S); FENDLER nr. 197 (B), Caracas, FUNCK et SCHLIM nr. 270 (RB).

Aspidium microchlaena Fée, 8 mém. 102. 1857; *Dryopteris microchlaena* C. Chr. Ind. 278. 1905 from Mexico, Orizaba, SCHAFFNER nr. 459 can, I think, with certainty be referred to this form of *D. submarginalis*. FOURNIER (Mex. pl. 1: 93) united it with *Aspidium microcarpon* Fée, 8 mém. 105, based on SCHAFFNER nr. 214 from Cordoba, Mexico. *Nephrodium crinitum* α Sod. is most probably the true *caripensis*.

4. forma *glauescens* (Sodiolo).

Syn. *Nephrodium crinitum* β *glauescens* Sod. Cr. vasc. quit. 251. 1893.

Under-surface more or less glaucous and often glandular. Sori often found furnished with a somewhat glandulose and ciliate indusium; segments acute and faintly toothed. Otherwise like f. *caripensis*.

Ecuador: Qualca, SODIOL (C).

Colombia: La Vega, LINDIG nr. 337 (B).

Venezuela: LANSBERG (B).

Costa Rica: WERCKLÉ nr. 16756 (C).

5. var. *tarapotensis* (Hook.).

Syn. *Nephrodium tarapotense* Hook. spec. fil. 4: 107. 1862.

A form with pinnæ resembling in shape exactly those of *D. Karstenii*, with which METTENIUS identified it; it differs from that species by more veins (10—12 to a side) and lack of articulated hairs on veins and costæ beneath; from typical *D. submarginalis* it differs by its very obtuse segments.

Peru: Mt. Campana, prope Tarapoto, SPRUCE nr. 4016 (Kew!).

6. var. *Lagerheimii* (Sod.).

Syn. *Nephrodium Lagerheimii* Sodirol, Cr. vasc. quit. 252. 1905.

Dryopteris Lagerheimii C. Chr. Ind. 273. 1905.

Large; stipe and rachis very scaly; scales of the stipe below about 5 cm long, those of the costæ beneath numerous and much broader than those of f. *caripensis*. Leaf 1 m long, pinnæ 20 cm long by 3 cm broad; segments rather falcate, finely toothed at the outer part; veins 15—17-jugate; indusium often present. In general habit not unlike *D. refulgens*.

Ecuador: Andes quitenses, LAGERHEIM, comm. SODIROL (S, C) — SPRUCE nr. 5294 (CC, H, L, RB, S).

Colombia: Cune, 1100 m, LINDIG nr. 295 (B).

Peru: Ruiz nr. 68 (B).

35. *Dryopteris Karstenii* (A. Br.) C. Chr. comb. nov. — Fig. 6.

Syn. *Aspidium Karstenii* A. Braun, Ind. sem. hort. Berol. app. 1857: 3.

Type-specimen in B!, cultivated in Hort. Berol. and raised from spores of plants collected by KARSTEN in Venezuela.

Closely related to *D. submarginalis* resembling it in the reddish scales, which form a tuft at the base of the stipes, but receding from it in the direction of *D. falciculata*. Compared with *D. submarginalis* the main characters of *D. Karstenii* are the following: Middle-sized; stipe 3½ dm long, like rachis clothed with hairs, hair-shaped scales and broader scales. Lamina 5—6 dm long, 2—2½ dm broad; largest pinnæ 10—12 cm long, 2—2½ cm broad; both sides throughout with short, scattered, reddish hairs, costæ beneath furnished with hairs and scales and with all intermediates between these two kinds of trichomes (fig. 3¹⁵). Segments close, slightly oblique, entire, broadly obtuse and often emarginate, upper basal one often quite free and a little shortened. Veins 6—8-jugate, sori medial or a little supra-medial furnished with a persistent, ciliate indusium.

— “Truncus erectus sensim supra terram elatius demum fere pedalis una cum phyllopodiis laxius dispositis paleaceus” (A. BRAUN l. c.), while the caudex of *D. caripensis* is described thus: “truncus erectus supra terram elevatus demum semipedalis et altior una cum phyllopodiis satis confertis dense paleaceo-lanatus paleis valde elongatis (fere pollicaribus) linearibus in pilum excurrentibus” (A. BRAUN l. c. pag. 2).

From *D. falciculata* and *D. nigrovenia* our species differs by the shape and structure of the scales.

Venezuela: MORITZ nr. 209, LANSBERG, GOLLMER, (B), Puerto Cabello, KARSTEN (C).

36. *Dryopteris deflexa* (Klf.) C. Chr. Ind. 261. 1905.

Syn. *Polypodium deflexum* Klf. Enum. 114. 1824.

Polypodium vestitum Raddi, Opusc. sci. Bol. 3: 288. 1819; Pl. Bras. 1: 24 tab. 36. 1825 (non Forst. 1786).

Nephrodium vestitum Bak. Syn. 265. 1867.

Aspidium Raddianum Mett. Pheg. u. Aspid. 91 nr. 221. 1858.

Nephrodium Raddianum Hk. sp. 4: 98 tab. 245. 1862.

Aspidium basilare Fée, Cr. vasc. Brés. 1: 135 tab. 43 fig. 2. 1869!

Type specimen from Brasilia, MERTENS communicavit (not seen). It is, however, sure that *P. deflexum* Klf. is identical with *P. vestitum* Raddi; the terms of the original diagnosis "costis venisque infra paleis lacero-ciliatis tectis, soris venae adproximatis" show it sufficiently. Whether *Polypodium lepigerum* Schrad. Gött. gel. Anz. 1824: 868 is the same, as suggested in Ind. Fil., is questionable. The short diagnosis agrees also with other species of this group; perhaps it is identical with var. *Aschersonii* (see below).

A very distinct species at once to be distinguished from allied species by the very scaly rachis and costæ and costulæ beneath; the scales are light flat, appressed, formed by large clear cells and with very long hairlike fringes on the margins (fig. 3¹¹). By this last character it differs from the more scaly forms of *D. submarginalis* f. *vera*, which are very similar *D. deflexa* in size and colour of the scales; still *D. deflexa* seems to want the dense tuft of scales at the base of the stipe. — Lower pinnæ reflexed; veins about 8 to each side, the basal pair reaching the margin above sinus. Sori inframedial, often close to the costa (*A. basilare* Fée), furnished with a small deciduous indusium. — Leaf-tissue hairless, underside sometimes slightly glandular.

A fairly constant species, apparently common in the mountains near Rio. I have seen the following specimens:

Rio: SELLOW nr. 680 (B); RIEDEL (B, H, W); MOSÉN nr. 97, 2704 (H, Rg), 2703 (Rg); GLAZIOU nr. 2373 (H), 2374 (C, H, Rg); GOELDI (C); MIKAN (Hb. PRESL).

Parahyba: GOELDI (C).

var. *Aschersonii* Mett. msc.

Differs from the type by the sessile pinnæ and persistent indusia; it is still more scaly than the type, and the scales are formed by small cells with very flexuose cell-walls.

Prov. Espirito Santo, an der Minasstrasse, Prinz zu WIED 1816 (B).

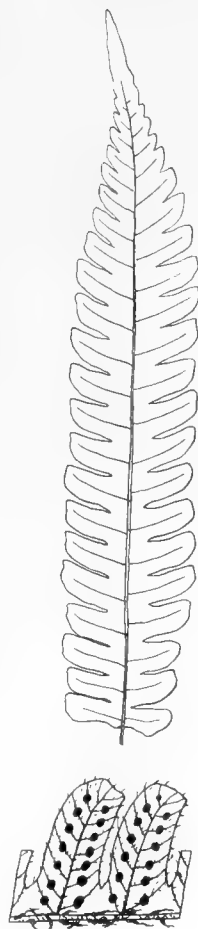


Fig. 6. *D. Karstenii* (A.Br.) C. Chr. Pinna $\times \frac{4}{5}$, and segments $\times 1\frac{1}{2}$ (MORITZ 209).

37. *Dryopteris Anniesii* Rosenstock, *Hedwigia* 46: 118. 1907. — Fig. 7.

Type from Brasilia, Rio Grande do Sul, Campestre do Seivaes, leg. JÜRGENS u. STIER nr. 158 (R!).

A large species; leaf 1 m or more long with numerous, alternate pinnæ, which are up to 20 cm long by $2\frac{1}{4}$ – $2\frac{1}{2}$ cm broad, the lower ones not or a little shortened. Segments patent or subfalcate, generally broadest towards the apex, separated by round, broad sinuses, the lower ones free or nearly so. Veins 12–15-jugate. Sori numerous subcostular or at least distinctly inframedial, reddish, furnished by a persistent, rather large, reddish, glabrous indusium. Leaf tissue of both surfaces glabrous, the underside often finely glandular, margins ciliate. — In general habit the species is not unlike *D. ctenitis*, probably its nearest ally, which it also resembles in the dense tuft of scales at the base of the stipe, its inframedial sori and persistent indusia, but it differs considerably by the structure of the scales; these are numerous, reddish, soft and clear, not unlike those of *D. deflexa*, but

their margins are toothed only not long-fimbriated (fig. 3¹³). — Stipe and rachis dark-coloured densely clothed with adpressed, darker scales.

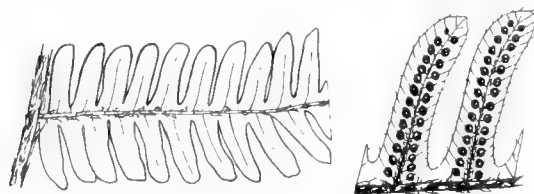


Fig. 7. *D. Anniesii* Ros. Base of pinna $\times \frac{1}{15}$ and segments $\times 1\frac{1}{2}$. (Orig.)

Paraná: Villa Nova, ANNIES nr. 89 (= Rosenstock: Fil. Exsicc. Austro-Brasil. nr. 89 (B, C, R, Rg, W). — Serro do Mar. inter Ypiranga et Volta Grande in terra silvosa, DUSÉN nr. 3625 (C, Rg).

var. *Ottonis* Rosenstock in herb.

Lamina ad basin sensim decrescens: pinnis inferioribus 3–4 jugis reflexis, infimis 4–5 cm longis; paleis costarum structura f. *typicae* sed ad marginem ciliis nonnullis instructis, fere ut in *D. deflexa*.

Sta Catharina: Joinville, MÜLLER (R); GERDER nr. 80a (R).

38. *Dryopteris fenestralis* sp. nov. — Fig. 8.

Type specimen from Brasilia: Rio, Petropolis, Alto do Imperador, leg. GLAZIOU nr. 7026, $2\frac{1}{3}$ 1878 (H, also B).

Stipitibus $4\frac{1}{2}$ decm longis, $1\frac{1}{2}$ cm crassis, rufis, ad basin paleis 2–3 cm longis, 3–4 mm latis densissime vestitis, supra cum rachi paleis sordide brunneis tenuissimis, 2–3 mm latis, 8–10 mm longis, laxe dispositis dense vestitis. Lamina ovato-lanceolata, ca. 7 decm longa, infra medium $3\frac{1}{2}$ –4 decm lata, atro-viridi, firmo-herbacea. Pinnis alternis, inferioribus petiolulatis, acuminatis, ad 20 cm longis, $3\frac{1}{2}$ cm latis, utrinque costis exceptis glabris, supra ad costas rufo-pilosis et squamis parvis longis filiformibus nonnullis instructis, infra ad costas costulas venasque squamis tenuibus latis vestitis, fere ad costam incisissimas vel inferioribus versus basin fere pinnatis. Laciniis falcatis basi recte truncata, apice acuto serrato, marginibus

parallelis, 4 mm latis, basali posteriore aequali vel parum reducta, anteriore sæpe valde prolongata lobata, rachi parallela vel eam tegente. Venis 15—20-jugis, parum obliquis. Soris parvis medialibus vel paulo inframedialibus, exindusiatis (?); sporangiis pilis articulatis intermixtis.

Allied to *D. Anniesii*, from which it differs by its numerous, not adpressed, dirty-brown, very large scales of stipe and rachis and by its small, medial sori without distinct indusia. The scales of the costæ beneath are larger than in any other species and formed by large, isodiametric cells with thin walls; their margins are irregularly toothed. Seen in the microscope a scale resembles not a little an old window, hence the specific name (fig. 3¹²).

To this species I refer a plant from Sta. Catharina: S. Joaquin, leg. SPANNAGEL nr. 174 (C, R); it is smaller but still more scaly and the scales larger and reddish-yellow; base of stipe with a dense tuft of very large, thin scales, the largest 3 cm long, 6 mm broad. It can be named var. *Spannagelii* Ros.

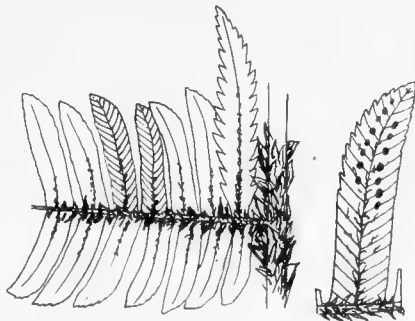


Fig. 8. *D. fenestralis* n. sp. Base of pinna $\times \frac{4}{5}$ and segment $\times 1\frac{1}{2}$. (Orig.)

39. *Dryopteris vellea* (Willd.) O. Ktze. Rev. 2: 814. 1891; C. Chr. Ind. 300.

Syn. *Aspidium velleum* Willd. sp. 5: 255. 1810.

Nephrodium velleum Desv. Prod. 261. 1827; Bak. Syn. 265.

Nephrodium aureovestitum Hk. sp. 4: 101 tab. 246. 1862.

This species was founded on PLUMIER tab. 49, which plate illustrates a plant from San Domingo; it is probable that *N. aureovestitum* Hk. based on LINDEN nr. 1901 from Cuba, Mt. Libanon is the same, but I have, however, not seen any Cuban specimen. It seems to be a rare species, I have seen only a single specimen from Jamaica, leg. JENMAN (W); according to JENMAN it is common in forests on the Manchester mountains at 2000 feet altitude.

D. vellea alone represents the subgenus in the West-Indies. It resembles in size *D. deflexa*, but in the scales much more *D. cirrhosa* var. *eriocaulis*, from which it differs by the glabrous leaf-tissue and fewer veins (8—10). The scales are reddish, entire with a pocket-shaped base, not hair-pointed (fig. 3ⁿ).

40. *Dryopteris cirrhosa* (Schum.) O. Ktze. Rev. 2: 812. 1891; C. Chr. Ind. 257.

Syn. *Aspidium cirrhosum* Schum. Vid. Selsk. Skr. 4: 231. 1827.

Nephrodium crinibulbon Hk. sp. 4: 92 tab. 244. 1862.

var. *eriocaulis* (Fée) C. Chr. comb. nov.

Syn. *Aspidium ericaulon* Fée, Cr. vasc. Br. 1: 136 tab. 4 fig. 1. 1869.

Nephrodium ericaulon Bak. Syn. 495. 1874.

Dryopteris ericaulis O. Ktze. Rev. 2: 812. 1891; C. Chr. Ind. 263.

Nephrodium ramentaceum Bak. Fl. bras. 1²: 473. 1870.

Type specimen of *D. cirrhosa* from Guinea, tropical West Africa, leg. THONNING (H!), of var. *eriocaulis* from Brazil, leg. GLAZIOU nr. 2369 (H, Herb. Cosson, Paris!).

This is the only species of the whole subgenus, which I do not hesitate to unite with an Old-World's species; the very handsome Brazilian *D. ericaulis* is nearly quite identical with the African *D. cirrhosa*; I see no other differences than the African form being less scaly and having more acute segments and smaller sori; in shape of scales, pubescence, texture, position of sori etc. the two forms quite agree.

The South-American form var. *eriocaulis* is very constant and the most beautiful and well-marked form of the whole subgenus; its essential characters are the following:

Stipe beneath with a tuft of scales, upwards like rachis very densely clothed with red-yellow, patent or often characteristically reflexed, entire, hairpointed scales. Lamina in size equal to *D. ctenitis* and *D. Anniesii*, shortly hairy throughout on both surfaces, beneath also glandulose; costæ beneath densely clothed with red scales, which generally stand at open angles to the costa and narrow from a sub-bullate base rather suddenly into a long narrow point with revolute but quite entire edges (fig. 3⁹). Segments oblique or subpatent, obtuse or subacute, often faintly toothed, 4¹/₂—5 mm broad. Veins 10—12. Sori inframedial or almost medial, furnished with a persistent, red indusium, which is shortly hairy and sparsely glandulose.

Brasilia: Parahyba, GOELDI (C) — Rio, GLAZIOU nr. 2369 (H), Sellow (B), LENORMAND (B) — São Paulo: Bella vista ad flumen Rio Pardo, WETTSTEIN u. SCHIFFNER 1901 (C, Hb. Wien) — Rio Tieté, GERDER nr. 80 (R).

Paraguay: HASSLER (C).

Besides these specimens I have seen numerous others originating from plants cultivated in Berlin 1862 and Kew 1868 under the names *Lastrea vestita* J. Sm. and *L. vulpina* Kl., which latter very proper name has not been published.

41. *Dryopteris nigrovenia* (Christ) C. Chr. Ind. 279. 1905 — Fig. 9a.

Syn. *Nephrodium nigrovenium* Christ, Bot. Gaz. 20: 545. 1895.

Aspidium setosum Kl. Linn. 20: 371. 1847 (description; non Sw.).

Aspidium deltoideum Fourn. Mex. pl. 1: 93. 1872 (non Sw.).

Type from Honduras: San Pedro Sula, Depart. Santa Barbara, 1000', leg. THIEME, ed. DONNELL SMITH nr. 5646 (W!, C).

An andine representative of *D. falciculata*, resembling it in size, habit and texture but differing by the following characters: Stem proportionally long, 5 dcm; lamina 5 dcm long 2 dcm broad, not ciliate and not glanduloso-pubescent, but both surfaces with scattered, short and thick hairs of the common type. Rachis and costæ beneath with several, small, dark-brown scales, which from an almost circular, subbullate basal part formed by large isodiametric cells and often furnished with some long cilies are suddenly narrowed into a long, narrow, entire apex, which is formed by longitudinal, narrow cells with thick cell-walls (fig. 3³). Segments subpatent or a little oblique, not falcate, generally distinctly serrate especially towards the mucronate apex, the posterior basal one of the lower pinnæ often somewhat shorter, the anterior basal one of the upper pinnæ generally enlarged. Veins 6—8 to each side, simple, sometimes dark-coloured. Sori medial or inframedial, small, with a subpersistent, glabrous indusium. — In some specimens the basiscop half of the basal pinnæ is much enlarged with deeply lobed segments and furcate veins.

D. nigrovenia is distributed from Mexico to Ecuador and it varies only a little; the form described as *A. setosum* Kl. differs only by its somewhat longer petioles of the lower pinnæ; to that form belong the specimens from Venezuela-Ecuador. I have seen the following specimens:

Mexico: San Luis Potosi, Hac. of Tamasopo, PRINGLE nr. 3957 (B, C, H, S, W); Vallée de Córdoba, Bourgeau nr. 1644 (B, H, S, W = *A. deltoideum* Fourn.), H. FINK nr. 55 part. (W).

Honduras, San Pedro Sula, THIEME ed. DONN. SMITH nr. 5646 (C, W).

Costa Rica, WERCKLÉ 1904 (C); Turrialba, A. et C. BRADE nr. 344 (R); Grenadilla Finca Hermes, A. et C. BRADE nr. 414 (R).

Panama: MAXON nr. 5172 (W).

Colombia: Santa Marta, H. H. SMITH nr. 2581 (C).

Venezuela: Tovar, MORITZ nr. 204 part, (B, C, S, = *A. setosum* Kl.), nr. 457 (B).

Ecuador: Andes quitenses, SPRUCE nr. 5721 (CC, RB).

Further I refer here a specimen from

Trinidad: FENDLER nr. 125 (W).

which, however, is very near *D. falciculata*, especially the specimens of that species from Parahyba and Guiana; thus it seems that the two species are connected by intermediate forms occurring in Northern South America, where they meet.

42. *Dryopteris Tonduzii* (Christ) C. Chr. Ind. 664. 1906.

Syn. *Aspidium Tonduzii* Christ, Prim. Fl. Costaric. III. 34. 1901.

Type from Costa Rica: Forêto de Tuis, TONDUZ nr. 11333 (C! also W).

Intermediate between *D. nigrovenia* and *D. Lindenii* resembling the former in venation the latter in size and colour. It differs from *D. nigrovenia* by its thin, light-green leaf, which is about 25 cm long by 13 cm broad, the leaf-tissue of both surfaces glabrous but costæ and costulæ with scattered, short, thick hairs. Stipe up to 25 cm long, slender, throughout glanduloso-pubescent and fibrillose by thin,

patent, glossy scales, the lower ones being larger, ovate-acuminate, the upper linear and hair-pointed with a bullate base. Rachis similarly slender, glanduloso-pubescent and somewhat scaly. Pinnæ 5–8 cm long, the costæ and costulæ beneath more or less glanduloso-pubescent and furnished with bullate-acuminate scales

much similar to those of *D. nigrovenia*. Segments oblique or falcate, the basal ones somewhat reduced, the larger serrate. Veins 5–6-jugate, very oblique, reaching the margins. Sori small, medial or supramedial, indusium small, fugacious, glandulose.

D. Lindeni is different by the veins not reaching the margins, *D. strigilosa* by different scales and glandular under surface.

Besides the type-specimen I refer here:

Costa Rica: Meseta, 2000 m, ALFARO nr. 16863 (C, RB), Navarro, WERCKLÉ (C).

Guatemala: Volcan Jumaytepeque, Depart. Santa Rosa, 6000 ft., HEYDE et LUX, ed. DONN. SMITH nr. 4425 (W).

43. *Dryopteris strigilosa* Davenport, Bot. Gaz. 21: 257. 1896; C. Chr. Ind. 295. — Fig. 10a.

Syn. *Aspidium strigosum* Dav. l. c.

Type from Mexico: Vera Cruz, dry calcareous cliffs, barranca of Metlac, 3000 ft., leg. PRINGLE nr. 6077 (C, H, S, W).

Near *D. Tonduzii*, resembling it in size, colour and texture, but the lower pinnæ incised to costa and the scales different. The scales of stipe and rachis, very numerous at the base of the stipe, are up to 1 cm long, patent, dark-brown, glossy, very narrow and scarcely widened at base, entire but the margins furnished with small, capitate glands (fig. 3^{4a}). When the scales fall they leave a scar by which the stipe becomes rough. Scales of the costæ beneath are similar but smaller

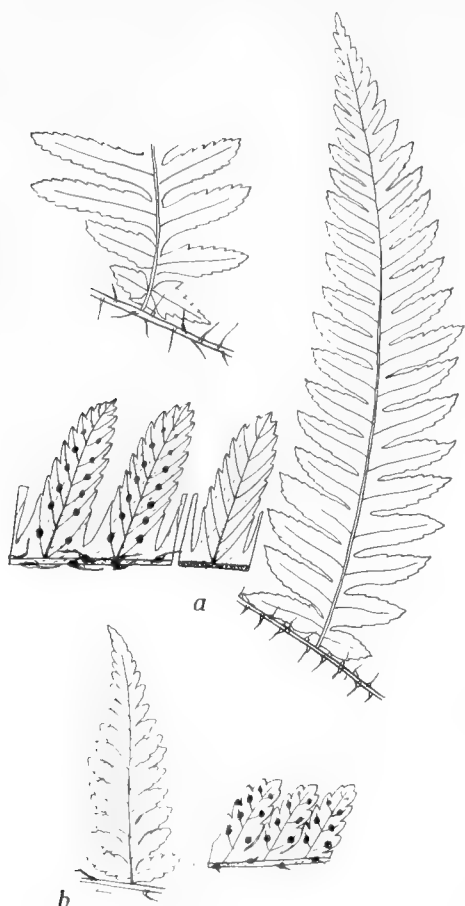


Fig. 9a. *D. nigrovenia* (Christ) C. Chr. Middle pinna and base of the second pinna from below, $\times \frac{4}{5}$, two segments seen from the underside and one seen from above, $\times 1\frac{1}{2}$ (orig.). — b. *D. Lindeni* (Kuhn) O. Ktze., middle pinna $\times \frac{4}{5}$ and three segments $\times 1\frac{1}{2}$ (orig.).

with a broader, still scarcely bullate base (fig. 3^{4b}). Whole leaf densely glandular-pubescent. Lower pinnæ about 4 cm long, broadest at base, fully pinnate at the lower half; segments (or pinnules) distinctly serrate, generally truncate at apex, veins 4–5 to each side, simple, reaching the margin; sori inframedial furnished with a glandulose indusium.

Polypodium subincisum Mart. et Gal. Mém. Acad. Brux. 15: 43. 1842 from Vera Cruz, GALEOTTI nr. 6290 (B, a single leaf only) is rather this species than the following, to which KUHN referred it.

var. *Cookii* Maxon in sched. (pro specie).

Exactly *D. strigilosa* in pubescence and scales but in cutting almost identical with *D. Tonduzii*; all pinnæ pinnatifid only and the segments entire or faintly toothed, oblique and acute, the basal ones shortened. Lower pinnæ with rather long petioles.

Guatemala: Alta Verapaz, near the Finca Sepacuite, on limestone cliffs, Cook and GRIGGS nr. 507 (W).

44. *Dryopteris Lindeni* (Kuhn) O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 275 Fig. 9b.

Syn. *Aspidium Lindeni* Kuhn, Linnaea 36: 116. 1869.

Nephrodium Lindeni Bak. Syn. 493. 1874.

Aspidium Lindeni Fourn. Mex. Pl. 1: 97. 1872.

Dryopteris Moreletii C. Chr. Ind. 278. 1905.

Type from Mexico: Tabasco, Teapa, leg. LINDEN nr. 1489 (Kew!, Mus. Paris!).

This beautiful species was described under the same name both by KUHN and FOURNIER. The type-specimen in Mus. Paris from Teapa, LINDEN sine num., after which FOURNIER named his new species, is exactly identical with the Kew specimen of LINDEN nr. 1489, which I regard as the type-number of KUHN's species.

Rhizome oblique; stipites fasciculated, 15—20 cm. long, slender, at base clothed with numerous, brown, entire, from a broad base narrowed into a hair-like apex, patent scales, upwards like rachis deciduously scaly by similar but shorter scales, which from a bullate base are suddenly contracted into a long hair-shaped, rather flaccid apex, 2—2 mm long. Lamina lanceolata, 15—25 cm long, 6—9 cm broad, thinly herbaceous, fresh-green, both surfaces (costæ above excepted) glabrous, costæ beneath with several small, light-brown, bullate scales. Pinnæ 15—20-jugate, short-stalked, the lower ones somewhat abbreviated, middle ones 3—4 cm long, 1—1¼ cm broad, broadest at the base, incised to a narrow wing to the costa into oblique, obtuse, distinctly serrated segments. Veins 2—3-jugate, very oblique and not reaching the margin, simple or more commonly furcate with a posterior branch running into the teeth or lobe and an anterior one which is very short and soriferous. Sori small, medial, medial apparently apical on the short anterior branch of the furcate veins or dorsal somewhat below the apex of the simple veins, furnished with a persistent, reniform, glandulose indusium.

D. Lindeni is certainly a most distinct species, by its scales not essentially different from *D. Tonduzii*; it is very closely related to *D. Salvini*, resembling it in colour, shape of scales and its peculiar venation, by which it at once can be distinguished from the three preceding species. Besides the type-specimen I have only seen one more, which is absolutely identical.

Guatemala; Alta Verapaz, Cubilquitz, 350 m, v. TUECKHEIM nr. II 839 (W).

45. *Dryopteris Salvini* (Bak.) O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 290. — Fig. 10b.
Syn. *Nephrodium Salvini* Bak. Syn. Fil. 274. 1867.

Type from Guatemala, SALVIN et GODMAN (Kew!), and identical specimens from the same country, Alta Verapaz, Cubilquitz, 350 m were collected by H. v. TÜRCKHEIM, ed. DONN. SMITH nr. 8647 (C, W).

In most characters: scales, venation, shape of lamina nearly identical with *D. Lindenii*, but a larger and more robust plant, lamina up to 50 cm long by 12 cm broad, the stipe and rachis rather strong; segments more deeply serrated with about 5 pairs of furcate or simple veins; basal segments nearly quite free. Indusium large, persistent.

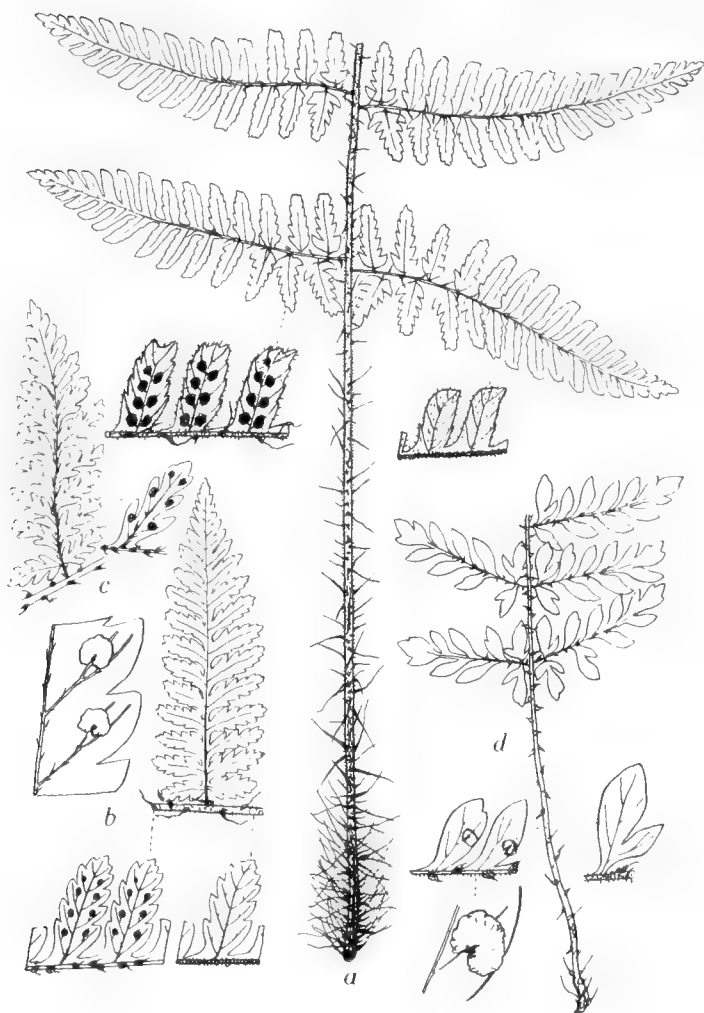


Fig. 10. a. *D. strigilosa* Dav. Stipe and base of lamina of a larger leaf, $\times \frac{4}{5}$, three segments seen from the underside and two others (from an upper pinna) seen from the upperside, $\times 1\frac{1}{2}$ (orig.). — b. *D. Salvini* (Bak.) O. Ktze. Middle pinna, $\times \frac{4}{5}$, two segments seen from the underside and one seen from the upperside, $\times 1\frac{1}{2}$; fragment, showing venation and indusia (J. D. S. 8647). — c. *D. lanceolata* (Bak.) O. Ktze., pinna $\times \frac{4}{5}$ and pinnula $\times 1\frac{1}{2}$ (orig.). d. *D. lanceolata* var. *tricholepis* (Bak.). Lower part of a leaf $\times \frac{4}{5}$, two segments seen from the underside with a sorus more magnified and a basal pinnula seen from the upperside, $\times 1\frac{1}{2}$. (J. D. S. 626.)

46. *Dryopteris lanceolata* (Bak.) O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 273. — Fig. 10c.

Syn. *Nephrodium lanceolatum* Bak. Syn. 498. 1874.

Type from Guatemala, Coban, SALVIN et GODMAN (Kew!), also Pansamalá, Depart. Verapaz, 3800 ft., TÜRCKHEIM ed. DONN. SMITH nr. 626 (B, C, W).

In size, texture and scales not materially different from *D. Salvini*, distinguished by its partly bipinnate lamina and larger, obtuse tertiary lobes. Pinnæ

rather distant, short-stalked, fully pinnate in the lower half; segments or pinnules oblique, deeply lobed with 2–3 oblique, obtuse lobes on each side, those of the anterior side the largest, the anterior side generally entire in the lower half and decurrent at base. Veins 2–4 to a side, simple, one to each lobe and not reaching the margin, each bearing a sorus, which is placed at the base of each lobe. Indusium very large, glabrous, brown, persistent. — Surfaces (costæ above excepted) glabrous, but costæ and often also costulæ beneath furnished with rather numerous bullate scales, which resemble those of the two preceding species but are larger and more numerous. Scales of stipe and rachis rather many, from a broad, subbullate base ovate-acuminate, brown, glossy.

var. *tricholepis* (Bak.) — Fig. 10 d.

Syn. *Nephrodium tricholepis* Bak. in Hemsley, Biol. Centr. Amer. 3: 651. 1885.

Dryopteris tricholepis C. Chr. Ind. 298. 1905.

A smaller plant than the type, otherwise scarcely different. Lamina 10–15 cm long, 3–5 cm broad at base where generally broadest (type 25×10 cm, somewhat narrowed below). Pinnæ pinnate; pinnules mostly fully entire, ligulate, or the larger ones with one or two obtuse lobes. Scales typical in shape; in several specimens they are of a peculiar, metallic lustre, not unlike the scales of *Polypodium moniliforme*. Sori often one or two only to each segment or pinnule.

Guatemala: Mts. of Verapaz, SALVIN (Kew!, type-specimen); Pansamalá, Alta Verapaz, 3800 ft., DONN. SMITH nr. 1563 (W) and v. TÜRCKHEIM ed. DONN. SMITH nr. 626 (B, C, W); near the Finca Sepacuite, Senaju, COOK and GRIGGS nr. 482 (W).

I have not seen TONDUZ nr. 13764 from Costa Rica, Nicoya, by CHRIST referred to *Aspidium tricholepis* (Bak.) (Bull. L'Herb. Boiss. II. 5: 259).

var. *deltoideo-lanceolata* n. var.

Lamina decidedly deltoid in outline, 20 cm long, 8 cm broad at base. Basal pinnæ the largest with the lower side enlarged with pinnules $1\frac{1}{2}$ cm long, 6 mm broad and nearly incised to the midrib into oblique, ligulate tertiary segments. Segments or pinnules of upper pinnæ mostly entire, the upper basal one large and broad.

A characteristic form, in most characters agreeing with var. *tricholepis* but tending to a tripinnate state and approaching the following species, which however is very different in venation and scales.

Guatemala: Alta Verapaz, Coban, H. v. TÜRCKHEIM nr. II. 1621 (W).

47. *Dryopteris Hemsleyana* (Bak.) C. Chr. Ind. 270. 1905.

Syn. *Polypodium Hemsleyanum* Bak.; Hemsley, Biol. Centr. Amer. 3: 660 tab. 108. 1885.

Type from Guatemala: Chilasco, GODMAN and SALVIN (Kew!).

In its most developed form a very distinct species but in minute characters not essentially different from *D. nigrovenia* and *D. Lindeni*. Stipe, rachis and costæ beneath with many, castaneous, narrow scales, which have a broad, but scarcely bullate base and a long filiform apex, which is sometimes ciliated. Stipes fasciculated from an erect rhizome, 25—35 cm long, castaneous, glossy. Lamina deltoid, 20—30 cm long, firmly herbaceous, dark-green, glabrous, nearly tripartite in the most developed form. Basal pair of pinnæ much the largest, up to 15 cm long, bipinnatifid, their basiscop pinnulæ much enlarged, 4—6 cm long; incised nearly to the costa into oblong, acute, entire or faintly toothed segments, their acroscop pinnulæ smaller often only 1 or $1\frac{1}{2}$ cm long, sometimes not quite free, serrated or lobed halfway down to the midrib. Other pinnæ pinnatifid or fully pinnate at the base, 6—8 cm long, 2— $2\frac{1}{2}$ cm broad, equal-sided, their segments acute, entire or toothed in the outer part. Veins 5—8, simple, reaching the margin. Sori infra-medial, small, apparently exindusiate.

Guatemala: Zamorora, Depart. Santa Rosa, 5500', HEYDE et LUX, ed. DONN. SMITH nr. 4662 (B, W) — Pansamalá, Depart. Alta Verapaz, 4000', H. v. TÜRCKHEIM, ed. DONNELL SMITH nr. 1057 (B, W). Costa Rica: Tablazo, 1900 m, BIOLLEY nr. 64 (C, CC, W); BRADE nr. 144 (R) — Surubres, BIOLLEY (W) — La Palma, WERCKLÉ nr. 17078, 17087 (C, CC, W) — Bord de l'aguacaliente 1300 m, PITTIER nr. 2416 (B) — Forests of Virris, near la banilla, 800 m, BIOLLEY (W). Panama: MAXON nr. 5412, 5435 (W).

48. *Dryopteris honesta* (Kze) C. Chr. Index 271. 1905. — Fig. 11 b.

Syn. *Polypodium honestum* Kunze, Linnaea 9: 49. 1834.

Phegopteris honesta Mett. Pheg. u. Aspid. nr. 59. 1858; Ettingsh. Farnkr. 164 tab. 106 fig. 3, 7, 13.

Polypodium fibrillosum Bak. Syn. 307. 1867.

Dryopteris fibrillosa C. Chr. Ind. 264. 1905.

Type from Peru, prope Pampayaco, leg. POEPPIG. — I have not seen an authentical specimen of the type-collection, but the three figures of ETTINGSHAUSEN quoted above, which agree excellently with the descriptions of KUNZE and METTENIUS, illustrate a form that is quite identical with a specimen in Herb. Mus. Paris of SPRUCE's collection nr. 4772 from Tarapoto, Peru, the type-number of *P. fibrillosum* Bak. The same form was collected in Bolivia, Antahuacana, O. BUCHTIEN nr. 2191 (R).

Stipe below densely clothed with patent 1—2 cm long, narrow and acute, castaneous or black-brown, faintly toothed scales. Pinnæ opposite, short-stalked, 10 cm long by 3 cm broad, at distances of 6—7 cm, the lowest pair deflexed with their lower side somewhat enlarged. Rachis and costæ beneath furnished with several, narrow, linear-acuminate, black-brown, faintly toothed scales (fig. 3¹⁷), costæ on both sides sparsely hairy or almost glabrous, surfaces otherwise glabrous. Texture firmly herbaceous, colour brownish, leaf-tissue finely pellucido-punctate. Segments broad, obtuse, shallowly repand or crenate, the basal posterior one of the

lower pinnæ nearly or quite free, lobato-pinnatifid. Veins about 7—8 to a side, once or twice forked, the branches occasionally anastomosing, not reaching the margin. Sori at the base of the anterior branch of the vein, small, a little oblong exindusiate.

In general habit, colour and texture very like *D. refulgens* but totally different in venation and scales and much more deeply cut.

49. *Dryopteris yungensis*
Christ et Ros.; Rosenstock
in Fletde. Repert. 5: 234.
1908. — Fig. 11a.

Type from Bolivia:
Yungas australis, Sirapuya
pr. Yanacachi, 2000 m, leg.
O. BUCHTIEN nr. 493 (auth.
specim. in RB!).

Closely related to *D. honesta* and perhaps not specifically distinct. It differs by the position of the sori, which are placed at the middle of the anterior branch of the forked vein, and by the structure of the scales of rachis and costæ beneath, which are broader, yellowish, the cells with large, clear lumina (fig. 3¹⁸). In all other respects the two species are nearly identical.

50. *Dryopteris leptosora*
C. Chr. Index 274. 1905. —
Fig. 11d.

Syn. *Nephrodium micro-*
sorum Hook. spec.
fil. 4: 106. 1862;
Syn. fil. 271.

Dryopteris micro-
sora O. Ktze. Rev.
2: 813. 1891.

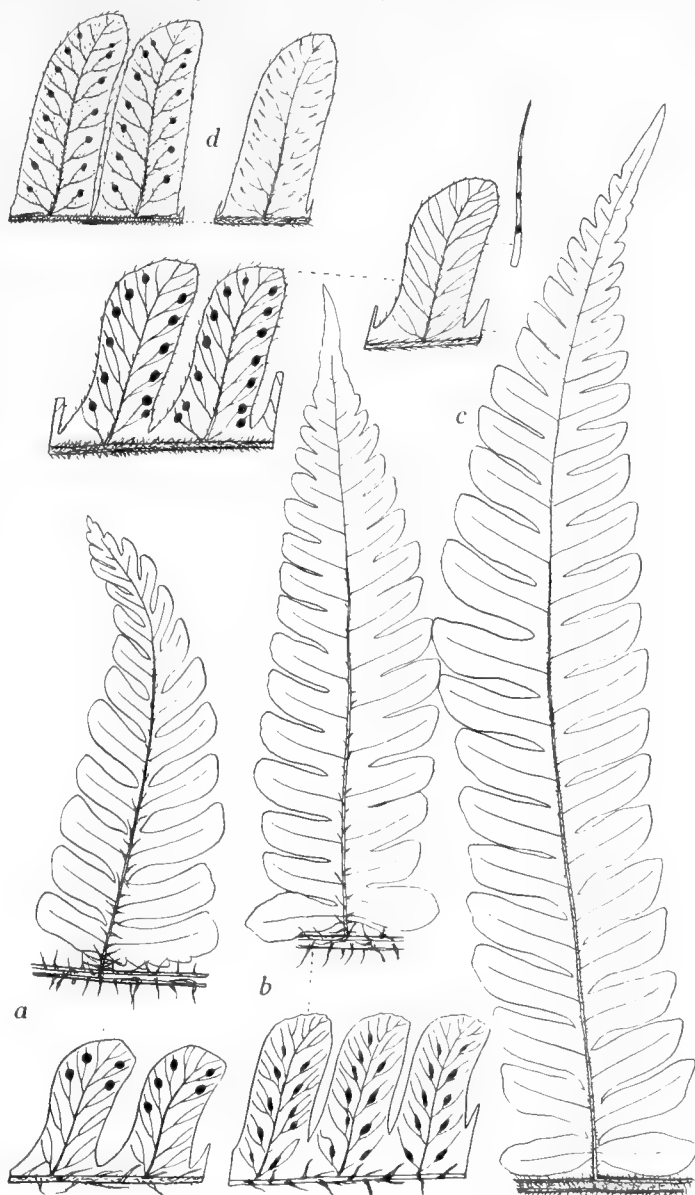


Fig. 11. a. *D. yungensis* Christ. et Ros. (orig.). — b. *D. honesta* (Kze.) C. Chr. (SPRUCE 4712) c. *D. platyloba* (Bak.) C. Chr. (orig.) d. *D. leptosora* C. Chr., two segments seen from the underside and one seen from above (orig.) — All pinnæ $\times \frac{1}{5}$, segments $\times 1\frac{1}{2}$.

Type from Ecuador, Chimborazo, SPRUCE sine num. (Kew!).

Stipe 35 cm long, $\frac{1}{2}$ cm thick, trisulcate above, throughout (at base densely) fibrillose by narrow, lanceolate, hair-pointed, toothed, brown, glossy scales. Lamina bipinnatifid or subbipinnate, 60—70 cm long, grass-green, firmly herbaceous, finely pellucido-punctate; rachis fibrillose like the stipe and densely brown-tomentose above by short, articulated hairs. Pinnæ 12—15 cm long, $3\frac{1}{2}$ —4 cm broad, short-stalked, the lower ones not reduced; upper surface glabrous, the costæ and basal part of the costulæ excepted, which are densely setose by antrorse, subulate, articulated hairs; costæ and costulæ beneath rather densely pubescent by patent, short subulate hairs, which are unicellular or consisting of two or rarely three cells, lower part of costæ fibrillose by narrow, brown, hair-like scales, leaf-tissue minutely pubescent by very small hairs. Pinnæ incised nearly or the lower ones quite to the rachis into close, patent, obtuse, entire or shallowly crenated segments, 6 mm broad; basal segments reduced. Veins distant, rather indistinct, 7—8-jugate, most of them forked with the branches not reaching the margin but terminating in a hydathod which is seen on the upperside within the edge as an oblong, pale-brown dot. Sori small, on the middle of the anterior branch of the forked vein, exindusiate, HOOKER says (l. c. 107): “involucre minute but apparently persistent reniformi-rotundate”; in the original specimen the sori are young and only a few of the sporangia developed; the receptacle bears rather many short, reddish, articulate hairs, which in the dried plant may be mistaken for a small indusium.

A very distinct species of uncertain relationship. It differs from the following species by its colour and especially by its upper pinnæ, which are not broadly adnate to rachis with the lower basal segments decurrent.

51. *Dryopteris platyloba* (Bak.) C. Chr. Index 285. 1905. — Fig. 11c.

Syn. *Polypodium rotundatum* Hk. spec. 4: 238. 1862 (non Willd.).

Polypodium platylobum Bak. Syn. 307. 1867.

Polypodium biseriale Bak. Syn. 309. 1867, pro parte.

Polypodium tarapotensis Bak. Syn. 505. 1874.

Dryopteris tarapotensis C. Chr. Ind. 297. 1905.

Type from Peru, Tarapoto, Mt. Guayrapurima, SPRUCE nr. 4656 (Kew!, also RB).

In Syn. Fil. BAKER cited SPRUCE nr. 4656 as type-number for his three species quoted above. I have the original-specimens of all three species for examination from Kew Herbarium, and I come here upon an instance of species-making, which fortunately is rather uncommon. Certainly the three specimens are not quite identical, that of *P. platylobum* being bipinnatifid with entire segments while the two others are bipinnate below with lobed segments, but the former specimen is only a smaller leaf of the same species of which the other two are a more developed state; there is not the slightest difference to find between the specimens as

to texture, colour, pubescence and other characters. The type-specimen of *P. tarapotense* and that referred to *P. biseriale* are fully identical or rather they belong to the same specimen; *P. tarapotense* was described from the upper half of a single leaf, of which the lower part with the stipe 7 years before was described as *P. biseriale*. The explanation of this BAKER's making a third species on the same collector-number is however quite clear; on the sheet with the upper part of a leaf described as *P. tarapotense* is affixed the lower part of a stipe with "large brown lanceolate scales with a sudden grey edge" (BAKER, l. c. 505). This stipe belongs to a species of *Alsophila* or *Cyathea*!

D. platyloba is a distinct species closely allied to *D. subincisa* but less cut. It agrees with different forms of that collective species by the upper pinnæ being broadly adnate to rachis with the lower basal segment decurrent, in the larger pinnate-pinnatifid pinnæ the basal posterior lobe is similarly adnate and decurrent on the costa. — Stipe 40–50 cm long, trisulcate above, densely clothed below with 1–1½ cm long, glossy, dark-brown, rigid, toothed, linear-lanceolate scales, upwards like rachis fibrillose by similar but smaller scales and rather densely pubescent by subulate, articulated hairs. Lamina up to 1 m long, lanceolate, grey-green or brownish-green when dry, paler beneath, thickly membranous. Pinnæ up to 20 cm long, 2½–4 cm broad, long-acuminate, the lower ones short-stalked, the upper adnate to rachis. Costæ and costulæ above setose by antrorse, subulate, articulated hairs, upperside otherwise glabrous; costæ rather hairy by patent hairs, which are partly short and unicellular, partly longer and pluricellular, subulate, leaf-tissue of underside very minutely and sparsely pubescent; scales of costæ very few, hairlike, brown. Most pinnæ pinnatifid only, still in large specimens fully pinnate at base, those of the basal pair with the basiscop side enlarged. Segments or pinnules 6–10 mm broad, obtuse, entire or deeply lobed, the posterior basal lobe decurrent and adnate to costa. Veins once forked in the entire segments, pinnate in the tertiary lobes not reaching the margin. Sori small, exindusiate, nearer the edge than the midrib, generally on the middle of the anterior branch of the forked vein, or near the apex of the simple veins of the tertiary lobes, 2–3 to each lobe.

52. *Dryopteris biserialis* (Bak.) C. Chr. Index 254. 1905.

Syn. *Polypodium biseriale* Bak. Syn. 309. 1867, pro parte.

Nephrodium subglabrum Sodiro, Cr. vasc. quit. 259. 1893.

Dryopteris subglabra C. Chr. Ind. 295. 1905.

Type from Ecuador, Mt. Tunguragua, SPRUCE sine num. (Kew!); prope San Nicolas, SODIRO (C).

As stated above the Peruvian specimen (SPRUCE 4656) referred to *P. biseriale* by BAKER belongs to the preceding species. I regard here the two other specimens which in Kew are referred to *P. biseriale* by BAKER as the type-specimens of a species, for which I use BAKER's name. It is a species closely allied to *P. platyloba*;

it differs a little in pubescence, the costæ beneath being furnished with some scattered, articulated hairs only, and especially by the position of the sori, which are placed near the base of the branches of the forked vein and nearer the costa and the margin. Generally both branches are soriferous and the sori therefore are in pairs, from which character BAKER took his specific name. The species is somewhat smaller than the preceding but fully tripinnatifid; in colour, texture, the adnate and decurrent segments and upper pinnæ the two species fully agree.

D. grandis (Pr.) subsp. *macroptera* (Klf.) (see Ind. Fil. 269) is a species of the same relationship as the two preceding ones; it is often bipinnatifid only but can be much larger, bipinnate with deeply lobed pinnules. It differs from the species mentioned by its glabrous rachis and costæ. It is common in South Brazil.

Species of uncertain position.

53. *Dryopteris hirsuto-setosa* Hieron. *Hedwigia* 46: 343 tab. 6 fig. 16. 1907.

Type from Ecuador: Baños-Pintuc, STÜBEL nr. 903 (B!).

A bipinnatifid species excellently described and figured by HIERONYMUS and I have nothing to add. It is certainly related to some of the sub-species referred to *D. subincisa* (Ind. Fil. 295), especially *Polypodium Blanchetianum* Kze. and *P. Karstenianum* Kl., with which "subspecies" it agrees in pubescence. As the two sub-species mentioned certainly are closely allied to the true *D. subincisa*, which can be regarded as the typical species of a proper group to which *D. platyloba* and *D. biserialis* also belong, it is very likely that *D. hirsuto-setosa* belongs to the same group. It differs from all the species in this paper referred to *Ctenitis* by its pubescence; the whole leaf is densely pilose by very long, soft, flexible, pluricellular hairs but apparently without scales.

The Brazilian *D. hirtula* (Kze.) C. Chr. is clothed with very similar hairs and is another species of a very doubtful systematic position. It is fully tripinnatifid and, therefore, excluded from the present monograph.

Unknown species of § *Ctenitis*.

1. *Phegopteris Blanchetiana* Fée, Gen. 245. 1850—52 — Bahia, BLANCHET nr. 2928.
2. *Aspidium obtusilobum* Fée, 8 mém. 105. 1857; *Dryopteris huatuscensis* C. Chr. Ind. 271. 1905. — Mexico, Huatusco, SCHAFFNER nr. 105.

This is perhaps a form of *D. submarginalis*.

3. *Phegopteris fluminensis* Fée, Cr. vasc. Br. 1: 97. 1869 — Rio de Janeiro, GLAZIOU nr. 965. — In Ind. Fil. referred to *D. deflexa*, but judging from the description it can be every other species.
4. *Aspidium nervatum* Fée, Cr. vasc. Br. 1: 136. 169. — Brazil, Serra os Orgaos, GLAZIOU nr. 1764. — In Ind. Fil. referred to *D. submarginalis*, it is perhaps the same as *D. pedicellata*.

Subgenus 4. *Lastrea* Bory, emend. C. Chr.

Biologiske Arbejder tilegnede Eug. Warming p. 79. 1911.

This subgenus is nearly identical with the group of *Dryopteris opposita* monographically dealt with in my former papers on American species of *Dryopteris* (quoted above pag. 55). I have there in some details mentioned the characters of the group, and I have only a little to add to my earlier treatment. Still my delimitation of the group must be somewhat changed, partly by including some species, which have not a decrescent lamina, partly by excluding some few species, which I now refer to the subgenus *Glaphyopteris*.

The character: a *lamina decrescens* is, like all other single characters, not available as standard-character, which all species of this subgenus have in common and which is found only here. It is evident that a species as *D. blanda* (Fée) C. Chr. is a near relative of *D. oligocarpa*, although it has not the lamina narrowed downwards. Our common *D. thelypteris* is another instance of a species having a non-attenuate leaf. Still such species are exceptions which affirm the rule. Out of the 118 species enumerated below only 3 or 4 have not auriculiform pinnæ. On the other hand species with a *lamina decrescens* are to be found also within other subgenera, f. inst. *Glaphyopteris* and *Steiropteris*. In some species of § *Cyclosorus*, f. inst. the typical *D. mollis*, the lamina is so much narrowed downwards as in most species of § *Lastrea*, and the same can be said on *D. Saffordii* a species of § *Eudryopteris*.

The best and most constant character of § *Lastrea* is the venation, as explained in my "Revision". The veins are always free, as a rule simple (in some few species normally furcate), the basal ones nearly always reaching the margins above sinus and not separated by a cartilaginous membrane as in species of § *Steiropteris* and *Cyclosorus*. The frequent occurrence of sessile glands on the leaf-tissue beneath and of aërophores at the base of the pinnæ are also characteristic for the subgenus. In this paper I have excluded those species having aërophores at the base of the segments and referred them to § *Glaphyopteris*, viz. *D. Canadazii*, *D. Thomsonii* and *D. macradenia*. Most species are rather hairy by simple and, as a rule, unicellular hairs; only in some very long-hairy species (*D. mertensioides*, *D. Ruiziana*, *D. nitens*, *D. multiformis* and some others) the long hairs consist of 2—4 cells, but such hairs are not very like the short, pluricellular hairs so characteristic in species of § *Ctenitis*. The sori are in several species exindusiate, in others furnished with a small, rarely persistent indusium.

Since the publication of my earlier papers I have examined numerous specimens of species belonging to § *Lastrea*, some of which belong to species previously unknown to me and some others to species new to science. In the following pages I give another supplement to my first paper, and the number of species is now increased to 120. As seen from that number the subgenus is extraordinarily

rich in species occurring in tropical America; each new collection received contains new forms. As the key in my "Revision" includes 82 species only I have worked out an entirely new key including all the species known to me. After the key follows a systematical enumeration of all species including descriptions of several new species or of species not seen, as I worked out my "Revision". As will be seen the order of the species is not exactly the same as in "Revision, and further that I have not arranged the species in sections and subsections. I think that my present arrangement is a natural one in that sense that species intimately related to each other are placed side by side. It is possible to separate out smaller groups of closely related species, but such groups are connected with other groups by several intermediate species that a thorough grouping of all the species should be very defective. As rather distinct small groups I shall here mention

1°) the group of *D. rudis*, which includes the species 151—160 and probably others. They are large-grown species with generally dark-green, coriaceous or papyraceous, more or less hairy but always eglandulose lamina, which downwards is suddenly narrowed, the lower 3—5 pinnæ being fully abortive and like small warts along the stipe; the basal segments of the lower pinnæ are similarly reduced; aërophore present; sori exindusiate. In most species the rhizome seems to be creeping. This group appears to be a most distinct one, its species being widely different from those allied to *D. oligocarpa* and *D. opposita*. Still the difference between these and such species as *D. Sprengelii* and *D. Christensenii* is only small and these species again are closely allied to *D. panamensis*, *D. pachyrachis*, *D. tablaizensis* and others, which no doubt are intimately related to *D. opposita* and *D. oligocarpa*. On the other hand *D. rudis* and its relatives are connected with the large bipinnate species *D. pteroidea* by a species as *D. euchlora*, and it is perhaps unnatural to place species as *D. Thomsonii*, *D. macradenia* in another subgenus, *Glaphyopteris*, as they are as to essential characters very near *D. rudis*.

2°) the group of *D. cheilanthoides*, including the species nr. 165—170. Large species of thick texture and numerous veins; glanduliform pinnæ as in the former group and aërophore as a rule present. Rhizome erect; hairs long, soft, pluricellular; underside often glandulose and viscid; basal segments of most species not much reduced, the lower one often prolonged; sori often indusiate. Species of a characteristic texture and colour, which I can not explain. This group is through *D. limbata* and *D. consanguinea* connected with *D. opposita* and *D. sancta*.

3°) The group of *D. rivularioides*, including the species nr. 132—137, characterized by a long-creeping rhizome and occasionally furcate veins. All the species are from South-Brazil and adjacent countries.

4°) The group of *D. sancta*, including the species nr. 56—62, small species of thin texture and often with unequal-sided pinnæ.

Key.

1. Tertiary veins 1—10-jugate. Lamina towards the base gradually, rarely abruptly attenuate, or, in some few species, not at all narrowed downwards.
2. Rhizome erect or decumbent with the stipites fasciculated. Veins nearly always simple.
3. Pinnæ rarely more than $1\frac{1}{2}$ cm broad, 10 cm long.
4. Lamina towards the base very gradually narrowed; reduced pinnæ numerous, stipe very short (type II). Basal segments, especially the upper one, generally longer and often broader than the others. Most species small.
5. Pinnæ entire or subentire, $\frac{1}{2}$ — $1\frac{1}{2}$ cm long.
 6. Pinnæ short-stalked, about 1 cm long; rachis without scales 54. *D. pusilla* (Mett.) O. Ktze.
 6. Pinnæ sessile, $1\frac{1}{2}$ cm long; rachis scaly 55. *D. brachypoda* (Bak.) C. Chr.
5. Pinnæ deeply lobed, or, as a rule, pinnatifid.
 6. Rachis scaly.
 7. Pinnæ scarcely 2 cm long, auricled on both sides of the base, subentire or subpinnatifid; stipe nearly none 55. *D. brachypoda* (Bak.) C. Chr.
 7. Pinnæ 3—4 cm long, regularly pinnatifid; stipe 3—4 cm long 102. *D. Funckii* (Mett.) O. Ktze.
 6. Rachis without scales.
 7. Veins not prominent above; lamina herbaceous or thinly membranous.
 8. Pinnæ unequal-sided, the anterior (upper) side being broader than the posterior one.
 9. Rhizome small, erect. Pinnæ pinnatifid, generally glandulose beneath.
 10. Indusium absent or minute. 56. *D. sancta* (L.) O. Ktze.
 10. Indusium large, persistent.
 11. Lamina herbaceous; sori about medial; reduced pinnæ few 57. *D. sanctiformis* n. sp.
 11. Lamina membranous to coriaceous; sori close to the margin 59. *D. consanguinea* (Fée) C. Chr.
 9. Stem epigæous, very long and scaly; pinnæ with a straight, cuneate, entire base on the lower side, not deeply cut, eglandulose 58. *D. longicaulis* (Bak.) C. Chr.
 8. Pinnæ equal-sided, but sometimes auricled at the upper base only.
 9. Leaf thinly herbaceous. Small species with pinnæ 2—3 cm long; veins 2—3-jugate.
 10. Segments entire, the margins not revolute.
 11. Sori supramedial; lamina scarcely 30 cm long 60. *D. delicatula* (Fée) C. Chr.
 11. Sori medial; lamina up to 50 cm long 61. *D. pseudosanta* C. Chr.
 10. Segments crenate with revolute margins 62. *D. physematioides* (Kuhn et Christ) C. Chr.
 9. Leaf firmly herbaceous or membranous. Most species larger with pinnæ 3—10 cm. long.

10. Segments short, obtuse; veins 3—7 to a side.
 11. Pinnæ obtuse 65. *D. coarctata* (Kze.) C. Chr.
 11. Pinnæ acuminate.
 12. Lamina glabrous or hairy on the rachis and costæ only.
 13. Sori medial. Under surface slightly or not glandular 64. *D. riopardensis* Ros.
 13. Sori near the margin. Under surface densely glandulose.
 14. Pinnæ close; segments entire, the basal ones not auricled
 63. *D. opposita* (Vahl) Urb.
 14. Pinnæ distant; segments toothed at the apex, the basal ones with an interne auricle
 59. *D. consanguinea* var. *aequalis* C. Chr.
 12. Lamina more or less hairy on the surfaces.
 13. Under surface glandular. Pinnæ auricled at the upper base
 65. *D. coarctata* (Kze.) C. Chr.
 13. Under surface not glandular. Pinnæ not auricled, shortly pubescent throughout; indusium very pilose
 66. *D. leucothrix* C. Chr.
10. Segments long, linear, falcate or very oblique; veins 7—9-jugate.
 11. Lamina downwards gradually narrowed.
 12. Under surface glandular
 67. *D. panamensis* (Pr.) C. Chr.
 12. Under surface not glandular
 144. *D. scalaris* (Christ) C. Chr.
 11. Lamina downwards abruptly narrowed, eglandulose..... 68. *D. silviensis* Hieron.
7. Veins prominent above, occasionally furcate. Lamina chartaceous or coriaceous.
 8. Lamina chartaceous or membranous, nearly glabrous; pinnæ as a rule unequal-sided 59. *D. consanguinea* (Fée) C. Chr.
 8. Lamina coriaceous, hairy on the ribs; pinnæ scarcely unequal-sided 69. *D. sculpturoides* (Fée) C. Chr.
4. Lamina towards the base shortly attenuate with 1—4 pairs of reduced pinnæ (type I), or, in some species, without auriculi-form pinnæ. Basal segments equal or shorter than the others, seldom a little longer. Middle-sized species.
5. All pinnæ sessile. Sori round.
 6. Rachis (and costæ beneath) without scales, at least the scales are very few and deciduous.
 7. Sporangia setose. Indusium absent or rarely found.
 8. Rachis and costæ beneath densely pulverulent by very short hairs, long hairs absent
 70. *D. concinna* (Willd.) O. Ktze.
 8. Rachis and costæ beneath sparsely pubescent by longer hairs.

9. Andine species. Pinnæ opposite, firm; veins about 8-jugate 71. *D. rufa* (Poir.) C. Chr.
9. South Brazilian species. Pinnæ alternate, herbaceous; veins about 5-jugate 72. *D. Stierii* (Ros.) C. Chr.
7. Sporangia glabrous.
8. Lamina herbaceous or membranous.
9. Rachis subglabrous or more or less pubescent by long hairs, not densely pulverulent by very short hairs.
10. Lower pinnæ not or a little reduced, not auriculiform.
11. Pinnæ 3—4 cm long; rachis and costæ on both sides pubescent. Indusium none 73. *D. blanda* (Fée) C. Chr.
11. Pinnæ 6—7 cm; rachis and costæ beneath glabrous. Indusium present 74. *D. tablana* (Christ) C. Chr.
10. 1—5 pairs of lower pinnæ much reduced, auriculiform or glanduliform.
11. Stipe at base with many long, brown, squarrose scales.
12. Pinnæ about 6 cm long, rigidly membranous or papyraceous.
13. Reduced pinnæ few (1—2 pairs). Veins above with long setæ 75. *D. Rimbachii* Ros.
13. Reduced pinnæ in 4—5 pairs. Veins above without long setæ.
14. Lower pinnæ gradually reduced. Upperside hairy, not glossy 76. *D. brachypus* (Sod.) C. Chr.
14. Lower pinnæ suddenly reduced. Upperside glabrous, glossy 77. *D. supranitens* Christ.
12. Pinnæ about 12 cm long, not glossy above, thinly membranous 78. *D. tablaziensis* Christ.
11. Stipe at base with a few scattered, short scales or scaleless.
12. Hairs of rachis beneath none or few, early deciduous.
13. Pinnæ unequal-sided, the anterior (upper) side being broader than the posterior one.
14. Pinnæ not deeply cut with a straight, entire, cuneate base on the lower side. Underside eglandulose. Veins 1—3-jugate 58. *D. longicaulis* (Bak.) C. Chr.
14. Pinnæ regularly pinnatifid on both sides. Underside glandulose. Veins 3—4-jugate 57. *D. sanctiformis* n. sp.

13. Pinnæ equal-sided.
 14. Lamina thinly herbaceous. Segments broad, very oblique.
 15. A distinct tuberculiform ærophore at the base of the pinnæ beneath. Sori infra-medial. Guadeloupe
 81. *D. hydrophila* (Fée) C. Chr.
 15. No distinct ærophore. Sori supramedial. Argentina
 82. *D. Lorentzii* (Hieron.) C. Chr.
 14. Lamina firmly herbaceous-membranaceous. Segments not very oblique.
 15. Veins 4–6-jugate; rachis and surfaces almost quite glabrous.
 16. A single pair of auriculi-form pinnæ
 74. *D. tablana* (Christ) C. Chr.
 16. 3–4 pairs of auriculi-form, often tripartite pinnæ ... 79. *D. Lindigii* C. Chr.
 15. Veins 8–10-jugate.
 16. Lamina downwards abruptly narrowed, chartaceous, sparsely hairy beneath on costæ and costulæ only
 80. *D. lustrata* (Hieron.) C. Chr.
 16. Lamina downwards gradually narrowed, firmly herbaceous or membranous.
 17. Indusium persistent, very setose. Rachis glabrous
 109. *D. palustris* (Mett.) O. Ktze.
 17. Indusium small, deciduous. Rachis sparsely hairy.
 18. Veins indistinct. Lamina firmly membranous, the underside nearly glabrous throughout, the upperside sparsely pubescent
 110. *D. Hieronymusii* C. Chr.

- 18. Veins distinct.
Lamina firmly
membranous, the
underside spar-
sely pubescent,
the upperside
subglabrous
- 83. *D. argentina* (Hieron.) C. Chr.
- 12. Hairs of rachis beneath persistent, often
many and sometimes pluricellular.
- 13. Indusium none or very small, rarely
seen.
- 14. Rhizome erect or obliquely erect.
- 15. Hairs of rachis long, patent,
often pluricellular.
- 16. Stipe more or less hairy,
not clothed with a dense
mass of woolly hairs.
- 17. Lamina herbaceous;
basal segments with-
out auricles
- 84. *D. oligocarpa* (H. B. W.) O. Ktze.
- 17. Lamina thickly mem-
branous; basal seg-
ments with an in-
terne auricle
- 170. *D. multiformis* n. sp.
- 16. Stipe and lower part of
rachis very densely hairy
by woolly, patent, long
hairs 166. *D. lanipes* C. Chr.
- 15. Hairs of rachis short, most-
ly adpressed, unicellular.
Andine species.
- 16. Veins 7—8 to a side in
the slightly oblique seg-
ments. . . 86. *D. rivulariformis* Ros.
- 16. Veins 10—11 to a side
in the falcate-ligulate
segments
- 112. *D. utañagensis* Hieron.
- 14. Rhizome decumbent or short-
creeping. South Brazil
- 87. *D. Regnelliana* C. Chr.
- 13. Indusium persistent.
- 14. Rachis densely pilose by long,
patent hairs
- 85. *D. pilosula* (Kl. et Karst.) Hieron.
- 14. Rachis short-hairy.
- 15. Indusium black. Guatemala
- 118. *D. melanochlaena* C. Chr.
- 15. Indusium pale. Rhizome
short-creeping.

16. Leaf-tissue of both sides glabrous. Indusium glabrous 88. *D. rioverdensis* C. Chr.
16. Leaf-tissue of both sides finely pubescent. Indusium setose 86. *D. rivulariformis* Ros.
9. Rachis densely pulverulent by very short hairs, long hairs being absent or few among the short ones.
10. Lamina towards base gradually and shortly attenuate. West-Indian and Andine species.
11. Indusium persistent, densely setose. Underside densely glandular. Jamaica 89. *D. Nockiana* (Jenm.) C. Chr.
11. Indusium none or small. Underside not or sparsely glandular.
12. Leaf throughout hairy by rather rigid hairs.
13. Veins above without solitary, long setæ..... 90. *D. muzensis* Hieron.
13. Veins above with solitary, long setæ..... 91. *D. columbiana* C. Chr.
12. Leaf glabrous or very inconspicuously hairy between the veins.
13. Hairs of costæ and veins minute; underside slightly glandulose. Sori supramedial with a glandulose indusium. Cuba..... 92. *D. piedrensis* C. Chr.
13. Hairs of costæ and veins rather long, rigid; underside eglandulose. Sori medial, exindusiate. Andes 93. *D. boqueronensis* Hieron.
10. Lamina towards base abruptly attenuate.
11. Andine species..... 94. *D. lepidula* Hieron.
11. Brazilian species..... 95. *D. Lindmani* C. Chr.
8. Lamina coriaceous or rigidly papyraceous.
9. Stipe and rachis glossy, atropurpureous.
10. Costæ beneath glabrous. 96. *D. laevigata* (Mett.) C. Chr.
10. Costæ beneath strigose.... 97. *D. Crossii* (Bak.) C. Chr.
9. Stipe and rachis stramineous or greyish.
10. Pinnæ scarcely $2\frac{1}{2}$ cm long, often subentire; veins 2—4-jugate. Stipe much longer than the lamina..... 99. *D. Millei* n. sp.
10. Pinnæ 4—10 cm long, deeply pinnatifid; veins 4—10-jugate. Stipe as a rule shorter than the lamina.
11. Rachis densely pulverulent by very short hairs, without long hairs.. 92. *D. piedrensis* C. Chr.
11. Rachis subglabrous or more or less hairy by longer hairs.
12. Lamina with a rounded base, the lower 1—2 pairs of pinnæ being auriculiform and rather closely placed.

13. Stipe at base with many long, brown scales 75. *D. Rimbachii* Ros.
13. Stipe at base with a few scattered short scales.
 14. Rachis and costæ beneath short-ly setose 130. *D. Pavoniana* (Kl.) C. Chr.
 14. Rachis and costæ villous by long, pluricellular hairs
 170. *D. multiformis* n. sp.
12. Several pairs of distant gradually or abruptly reduced pinnæ.
 13. Stipe very densely clothed with woolly hairs..... 166. *D. lanipes* C. Chr.
 13. Stipe glabrous or sparsely pubescent.
 14. Lamina gradually narrowed downwards
 98. *D. Galanderi* (Hieron.) C. Chr.
 14. Lamina abruptly narrowed
 80. *D. lustrata* (Hieron.) C. Chr.
6. Rachis (and costæ beneath) more or less scaly.
 7. Scales rather few, small and narrow-subulate, brown.
 8. Whole leaf throughout hairy by short pseudo-stellate hairs, i. e. the hairs are often found in bundles of 2—5 originating from the same epidermal cell
 100. *D. phacelothrix* C. Chr. et Ros. n. sp.
 8. Leaf with ordinary simple hairs.
 9. Lamina coriaceous. Segments broadly triangular
 101. *D. caucaensis* (Hieron.) C. Chr.
 9. Lamina firmly herbaceous. Segments oblong
 102. *D. Funckii* (Mett.) O. Ktze.
 7. Scales of rachis very numerous, broad, yellowish, glandulose 103. *D. velata* (Kze.) O. Ktze.
 5. Lower pinnæ petiolulate.
 6. Pinnæ entire or subentire, $1\frac{1}{2}$ — $1\frac{1}{2}$ cm long. Indusium persistent, densely setose 54. *D. pusilla* (Mett.) O. Ktze.
 6. Pinnæ 2— cm long, deeply lobed or pinnatifid. Indusium absent or minute.
 7. Pinnæ with a truncate base. Sori oblong-linear
 104. *D. aspidioides* (Willd.) C. Chr.
 7. Pinnæ with a cuneate base. Sori oblong-linear or round
 105. *D. ptarmica* (Kze.) O. Ktze.
 3. Pinnæ 2—5 cm broad, 10—15 cm long, the lower 3—5 pairs as a rule reduced (type III). Most species large.
 4. Sori all round.
 5. Rachis without scales. Segments rarely 5 mm broad.
 6. Leaf-tissue glabrous, at least not densely pubescent by adpressed hairs.
 7. Lamina papyraceous or membranous.
 8. Pinnæ pinnatifid only; most segments entire.
 9. Rachis glabrous beneath or deciduously hairy.
 10. Lamina beneath with large, red glands. Sori medial, indusium large, glandulose.
 106. *D. pachyrachis* (Kze.) O. Ktze.

10. Lamina eglandulose.
11. Costæ beneath without scales.
 12. Indusium very large, persistent; sori medial. 106. *D. pachyrachis* (Kze.) O. Ktze.
 12. Indusium small or none.
 13. Sori medial, exindusiate 146. *D. Bradei* Christ.
 13. Sori supramedial or submarginal.
 14. Indusium present; pinnæ about $1\frac{1}{2}$ cm broad; veins above sparsely setose 110. *D. Hieronymusii* C. Chr.
 14. Indusium none; pinnæ 2 cm broad; veins above glabrous 111. *D. roraimensis* (Bak.) C. Chr.
11. Costæ beneath with scattered, brown scales.
 12. Segments with open spaces between; indusium very large, persistent, glandular 106. *D. pachyrachis* var. *Sprucei* (Bak.) C. Chr.
 12. Segments close, acute; indusium smaller, not glandular 113. *D. illicita* Christ.
9. Rachis rather pubescent. Underside eglandulose.
 10. Sori medial. Lamina papyraceous or subcoriaceous.
 11. Segments 2 mm broad, falcate-ligulate 112. *D. utaëgensis* Hieron.
 11. Segments 3—4 mm broad, patent.
 12. Basal segments parallel to rachis 106. *D. pachyrachis* (Kze.) O. Ktze.
 12. Basal segments (of larger pinnæ) overlapping the rachis or with interne auricles.
 13. Rachis short-hairy; costæ beneath without scales; indusium persistent 107. *D. supina* (Sod.) C. Chr.
 13. Rachis long-hairy; costæ beneath somewhat scaly; indusium none 170. *D. multififormis* n. sp.
 10. Sori near the margin. Lamina firmly membranous.
 11. Lamina gradually narrowed downwards with 2—3 pairs of auriculiform pinnæ. Veins 8—9-jugate 120. *D. demerarana* (Bak.) C. Chr.
 11. Lamina abruptly attenuate with several pairs of glanduliform pinnæ. Veins 10—14-jugate 148. *D. Christensenii* Christ.
8. Pinnæ with the lower segments free; segments crenate 114. *D. atropurpurea* Hieron.
7. Lamina thinly herbaceous.
 8. Costæ beneath glabrous or sparsely pubescent by unicellular hairs, without scales.
 9. Sori medial. Rachis glabrous or nearly so; under surface generally glandulose.
 10. Indusium glabrous, glandulose.
 11. Jamaica . . . 106. *D. pachyrachis* var. *Jenmani* (Bak.)

11. South Brazil 108. *D. tenerrima* (Fée) Ros.
10. Indusium densely setose 109. *D. palustris* (Mett.) O. Ktze.
9. Sori supramedial. Rachis pubescent.
 10. Stipe and rachis strong. Basal segments enlarged 63. *D. opposita* var. *amphioxypteris* (Sod.) C. Chr.
 10. Stipe and rachis very slender. Basal segments equal 115. *D. recumbens* Ros.
8. Costæ beneath densely soft-hairy by long, patent, pluricellular hairs and sparsely scaly
 123. *D. nitens* (Desv.) C. Chr.
6. Leaf-tissue, especially above, densely and finely puberulous by adpressed hairs.
7. West-Indian species. Indusium small, setose or absent.
 8. Segments 3—5 mm broad, both surfaces densely downy. Stipe and rachis slender, not scaly
 116. *D. rustica* (Fée) C. Chr.
 8. Segments 5—7 mm broad, underside subglabrous. Stipe strong, scaly 117. *D. Germaniana* (Fée) C. Chr.
7. Continental species.
 8. Indusium persistent.
 9. Indusium black, slightly ciliate. Segments entire; veins 7—10 118. *D. melanochlaena* C. Chr.
 9. Indusium pale, setose. Segments toothed upwards. Veins 10—11 119. *D. atrorubens* (Mett.) C. Chr.
 8. Indusium absent.
 9. Sori near the margin. Lower pinnæ auriculiform
 120. *D. demerarana* (Bak.) C. Chr.
 9. Sori about medial. Lowermost pinnæ glanduliform 147. *D. Rusbyi* C. Chr.
5. Rachis more or less scaly. Segments often 5—7 mm broad.
6. Pinnæ incised two-thirds of the way down to the costa only, glabrous 122. *D. Moritziana* Urban.
6. Pinnæ incised to a narrow wing to the costa.
7. Lamina (costæ above excepted) glabrous. Basal segments enlarged 113. *D. illicita* Christ.
7. Lamina pubescent, at least on the costæ and costulæ beneath. Basal segments not enlarged.
8. Sori medial or inframedial.
 9. Lamina firmly herbaceous to chartaceous. Costæ beneath strigose or hairy by patent, rather stiff, unicellular hairs.
 10. Leaf-tissue glabrous. Sori exindusiate 147. *D. Bradei* Christ.
 10. Leaf-tissue finely pubescent. Costæ beneath patently hairy 121. *D. dominicensis* C. Chr.
 9. Lamina very thin; costæ beneath and margins with numerous soft, patent, pluricellular hairs
 123. *D. nitens* (Desv.) C. Chr.
8. Sori supramedial. Leaf-tissue finely pubescent above.
 9. Rachis very sparsely scaly. Pinnæ distant, the lower 2—3 pairs reduced. Veins 7—9.
 120. *D. demerarana* (Bak.) C. Chr.
 9. Rachis very scaly. Pinnæ closer, about 10 pairs reduced. Veins 10—15. . . 150. *D. corazonensis* (Sod.) C. Chr.

4. Basal sori oblong or linear, the upper ones roundish, oblong or linear.
5. Lamina gradually narrowed downwards.
 6. Basal veins running to the sinus. Reduced pinnæ few. Veins 6—7-jugate 124. *D. diplazioides* (Desv.) Urban.
 6. Basal veins running to the margin above sinus. Reduced pinnæ several. Veins 10—15-jugate.
 7. Veins immersed, 10—12-jugate. Leaf throughout shortly pubescent, 50—60 cm long. Sori medial, oblong-linear 125. *D. consimilis* (Fée) C. Chr.
 7. Veins raised above, 10—15-jugate. Leaf sparsely pubescent by longer hairs, 80—100 cm long. Sori supra-medial, roundish or oblong... 126. *D. heteroclita* (Desv.) C. Chr.
5. Lamina downwards abruptly narrowed.
 6. Lamina finely puberulous throughout. Sori exindusiate 127. *D. atrovirens* C. Chr.
 6. Lamina glabrous between the veins. Indusium small. 128. *D. leptogrammoides* Ros.
2. Rhizome wide-creeping with distant stipites. Veins not rarely furcate.
3. Lamina coriaceous. Reduced pinnæ none or 1—2-jugate.
 4. Sporangia setose. Jamaica 129. *D. firma* (Jenm.) C. Chr.
 4. Sporangia glabrous. Andes 130. *D. Pavoniana* (Kl.) C. Chr.
3. Lamina herbaceous or membranous. Sporangia glabrous.
 4. Lower 3—8 pairs of pinnæ reduced.
 5. Rachis and costæ beneath sparsely scaly. Andine species 131. *D. Rosenstockii* C. Chr.
 5. Rachis and costæ without scales. South-Brazilian species.
 6. Leaf-tissue glabrous.
 7. Pinnæ up to 8 cm long, scarcely narrowed towards their base 132. *D. Santa Catharinae* Ros.
 7. Pinnæ up to 18 cm long; narrowed gradually towards their base 133. *D. Jürgensii* (Ros.) C. Chr.
 6. Leaf-tissue beneath pubescent.
 7. Lamina abruptly narrowed downwards.. 134. *D. Mosenii* C. Chr.
 7. Lamina gradually narrowed downwards.
 8. Pinnæ deltoideo-oblong. Veins often furcate 135. *D. rivularioides* (Fée) C. Chr.
 8. Pinnæ linear-lanceolate. Veins simple .. 136. *D. scariosa* Ros.
 4. Lower pinnæ not reduced at least not auriculiform. Most veins furcate. Lower segments of larger pinnæ quite free. Veins simple 137. *D. tremula* Christ.
 1. Veins 12—25-jugate, generally close.
 2. Lamina bipinnatifid; no segments free.
 3. Sporangia glabrous.
 4. Argentine and South-Brazilian species.
 5. Aërophore none. Rhizome creeping. Costæ more or less pilose.
 6. Veins 14—16-jugate, pellucid. Lamina thinly herbaceous 138. *D. siambonensis* (Hieron.) C. Chr.
 6. Veins 16—18-jugate. Lamina firm 139. *D. achalensis* (Hieron.) C. Chr.
 5. Aërophore present. Rhizome erect. Lamina with some few scattered long hairs or perfectly glabrous.

6. Margins of segments revolute covering the sori. Lamina coriaceous, glandular beneath. Veins simple. Lower basal segments prolonged 169. *D. cheilanthoides* (Kze.) C. Chr.
6. Margins plane. Lamina membranous, eglandulose. Veins often furcate. Lower basal segments not prolonged 162. *D. Glaziovii* (Christ.) C. Chr.
4. West Indian and Andine species.
 5. Lamina herbaceous or membranous; margins plane or slightly revolute.
 6. Rachis not or very slightly scaly.
 7. Segments entire or inconspicuously toothed, the basal ones without auricles.
 8. Rachis glabrous or with few, deciduous hairs.
 9. Lamina glandulose beneath 140. *D. Sprengelii* (Klf.) O. Ktze.
 9. Lamina eglandulose.
 10. Segments 2—4 mm broad. Leaf-tissue of both sides minutely puberulous. Sori close to the edge. Veins 10—15-jugate.
 11. 3—4 pairs of lower pinnæ auriculiform. Segments very close entire 141. *D. struthiopteroides* C. Chr.
 11. 3—4 pairs of lower pinnæ glanduliform. Segments not close, often serrate. 143. *D. conformis* (Sod.) C. Chr.
 10. Segments 5—7 mm broad. Leaf entirely glabrous; costæ beneath with some scattered light-brown scales. 161. *D. euchlora* (Sod.) C. Chr.
 8. Rachis and costæ more or less pilose.
 9. Sori indusiate. Lamina glandulose beneath 142. *D. Mercurii* (A. Br.) Hieron.
 9. Sori exindusiate, or indusium very small. Lamina eglandulose.
 10. Lower pinnæ very gradually reduced.
 11. Sori oblong or linear see nr. 125. *D. consimilis* and nr. 126. *D. heteroclita*.
 11. Sori round. 144. *D. scalaris* (Christ.) C. Chr.
 10. Lower pinnæ abruptly reduced, the lowermost glanduliform.
 11. Leaf-tissue of upperside glabrous.
 12. Sori medial. Rachis with some few scales. 146. *D. Bradei* Christ.
 12. Sori supramedial. Rachis densely ochraceo-tomentose without scales 154. *D. rudis* (Kze.) C. Chr.
 11. Leaf-tissue of upperside finely pubescent.
 12. Sori medial. Costæ on both sides setose. Bolivia. 147. *D. Rusbyi* C. Chr.
 12. Sori near the margin. Rachis and costæ beneath very shortly and densely pulverulent 148. *D. Christensenii* Christ.
 7. Segments crenate or more or less lobed.
 8. Basal segments with an interne auricle which overlaps the rachis.

9. Sori close to the margin. West-Indian species
149. *D. limbata* (Kze.) O. Ktze.
9. Sori about medial. Ecuador 170. *D. multiformis* n. sp.
8. Basal segments without interne auricle.
9. Lamina rather gradually reduced downwards
140. *D. Sprengelii* var. *Sherringii* (Jenm.).
9. Lamina abruptly reduced, the lowermost pinnæ glanduliform.
10. Both surfaces finely pubescent. Segments 2—4 mm broad 143. *D. conformis* (Sod.) C. Chr.
10. Lamina quite glabrous. Segments 5—7 mm broad 161. *D. euchlora* (Sod.) C. Chr.
6. Rachis rather scaly. Lamina suddenly with 5—8 pairs of auriculiform pinnæ 150. *D. corazonensis* (Sod.) C. Chr.
5. Lamina coriaceous or chartaceous with the margins more or less revolute, downwards abruptly narrowed with 2—5 pairs of distant, glanduliform pinnæ. Costæ beneath often with some few scales.
6. The revolute margins not covering the sori. Basal segments of larger pinnæ generally much reduced.
7. Costæ beneath glabrous or more or less pilose by short, unicellular hairs.
8. Lamina quite glabrous 151. *D. semilunata* (Sod.) C. Chr.
8. Lamina (at least rachis) more or less pilose.
9. Pinnæ about 5 cm long with very oblique segments 152. *D. canelensis* Ros.
9. Pinnæ 10—20 cm long, the segments often falcate but not very oblique.
10. Costæ beneath setose by more or less antrorse stiff hairs and generally furnished with scattered small, brown scales. Indusium none.
11. Sori medial. Largest pinnæ scarcely 12 cm long. Lamina (costa excepted) subglabrous
153. *D. nervosa* (Kl.) C. Chr.
11. Sori supramedial. Largest pinnæ 15—20 cm long. Lamina, especially beneath, setose.
12. Segments rather close with acute sinuses between 154. *D. rudis* (Kze.) C. Chr.
12. Segments patent separated by broad open sinuses 155. *D. Engelii* Hieron.
10. Costæ beneath pilose by patent hairs.
11. Largest pinnæ 10—18 cm long, horizontal or ascendent, sparsely strigose beneath on costæ and costulæ only. Receptacle glabrous. Segments close, oblique or sub-falcate.
12. Basal segments without auricle.
13. Veins 11—12-jugate. Segments obtuse 158. *D. strigifera* Hieron.
13. Veins 14—15-jugate. Segments acute 159. *D. Brausei* Hieron.
12. Basal segments with an interne auricle
170. *D. multiformis* n. sp.

11. Largest pinnæ up to 30 cm long, often pendent, rather densely hairy by shorter hairs and longer patent "strigæ" on the costæ beneath; segments patent separated by broad sinuses. Receptacle with long hairs 160. *D. piloso-hispida* (Hook.) C. Chr.
7. Costæ beneath patently and softly hairy by long pluricellular hairs.
8. Stipe glabrous or shortly pubescent.
9. Basal segments not auricled, sori close to the edge.
10. Pinnæ 10—15 cm long, scarcely 2 cm broad; costæ beneath with numerous long, pluricellular hairs 168. *D. mertensioides* C. Chr.
10. Pinnæ up to 30 cm long, 3—4 cm broad; hairs shorter, seldom pluricellular
160. *D. piloso-hispida* (Hk.) C. Chr.
9. Basal segments with an auricle at the inner side. Sori about medial.
10. Segments linear-falcate with revolute margins, obtuse, the basal ones considerably reduced. Costæ beneath with several large, light-brown scales 165. *D. Ruiziana* (Kl.) C. Chr.
10. Segments from a broad base tapering to the acute point, the lower basal one not or a little reduced. Costæ beneath very sparsely scaly
170. *D. multififormis* n. sp.
8. Stipe, especially below, with a dense mass of woolly hairs 166. *D. lanipes* C. Chr.
6. Sori, at least the upper ones, covered by the revolute margins of the segment. Basal segments not or a little reduced, the lower one often prolonged.
7. Segments crenate 167. *D. horrens* Hieron.
7. Segments entire.
8. Margins densely ciliate; lower basal segment not prolonged and not auricled ... 168. *D. mertensioides* C. Chr.
8. Margins scarcely ciliate; lower basal segment always prolonged and auricled at the inner side
169. *D. cheilanthoides* (Kze.) C. Chr.
3. Sporangia setose. Species closely allied to *D. rudis*.
4. South Brazil 157. *D. Heineri* C. Chr.
4. Peru 156. *D. peruviana* Ros.
2. Lamina bipinnate or the lower segments of the larger pinnæ quite free. —
3. Basal segments or pinnules much reduced, not auricled. Hairs unicellular.
4. Lamina rigidly coriaceous, quite glabrous 151. *D. semilunata* (Sod.) C. Chr.
4. Lamina herbaceous or membranous, at least the costæ pubescent.
5. Only the basal segments free 161. *D. euchlora* (Sod.) C. Chr.
5. Several free pinnules.
6. Pinnulæ (and segments) about 2 cm long, 2=3 mm broad, finely pubescent throughout, grass-green, herbaceous. Veins simple 145. *D. Bonapartii* Ros.
6. Pinnulæ 3—5 cm long, $\frac{1}{2}$ cm broad, often deeply lobed, membranous, dark-green. Veins furcate.

7. Rachises and costæ glabrous or sparsely setose.
 8. Brazilian species 162. *D. Glaziovii* (Christ) C. Chr.
 8. Andine species. Sori close to the edge
 163. *D. pteroiidea* (Kl.) C. Chr.
 7. Rachises and costæ villous. Sori about medial
 164. *D. cochaensis* n. sp.
 3. Basal segments, at least the lower one, not reduced and auricled
 at the inner side. Hairs long, pluricellular 170. *D. multiformis* n. sp.

54. *Dryopteris pusilla* (Mett.) O. Ktze. Rev. Gen. Pl. 2: 813. 1891; C. Chr. Ind. 287. — Fig. 12 a.

Syn. *Aspidium pusillum* Mett. Ann. sc. nat. V. 2: 245. 1864.

Type from Colombia, Fusagasuga, 1900 m, leg. LINDIG nr. 92 (B!), Bogotá, STÜBEL nr. 413 part. et 427 (B).

Peru, St. Gavan, LECHLER nr. 2242 (B); Sachapeta, LECHLER nr. 2691 (B).

A species of doubtful position, not nearly related to any other. It is possible that it belongs to § *Cyclosorus*, which the very hairy, persistent indusia seem to indicate, still I place it here provisionally. The lamina is narrowed downwards gradually and the stipe is very short, but the lowermost pinnæ are not so small as could be expected in a species belonging to the group of *D. opposita*.

55. *Dryopteris brachypoda* (Bak.) C. Chr. Ind. 255. 1905. — Fig. 12 b.

Syn. *Nephrodium brachypodum* Bak. Timehri 5: 213. 1886; Trans. Linn. Soc.

II. Bot. 2: 290. 1887; Jenman, W. Ind. and Guiana Ferns 207.

Type from British Guiana, upper slope of Mt. Roraima, IM THURN nr. 275 (Kew!).

A small species in general habit very much resembling *D. sagittata*, but a true *Lastrea* intermediate between *D. pusilla* and *D. coarctata*.

Rhizome erect, sparsely scaly. Leaves densely fasciculate, practically without stipe, lanceolate, 15 cm long, 2½ cm broad at the middle, narrowed very gradually towards the base through several pairs of dwindling pinnæ, firmly membranous, dark-green; rachis strigose, especially on the sides, glabrous in the furrow above, and clothed with many small, brown scales. Pinnæ sessile, alternate, horizontal, the largest 1½ cm long, 5–6 mm broad, obtuse, auricled on both sides of the base, slightly pilose on the costæ and main veins beneath, underside finely glandulose, the upper and lower ones entire, the middle and largest ones crenate or lobed one third of the way to the midrib, margins somewhat revolute. Secondary veins forked in the entire pinnæ, in the lobed pinnæ with 2–3 tertiary simple veins. Sori supramedial, furnished with rather large, persistent, at least red-brown, subglabrous indusia. Sporangia glabrous.

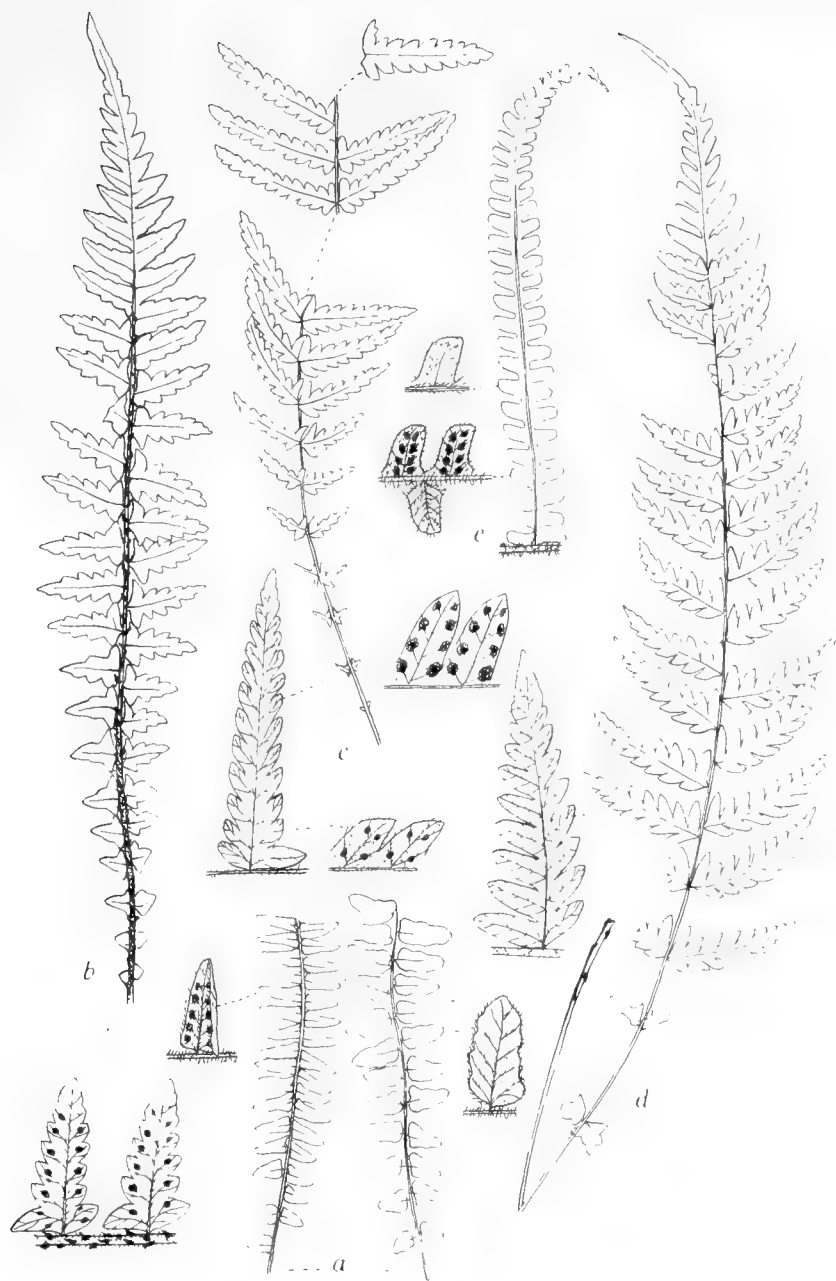


Fig. 12. *a.* *D. pusilla* (Mett.) O. Ktze. Lower part of two leaves, $\times \frac{4}{5}$, and two pinnae, fertile and sterile, $\times 1\frac{1}{2}$. (orig.). — *b.* *D. brachypoda* (Bak.) C. Chr., entire leaf, $\times \frac{4}{5}$, and two pinnae, $\times 1\frac{1}{2}$ (orig.). — *c.* *D. pseudosancta* C. Chr., base and middle part of a leaf, $\times \frac{4}{5}$, pinna, $\times 1\frac{1}{2}$ and two segments, $\times 3$ (orig.). — *d.* *D. sanctiformis* n. sp., entire leaf, $\times \frac{4}{5}$, pinna $\times 1\frac{1}{2}$ and two segments $\times 3$ (orig.). — *e.* *D. leucothrix* C. Chr., pinna $\times \frac{4}{5}$, three segments seen from the underside and one seen from above, $\times 1\frac{1}{2}$ (orig.).

This is a more robust plant than *D. pusilla* with all free pinnæ auricled on both sides of the base and the central ones lobed with subpinnate secondary veins. *D. coarctata* has most pinnæ regularly pinnatifid and a scaleless rachis. *D. sagittata* is apparently similar, but it belongs to § *Goniopteris* and its hairs are stellate.

56. ***D. sancta*** (L.) O. Ktze.; C. Chr. Revision nr. 32 fig. 20; Smiths. Misc. Coll. 52: 378.

Area: Dominica, Porto Rico, Haïti, Jamaica, Cuba, Guatemala.

57. ***Dryopteris sanctiformis*** n. sp. — Fig. 12d.

Type from Panama, eastern slope of Chiriqui Volcano, 2100—2300 m; leg. MAXON nr. 5294 (W).

Rhizomate erecto, breve, dense radicante, squamis parvis ovatis parce onusto. Stipitibus dense fasciculatis, gracilibus, rubrescentibus, nitidis, ad basin sparse paleaceis, glabris, 6—10 cm longis. Lamina anguste lanceolata, ad 22 cm longa, medio 3—4 cm lata, versus basin breviter attenuata, gramineo-viridi, firmo-herbacea, bipinnatifida; rachi gracili, brevissime et tenuissime puberula. Pinnis multijugis, sessilibus, alternis vel inferioribus suboppositis, falcatis, infimis 2—3 paris auriculiformibus, medialibus maximis, 2—2½ cm longis, basi 5—8 mm latis, acutis, supra ad costas sparse setosis, infra ubique glandulosis, inaequaliter pinnatifidis. Laciniis 6—7-jugis, ovato-oblongis, obliquis, acutis, anterioribus majoribus, basali anteriore semper maxima, posterioribus minoribus, basali minima. Venis simplicibus, 3—4-jugis. Soris supramedialibus, indusio late affixo glanduloso persistente obtectis. Sporangiiis glabris.

This new species resembles very much certain forms of *D. sancta*, mainly in size and its unequal-sided pinnæ; it differs by its indusiate sori and its lamina being shortly narrowed downwards about as in *D. concinna* or *D. Lindigii*, but its real affinity is with *D. sancta* and *D. pseudosancta*. The indusium is peculiar; it is glossy, red-brown and impressed in the centre, while the free edges are greenish or whitish and glandular; it is not exactly reniform but rather variable in shape.

To this species I refer SPRUCE nr. 5297 (RB) from Ecuador that is rather typical and further two other specimens, which are not glandular beneath and somewhat more pubescent but agree in size and shape of pinnæ:

Venezuela, EGGERS nr. 13260 (H).

Ecuador, Andes of Quito, SPRUCE nr. 5297 A (H, L).

58. ***Dryopteris longicaulis*** (Bak.) C. Chr. Index 275. 1905.

Syn. *Nephrodium longicaule* Baker, Journ. Bot. 1881: 204, et in Hook. Icon. pl. tab. 1658.

Type from Colombia, Province of Antioquia, leg. KALBREYER nr. 1454 (Kew!).

A remarkable species well figured by BAKER. It belongs to the small group of *D. sancta* by its narrow leaf and unequal-sided pinnæ differing from all other

species by its very long, epigæous caudex, about 20 cm long, $\frac{1}{2}$ cm thick, clothed throughout with squarrose, glossy, castaneous, lanceolate-acuminate, narrow, pubescent scales. Stipe glossy, glabrous, about 5—6 cm long to the lowermost pair of auriculiform pinnæ. Lamina 40—50 cm long, 5 cm broad, downwards rather suddenly narrowed, light-green when alive, brownish when dried, firmly herbaceous, the rachis slender, very slightly strigose above. Pinnæ very numerous, sessile, the lower 4—5 pairs opposite, very remote and very small, auriculiform; above them a single pair of larger, auriculiform pinnæ, the next following scarcely shortened; middle pinnæ alternate, 3 cm long, 7—8 mm broad, acute, auricled at the upper base, cuneate on the lower side, very finely pubescent on the upper surface, glabrous and eglandulose beneath, entire in the outer third, the upper margin lobed in the lower two-thirds about $\frac{1}{3}$ of the way to the midrib, the lower one lobed in the centre only. Upper basal lobe the largest, making the pinnæ auricled, central lobes about as broad as long, obtuse. Veins pinnate in the lobes with 1—3 pairs of tertiary simple veins. Sori medial, small; indusium subpersistent, brown, glabrous. Sporangia glabrous.

In general habit this species resembles not a little a species of *Asplenium*, f. inst. *A. unilaterale* Lam., because the lower half of the basiscop side of the pinnæ is nearly cut away. The peculiar scaly stem not considered it differs from *D. sancta* and *D. sanctiformis*, its nearest relatives, by the less cut pinnæ and the lamina not being gradually but suddenly narrowed downwards.

59. *D. consanguinea* (Fée) C. Chr. Revision nr. 33 fig. 21; Smiths. Misc. Coll. 52: 380.

Area: Trinidad, Grenada, Martinique, Dominica, Guadeloupe. Jamaica (var. *aequalis*). Panama.

This species may be a small form of *D. limbata*. I refer to it a peculiar variety from Panama, Chiriqui, 1000—1300 m; MAXON nr. 5243a (W). It is in habit perfectly typical, but the underside is eglandulose and the sori are very small, consisting of some few (3—4) sporangia, without trace of indusia and placed very close to the margin nearly apical on the veins.

60. *D. delicatula* (Fée) C. Chr. Revision nr. 30.

Area: Guadeloupe. — The lamina is about 30 cm long, not 3 cm as erroneously stated in "Revision" 295.

D. hydropbila (Fée) C. Chr. and *D. caribaea* (Jenm.) C. Chr. do not belong here.

61. *D. pseudosancta* C. Chr. Smiths. Misc. Coll. 52: 378. — Fig. 12c.

Area: Costa Rica, Guatemala.

Dryopteris negligens (Jenm.) C. Chr. Index 279. — *Nephrodium negligens* Jenm. Bull. Bot. Dept. Jam. n. s. 3: 20. 1896. — Jamaica, is according to a photograph of the type-specimen in JENMAN'S type-herbarium in New York Bot. Garden, together with a small fragment kindly sent me by Miss SLOSSON a small species allied to *D. sancta*, but not unlike small forms of *D. oligocarpa*; it agrees with the latter in pubescence; some of the pinnæ (3×1 cm) are rather unequal-sided. It may be a valid species.

62. **D. physematioides** (Kuhn et Christ) C. Chr., Revision nr. 31.

Area: San Domingo.

63. **D. opposita** (Vahl) Urban; C. Chr. Revision nr. 25 fig. 15, 16; Smiths. Misc. Coll. 52: 375.

Area. Type: Lesser Antilles, common. Different forms occur in Jamaica, Cuba, Florida, Mexico, Colombia—Peru. var. *rivulorum* (Raddi): Southern Brazil.

I now regard *D. amphioxypteris* (Sod.) C. Chr., Revision nr. 52 as a variety of *D. opposita*, at least I can not distinguish specifically from that species an authentic specimen in Kew. It has the basal segments considerably enlarged and often lobed, and it has 8—10 veins to a side, but as to other characters it agrees very well with the typical form.

64. **D. riopardensis** Ros.; C. Chr., Revision nr. 26 fig. 17.

Area: South Brazil, Rio Grande do Sul.

This is most probably a form of *D. opposita* var. *rivulorum*.

65. **D. coarctata** (Kze.) C. Chr., Revision nr. 27 fig. 18.

Area: Venezuela—Colombia. Cuba, WRIGHT nr. 3925 (S, W), previously determined as *D. sagittata*, which it resembles in general habit but totally different in pubescence.

66. **D. leucothrix** C. Chr. Smiths. Misc. Coll. 52: 377. — Fig. 12 e.

Area: Bolivia.

67. **D. panamensis** (Pr.) C. Chr., Revision nr. 28 fig. 19. Smiths. Misc. Coll. 52: 376.

Area: Porto Rico, Jamaica, Cuba; Mexico to Panama, very common.

Additional synonyms of this variable species are, according to the original specimens, the following:

Lastrea Leiboldiana Pr. Epim. 41. 1849, Mexico, LEIBOLD (Hb. PRESL!), exactly identical with *P. litigiosum* Liebm.

Aspidium Ghiesbreghtii Fourn. Mex. Pl. 1: 94, 1872, Mexico, GHIESBREGHT, BOURGEAU nr. 3615 (Mus. Paris!).

Nephrodium caribaeum Jenm. Journ. Bot. 1886: 270; Bull. Dept. Jam. n. s. 3: 21. 1896.

Dryopteris caribaea C. Chr. Ind. 257. 1905.

Jamaica; Mt. Diabolo, SHERRING (Kew, authentic specimen; New York Bot. Gard., a leaf of JENMAN's type collection received from Miss SLOSSON).

Nearly identical with the narrow form of *D. panamensis* upon which PRESL founded his new species. JENMAN does not mention the glandular underside.

Polypodium gracilentum Jenm., Bull. Dept. Jam. n. s. 4: 129. 1897; *Dryopteris gracilenta* C. Chr. Ind. 268, which I believed (Revision p. 294) to be *D. panamensis* is not that species according to two pinnæ of the type-specimen in the JENMAN collection in New York Bot. Garden, kindly sent me by Miss MARGARET SLOSSON. It may be a distinct species, allied to *D. scalpturoides*. The sori are apparently indusiate, but the indusia are very small with long hairs, which nearly completely conceal the young sporangia.

68. *D. silviensis* Hieron.; C. Chr. Revision nr. 29.

Area: Colombia.

69. *D. scalpturoides* (Fée) C. Chr., Revision nr. 34, fig. 22; Smiths. Misc. Coll. 52: 381.

Area: Cuba, Jamaica.

70. *D. concinna* (Willd.) O. Ktze.; C. Chr. Rev. nr. 1 fig. 2; Smiths. Misc. Coll. 52: 369.

Area: Guadeloupe, Jamaica, Cuba, Mexico to Ecuador.

71. *Dryopteris rufa* (Poir.) C. Chr. Ind. 290. 1905.

Syn: *Polypodium ruffum* Poir. Enc. méth. 5: 532. 1804.

Type from Peru, Lima (Herb. LAMARCK, Mus. bot. Paris!); another specimen from Lima, WATSON nr. 818 (RB) is perfectly typical.

In my "Revision" pag. 274 I suggested this to be the Chilene form of *D. argentina*, which is not the case. *D. rufa* is a distinct species, in general habit resembling *D. concinna* and *D. oligocarpa*, well-marked by its setose sporangia as in *D. concinna*, but rachis and costæ of both sides are sparsely pubescent by short hairs, not densely pulverulent as in *D. concinna*; leaf otherwise glabrous. — Lamina 40—50 cm long by 10 cm wide, narrowed downwards as in the two species mentioned; pinnæ opposite, firm; segments close, patent, acute; veins about 8 to a side; sori about medial; indusium not seen.

72. *D. Stierii* (Ros.) C. Chr., Revision nr. 2 fig. 3.

Area: Brazil, Rio Grande do Sul.

73. *Dryopteris blanda* (Fée) C. Chr. Index 254. 1905. — Fig. 13.

Syn. *Phegopteris blanda* Fée, 8. mém. 91. 1857.

Phegopteris caespitosa Fourn. Mex. Pl. 1: 89. 1872.

Polypodium caespitosum Bak. Syn. Fil. 305. 1874.

Dryopteris caespitosa C. Chr. Index 256. 1905.

Type from Mexico, Mirador, leg. SCHAFFNER nr. 222 (specim. auth. in Kew!).

A small species, not unlike small forms of *D. oligocarpa*, but the lower pinnæ not or very slightly reduced. — Rhizome erect or decumbent, small, scales very few. Stipes very slender, 15—20 cm long, stramineous, glabrous. Lamina 12—15 cm

long by 7 cm broad at the middle, ovate-lanceolate, acuminate, somewhat narrowed downwards, herbaceous, dark-green. Rachis slender, short-hairy. Pinnæ 7—8 to a



Fig. 13. *D. blanda* (Fée) C. Chr. Pinna $\times \frac{4}{5}$ and segments $\times 1\frac{1}{2}$ (BOURGEAU 2005).

side, sessile, suddenly narrowed into a short, entire apex, $3\frac{1}{2}$ cm long, $1\frac{1}{2}$ cm broad, the lowest pair generally somewhat shortened and reflexed; costæ on both sides sparsely pubescent by a few longer and more numerous very short hairs, surfaces otherwise glabrous. Segments 9—10-jugate, oblong oblique, connected by a wing $1\frac{1}{4}$ mm broad, the sinuses between acute, entire or slightly repand, the basal ones of lower pinnæ shortened, of upper ones

equal or a little prolonged. Veins 5—6 to a side, simple, oblique. Sori a little below the middle of the vein, exindusiate. Sporangia glabrous.

A very distinct species not easily to be confounded with others. It is not probable that FOURNIER had seen an authentic specimen of *Ph. blanda*, since he described his *Ph. caespitosa* as new; the two are exactly the same species. FÉE described the rhizome as “rampant”, but on the label to the specimen at Kew is written: “rhizome decumb. terminal fronds”.

Specimens seen:

Mexico: Vera Cruz, JARED G. SCHMITH nr. 63 (W); Cordova, BOURGEAU nr. 2005 (H, W, Mus. Paris. *Ph. caespitosa* Fourn.), H. FINK nr. 96a (W).

Guatemala: vicinity of Cacao, BARBER nr. 171 (W, pinnæ acuminate). Alta Verapaz, near the Finca Sepacuite, COOK and GRIGGS nr. 57 (W).

Costa Rica: WERCKLÉ nr. 130 (C) — Grenadilla, Finca Hermes, BRADE nr. 412 (R).

74. *Dryopteris tablana* (Christ) C. Chr. Ind. 297. 1905. — Fig. 14.

Syn. *Aspidium tablanum* Christ, Bull. l'Herb. Boiss. II. 5: 727. 1905.

Type from Mexico: Chiapas, San Pablo, tierra templada “auf Leimboden”, leg. MUNCH nr. 146 (C!). — The Costa Rican specimen mentioned by CHRIST belongs to *D. blanda*.

Very like *D. blanda*, but larger: stipe 20 cm long, lamina 25×12 cm, slightly ciliate and costæ above strigose, otherwise entirely glabrous. Pinnæ opposite, linear, shortly acuminate, $6-6\frac{1}{2}$ cm long, 12—13 mm broad, the lower ones reflexed and not shortened, but below them I find in a single leaf a pair of small auricles. Segments very obtuse, almost with a truncate apex that is crenato-dentate. Veins 4—5-jugate, not very oblique. Sori supramedial with a distinct, glabrous indusium. Sporangia glabrous. — Rhizome erect, 1 cm thick, 8 cm high, scaleless.



Fig. 14. *D. tablana* (Christ) C. Chr. Pinna $\times \frac{4}{5}$ and segments $\times 1\frac{1}{2}$, (orig.)

75. *Dryopteris Rimbachii* Ros. Fedde, Repert. 7: 147. 1909.

Type from Ecuador, Mt. Tunguragua, 2500 m, leg. RIMBACH nr. 119 (R! CC).

Well-marked by its coriaceous lamina and long basal scales; the leaf is very shortly narrowed towards the base and bears 2—3 pairs of close, auriculiform pinnæ. In general habit it resembles closely *D. Pavoniana*, specimens of which were collected at the same locality by Dr. RIMBACH (nr. 118, R, CC), but that species has a long-creeping, branched rhizome and wants the long basal scales and auriculiform pinnæ.

76. *Dryopteris brachypus* (Sod.) C. Chr. comb. nov.

Syn. *Nephrodium brachypus* Sod. Rec. 43. 1883; Cr. vasc. quit. 228. 1893.

Type from Ecuador, ad viam Quito-Guayaquil, leg. SODIRO (specim. auth. in Kew!).

Rhizome erect, up to 15 cm long, 1 cm thick, like the base of stipe densely clothed with narrow, subulate, brown, firm, finely hairy, squarrose scales; similar scales are scattered along the stipe and rachis, but they are deciduous and not always found. Stipites fasciculated, about 10 cm long, finely hairy. Lamina 40—70 cm long, 10—15 cm broad, downwards gradually but rather shortly attenuate, firmly membranous, not glossy above, pale-green beneath, the upperside finely pubescent by adpressed hairs, the channelled costæ more decidedly strigose, the underside nearly glabrous; rachis sulcate and hairy above. Middle pinnæ 6—9 cm long, $1\frac{1}{2}$ — $1\frac{3}{4}$ cm broad, shortly acuminate, the lower ones gradually shortened, the lowermost auriculiform, about 1 cm long. Segments a little oblique, acute, larger ones often faintly toothed, the basal ones equal. Veins 5—7-jugate, simple, rather indistinct. Sori close to the margin with glabrous sporangia; indusium not seen; according to SODIRO it is puberulous.

D. brachypus is in general habit not unlike *D. oligocarpa*; it is best characterized by its numerous basal, squarrose scales, agreeing in this character with the two following species; these three species (and perhaps *D. Rimbachii*) form a proper little group marked by the said character.

77. *Dryopteris supranitens* Christ, Fedde, Repert. 8: 19. 1910.

Type from Costa Rica, Tablazo, Finca Haberl, A. et C. BRADE nr. 270 (R!); specimens from Panama, Chiriqui, MAXON nr. 5244 (W) are very alike.

Closely related to *tablaziensis* with similar basal scales; it differs by its rigid texture, glossy upperside, narrower lamina that is suddenly narrowed downwards with 4—5 pairs of distant glanduliform pinnæ (type IV), by its sparse pubescence and long-acuminated pinnæ. From *D. brachypus* it differs in shape of lamina and nearly glabrous upperside. — A peculiar form with the segments distinctly serrated is MAXON's nr. 5524 from Panama (W).

78. *D. tablaziensis* Christ; C. Chr. Revision nr. 7 fig. 6.

Area: Costa Rica and Panama (MAXON nr. 5287 and 5336, W).

The rhizome is erect, about 10 cm high, 1 cm thick. In MAXON's specimens I find small, castaneous scales on the costæ beneath.

79. *D. Lindigii* C. Chr., Revision nr. 15.

Area: Costa Rica, Colombia, Venezuela.

80. *D. lustrata* (Hieron.) C. Chr., Revision nr. 14.

Area: Colombia, Peru.

81. *Dryopteris hydrophila* (Fée) C. Chr. Index 271. 1905.

Syn. *Phegopteris hydrophila* Fée, 11 mém. 56 tab. 13 fig. 3. 1866.

Polypodium hydrophilum Bak. Ann. of Bot. 5: 456. 1891.

Type from Guadeloupe, leg. L'HERMINIER (Herb. COSSON, Paris!); MAZÉ nr. 363 et 843 (Kew!).

In my Revision pag. 294—295 I believed that *Ph. hydrophila* and *Ph. delicatula* were forms of one species; this is not the case, *Ph. hydrophila* being a distinct species closely related to *D. oligocarpa*, while *Ph. delicatula* is allied to *D. sancta*. *D. hydrophila* was well figured by Fée; it resembles in general habit very closely *D. oligocarpa* and *D. Lorentzii* differing from both by its inframedial sori, which are furnished with minute, ciliate indusia, not seen by FÉE. Rachis slender, subglabrous beneath; pinnae with a large, tuberculiform aërophore at the base beneath, broadest at the base, very finely and rather sparsely pubescent above, the costæ and veins beneath shortly and sparsely hairy. Segments very oblique, broad, acute, the basal ones equal-sized, the upper one parallel to rachis, the lower one much oblique. Veins 5-jugate, distant, oblique. — FOURNIER (Pl. mex. 1: 89) referred to this species a plant from Mexico, San Luis Potosi, VIRLET nr. 44.

82. *D. Lorentzii* (Hieron.) C. Chr., Revision nr. 3 fig. 4.

Area: Argentina.

83. *D. argentina* (Hieron.) C. Chr., Revision nr. 4.

Area: Argentina, Chile—Peru, Bolivia.

84. *D. oligocarpa* (H. B. Willd.) O. Ktze.; C. Chr. Revision nr. 5 fig. 5; Smiths. Misc. Coll. 52: 370.

Area: Mexico—Bolivia; Cuba, Jamaica, Haïti, St. Kitts. — South Brazil (var. *retusa* (Sw.)). Beautiful specimens of this species were recently collected in Panama by MAXON, nr. 4936, 4970, 5242 (W). It varies considerably in pubescence and I now regard *D. navarrensis* Christ; C. Chr. Misc. Coll. 52: 371 as a very hairy variety with several of the longer hairs consisting of 2—4 cells.

85. **D. pilosula** (Kl. & Karst.) Hieron.; C. Chr. Revision nr. 6.

Area: Mexico—Peru.

This, also, is most probably only a variety of *D. oligocarpa*.

86. **Dryopteris rivulariformis** Ros., Fedde, Repert. 6: 316. 1909.

Syn. *Dryopteris stenophylla* Ros., Fedde, Repert. 5: 233. 1908 (non C. Chr.).

Type from Bolivia, Yungas, Sirupaya pr. Yanacachi, leg. BUCHTIEN nr. 495.

Of this I have seen only a single pinna, kindly sent me by Dr. ROSENSTOCK.

It appears to be a good species.

87. **D. Regnelliana** C. Chr., Revision nr. 20 fig. 12.

Area: Brazil, Minas Geraes.

88. **D. rioverdensis** C. Chr., Revision nr. 19 fig. 11.

Area: Brazil, Minas Geraes.

89. **D. Nockiana** (Jenm.) C. Chr., Revision nr. 8 fig. 7; Smiths. Misc. Coll. 52: 371.

Area: Jamaica.

90. **D. muzensis** Hieron.; C. Chr. Revision nr. 10.

Area: Colombia.

91. **D. columbiana** C. Chr. Revision nr. 9 fig. 8; Smiths. Misc. Coll. 52: 372.

Area: Colombia—Panama.

Specimens of MAXON's recent collection from Panama (nr. 5202, W) shows that this is clearly distinct from *D. oligocarpa*.

92. **D. piedrensis** C. Chr., Smiths. Misc. Coll. 52: 372.

Area: Cuba, Gran Piedra, Oriente, MAXON nr. 4041 (W); J. A. SHAFER nr. 8954 (C. Chr.).

93. **D. boqueronensis** Hieron.; C. Chr. Revision nr. 11.

Area: Colombia.

94. **D. lepidula** Hieron.; C. Chr. Revision nr. 12.

Area: Colombia.

95. **D. Lindmani** C. Chr., Revision nr. 13 fig. 9.

Area: Brazil: Sao Paulo, MOSÉN; HEINER nr. 523 (Rg).

96. **D. laevigata** (Mett.) C. Chr., Revision nr. 17 fig. 10.

Area: Peru.

97. *Dryopteris Crossii* (Bak.) C. Chr. Index 259. 1905.

Syn. *Polypodium Crossii* Bak. Ann. of Bot. 5: 455. 1891.

Type from Ecuador, Sierra de Roritroche, Andes of Loja, leg. Cross, Oct. 1861 (Kew!).

A very fine plant very near (too near?) to *D. laevigata*, which it resembles by its purplish-castaneous stipe and rachis, size and texture of lamina, but the lamina is more gradually narrowed downwards and the costæ are strigosely hairy beneath.

Rhizome? Stipe to the lowest pair of glanduliform pinnæ up to 15 cm long, like rachis purplish-castaneous, glossy and quite glabrous. Lamina lanceolate, 25—35 cm long, 6—7 cm broad at the middle, narrowed towards both ends, coriaceous, brown when dry. Pinnæ close, sessile, subopposite or alternate horizontal or slightly ascending, the lower 3—4 pairs very small, like tubercles on the stipe at distances of 3—4 cm; above them 2—3 pairs of gradually larger auriculiform pinnæ, middle ones the largest, 3—3½ cm long, about 8 mm broad, acute but scarcely acuminate, glabrous above, strigose on the costa beneath, furnished with a large, 2—3 mm long aërophore at the base beneath, incised to a wing 1—1½ mm broad. Segments a little oblique, close, obtuse but the margins being revolute apparently acute, and triangular of shape, the basal ones equal or a little longer. Veins about 6 to a side; sori a little above the middle of the vein, exindusiate, small, consisting of a few sporangia only.

98. *D. Galanderi* (Hieron.) C. Chr. Revision nr. 16.

Area: Argentina, Minas Geraes.

99. *Dryopteris Millei* sp. nov.

Ecuador, in pascuis gelidis Andium Paluguillo, 3300 m, leg. A. MILLE nr. 125 (RB!).

Rhizomate breve, decumbente, apice squamis lanceolatis castaneis hirtis onusto. Stipitibus longissimis, usque ad 40 cm longis, fasciculatis, brunneo-stramineis basi fusciscentibus et sparse squamosis, nitidis, decidue hirtis denique glabris, vix ultra 1½ mm crassis. Lamina lanceolata, utrinque attenuata, raro ad 20 cm longa, medio 2—5 cm lata, rigide coriacea, pallide viridi. Rachis straminea, pilis albidis patentibus decidue hirta. Pinnis 10—20-jugis, inferioribus sensim reductis more *D. oligocarpicae* (typus I), saepe reflexis, medialibus maximis, raro ultra 2 cm longis, sessilibus, horizontalibus vel leviter falcatis, apice integro acuto, ciliatis, costis utrinque et costulis infra setis antrorsis setulosis, inter venas glabris et eglandulosis, infra apicem integrum vel serratum ad alam 2 mm latam pinnatifidis. Laciniis 5—6-jugis, ovatis, subobtusis, obliquis, marginibus dense ciliatis revolutis, basalibus aequalibus vel parum auctis. Venis 2—4-jugis, indivisis. Soris infra medium venularum sitis, rotundis, exindusiatis; sporangiis glabris.

A distinct new small species, remarkable by its very long stipe and small lamina of coriaceous texture. It is related to *D. laevigata*, from which it differs by its stramineous rachis and setulose costæ.

100. *Dryopteris phacelothrix* C. Chr. et Ros. n. sp. — Ros. Fedde, Rept 11: 56. 1912.

Bolivia: Yungas borealis, Unduavi, 3300 m, leg. O. BUCHTIEN nr. 2707 et 2709 (R).

Lastrea rhizomate oblique adscendente seu suberecto, paleis ferrugineo-brunneis, lanceolatis, dorso margineque pilosulis vestitis. Stipitibus fasciculatis, c. 15 cm longis, 2 mm crassis, stramineis, paleis adpressis, iis rhizomatis similibus ornatis pilisque brevibus, strictis, 2—6 fasciculatis vel simplicibus dense hirtis. Laminis elongato-ovalibus utroque breviter et gradatim attenuatis, ad 50 cm longis, 12 cm latis, pinnato-pinnatifidis, subcoriaceo-chartaceis, utrinque viridibus, in rachibus, costis nervisque densius, inter nervos parcius hirtis-pilosis, pilis iis stipitis similibus, rachibus costisque paleis ovato-lanceolatis brunneis persistentibus adspersis. Pinnis infra apicem brevem pinnatifidum c. 30-jugis, subapproximatis, infra oppositis, sursum alternis, recte patentibus, subsessilibus, profunde pinnatifidis, medialibus c. 5½ cm longis, 1¼ cm latis, e basi subæquali vel anteriore paullisper adaucta lineari lanceolatis, breviter acuminatis, inframedialibus sensim brevioribus, inferioribus remotis et citius abbreviatis, infimis auriculiformibus, superioribus sensim diminutis et minus profunde incisis, postremis basi lata sessilibus, demum confluentibus et in apicem laminae secretum sensim transeuntibus. Segmentis pinnarum majorum subrecte patentibus, linearibus, obtusis, c. 5 mm longis, 2 mm latis, margine crenulato, subrevoluto seu integerrimo, plano, sinibus acutis interstinctis, ala 1 mm fere lata confluentibus; venulis lateralibus ad 6-jugis, supra prominentibus, subtus prominulis, simplicibus, parallelis, basalibus longe supra sinum marginem attingentibus. Soris medialibus, exindusiatis, pilis fasciculatis sporangiis intermixtis. (ROSENSTOCK descripsit.)

I am due to Dr. ROSENSTOCK a specimen of this species unique by its peculiar pubescence. Without a minute examination several of the hairs appear to be stellate but the whole aspect of the species is that of *Lastrea* not of *Goniopteris*. By microscopical analysis I found, however, that the hairs are not stellate, i. e. branched, but fasciculate, as Dr. ROSENSTOCK expresses it, i. e. 2—6 hairs spring out from the same epidermal cell from nearly the same point.

101. *D. caucaensis* (Hieron.) C. Chr., Revision nr. 21 fig. 13.

Area: Costa Rica—Colombia, and it is now also found in Bolivia, BUCHTIEN nr. 2697 (R).

102. *D. Funckii* (Mett.) O. Ktze.; C. Chr. Revision nr. 35 fig. 23.

Area: Venezuela—Colombia and it is now also recorded from Costa Rica: Volcano Barba, BRADE nr. 289 (R). — The species has its proper position here.

103. *D. velata* (Kze.) O. Ktze.; C. Chr. Revision nr. 22; Smiths. Misc. Coll. 52: 373.

Area: Cuba.

104. *D. aspidioides* (Willd.) C. Chr., Revision nr. 23.

Area: Costa Rica—Peru.

105. *D. ptarmica* (Kze.) O. Ktze.; C. Chr. Revision nr. 24 fig. 14.

Area: Southern Brazil.

106. *D. pachyrachis* (Kze.) O. Ktze., C. Chr. Revision nr. 44 fig. 31; Smiths. Misc. Coll. 52: 382.

Area: Panama—Venezuela—Ecuador. Southern Brazil (var. *platyrachis* (Fée) C. Chr.). — Jamaica, St. Vincent (var. *Jenmani* (Bak.) C. Chr.).

The specimens from Panama collected by MAXON nr. 5274 (W) are very similar to var. *bogotensis* C. Chr. Revision 306, which differs from typical *D. pachyrachis* by its pubescent rachis and upperside. Similar more hairy forms occur in Ecuador and I think I was right in referring *Nephrodium crassipes* Sod. and *N. stramineum* Sod. to *D. pachyrachis*. A third variety is no doubt

Nephrodium Sprucei Bak. Syn. Fil. 269, 1867.

Dryopteris Sprucei O. Ktze. Rev. 2: 813. 1891; C. Chr. Index 294.

Ecuador, Mt. Tunguragua, SPRUCE nr. 5299, Llalla, SPRUCE nr. 5299 A, Montaña de Canelos, SPRUCE nr. 5301 (all Kew!).

BAKER founded his new species on these three specimens, which were all referred to *N. resinoso-foetidum* by HOOKER; they are, however, not quite identical. Best agreeing with BAKER's description are nr. 5299 A and 5301. Nr. 5301 is as to all characters the eglandulose underside excepted typical *D. pachyrachis*; nr. 5299 A is nearly the same, but the upperside is rather pubescent, the underside and indusium sparsely glandular with the characteristic large, red glands, and the costæ beneath bear some few thin, brown scales. Nr. 5299 looks very different, but it is certainly a large-growing, thin-leaved form of *D. pachyrachis*. Its pinnæ are 20 cm long, 3½ cm broad, rather densely hairy above, glabrous and slightly glandular beneath, the costæ rather scaly, and the irregular (sometimes subathyroid) very large indusia slightly ciliate and glandular by large, red glands. Segments 5 mm broad with about 10 veins to a side.

The var. *Jenmani* (Bak.) C. Chr. may be specifically different (see Smiths. Misc. Coll. 52: 382—383); still it differs scarcely more from the type than do some of the Andine forms. The most different form is the large, very thin-leaved and densely glandulose variety that I erroneously in "Revision" referred to *D. Germaniana* as var. *glandulosa*. It was also collected by W. HARRIS nr. 7485 (B), Jamaica, Blue Mountain Peak.

107. *D. supina* (Sod.) C. Chr., Revision nr. 46 fig. 32.

Area: Ecuador—Colombia.

The var. *Biolleyi* Christ is a small form of *D. cheilanthoides*.

108. *D. tenerrima* (Fée) Ros.; C. Chr. Revision nr. 50.

Area: Southern Brazil.

109. *D. palustris* (Mett.) O. Ktze.; C. Chr. Revision nr. 49 fig. 33.

Area: Southern Brazil.

110. *D. Hieronymusii* C. Chr., Revision nr. 45.

Area: Colombia.

111. *D. roraimensis* (Bak.) C. Chr.; Smiths. Misc. Coll. 52: 383. — Fig. 15 a.

Area: Guiana, Mt. Roraima, in THURN nr. 168 (Kew!).

In the type-specimen the segments are scarcely falcate but often faintly crenate.

112. *D. utaëgensis* Hieron.; C. Chr. Revision nr. 47.

Area: Ecuador.

113. *Dryopteris illicita* Christ, Bull. Soc. bot. Gèneve II. 1: 225. 1909.

Type from Costa Rica, La Palma, WERCKLÉ (not seen).

To this species I refer specimens from Candelaria, Costa Rica, A. et C. BRADÉ nr. 410 (R). It is a very large species remarkably resembling a glabrous form of *D. patens*, but the venation and the decrescent lamina show that it is a member of the subgenus *Lastrea*. The thick, quadrangular, stramineous rachis and the costæ are sparsely pubescent above, otherwise the leaf is quite glabrous and eglandulose, but the costæ and costulæ bear beneath some thin, light-brown scales. Segments close, subfalcate, acute, the basal ones both prolonged and parallel to rachis as in *D. patens*. Veins about 10 to a side. Sori supramedial with subpersistent, glabrous indusia.

114. *D. atropurpurea* Hieron.; C. Chr. Revision nr. 48.

Area: Colombia. — See remarks under *D. euchlora*.

115. *D. recumbens* Ros.; C. Chr. Revision nr. 51.

Area: South Brazil.

116. *D. rustica* (Fée) C. Chr., Revision nr. 53; Smiths. Misc. Coll. 52: 383.

Area: Guadeloupe, St. Vincent. Jamaica (var. *nimbata* (Jenm.), indusiate; authentic specimen in Kew!).

117. **D. Germaniana** (Fée) C. Chr., Revision nr. 55, Smiths. Misc. Coll. 52: 384.

Area: Guadeloupe, Porto Rico (t. W. R. MAXON), Cuba.

The variety *glandulosa* Revision 311 belongs to *D. pachyrachis*.

118. **D. melanochlaena** C. Chr. Smiths. Misc. Coll. 52: 384. — Fig. 15 b.

Area: Guatemala.

119. **D. atrorubens** (Mett.) C. Chr., Revision nr. 54 fig. 34.

Area: Peru.

120. **D. demerarana** (Bak.) C. Chr., Smiths. Misc. Coll. 52: 385. — Fig. 15 d.

Area: Guiana, Mt. Roraima, IM THURN nr. 356 (Kew!).

Best marked by its subopposite very distant pinnæ. Closely allied to *D. rustica*.

121. **D. dominicensis** C. Chr. Smiths. Misc. Coll. 52: 384. — Fig. 15 c.

Area: Dominica; Martinique, HUSNOT nr. 356 (CC); Guadeloupe, MAZÉ nr. 647.

122. **D. Moritziana** Urban; C. Chr. Revision nr. 56 fig. 35.

Area: Venezuela.

123. **Dryopteris nitens** (Desv.) C. Chr. comb. nov. — Fig. 15 e.

Syn. *Polypodium nitens* Desv. Prodr. 240. 1827.

Dryopteris bañiensis Ros. Fedde, Repert. 7: 301. 1909!

Type from Peru, (Herb. Mus. Paris!). — Ecuador, Baños, SPRUCE sine num. (RB).

This was founded on an incomplete specimen but is apparently a very distinct species. The original specimen consists of the lower half of a lamina with a part of the stipe, which is stramineous and glabrous. Lowest pair of pinnæ deflexed and below them a pair of auriculiform pinnæ. Pinnæ opposite, sessile, 12 cm long by 2 cm broad, acuminate, thinly herbaceous, the costæ above pilose, costæ, costulæ and veins beneath long- and soft-hairy by whitish, thin, pluricellular hairs, the costæ besides furnished with a few small, red, thin scales. Rachis stramineous, deciduously hairy. Segments close, connected by a wing $1\frac{1}{2}$ mm broad, bluntly rounded at the apex, the margins obscurely crenate, ciliate, 6–7 mm broad, the lower pair not reduced. Veins simple, 8-jugate. Sori medial, reddish, exindusiate.

D. bañiensis Ros. based on a single leaf with a pinnatifid apex is quite the same.

124. **D. diplazioides** (Desv.) Urban; C. Chr. Revision nr. 58.

Area: Mexico—Colombia, Bolivia (BUCHTIEN nr. 1126, 1130, R). San Domingo, Guadeloupe. South Brazil (var. *brevisora* Ros.).

125. *D. consimilis* (Fée) C. Chr., Revision nr. 59 fig. 37, Smiths. Misc. Coll. 52: 386.
Area: Guadeloupe, Jamaica.
126. *D. heteroclita* (Desv.) C. Chr., Revision nr. 60 (vix fig. 38 = *D. consimilis*?);
Smiths. Misc. Coll. 52: 386.
Area: Jamaica.

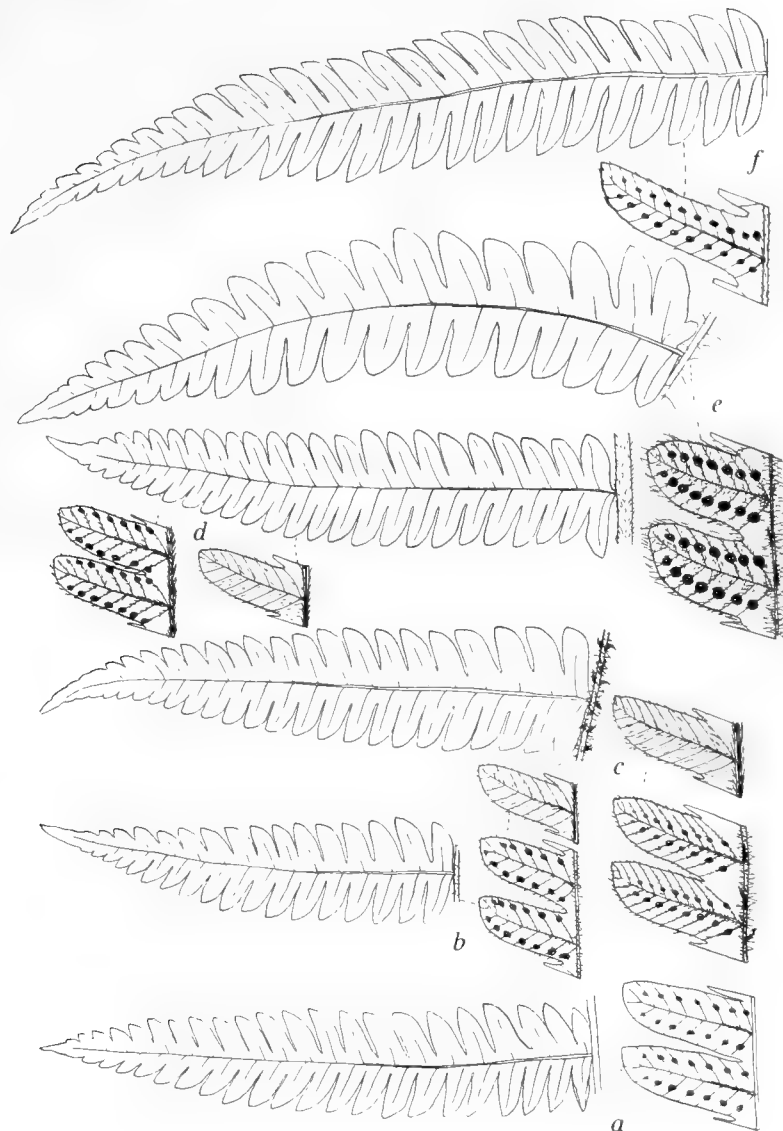


Fig. 15. a. *D. roraimensis* (Bak.) C. Chr. (orig.). — b. *D. melanochlæna* C. Chr. (orig.). — c. *D. dominicensis* C. Chr. (orig.). — d. *D. demerarana* (Bak.) C. Chr. (orig.). — e. *D. nitens* (Desv.) C. Chr. (orig.). — f. *D. Bradei* Christ (orig.). — All pinnae $\times 4\frac{1}{5}$, segments $\times 1\frac{1}{2}$, those without sori seen from above.

127. *D. atrovirens* C. Chr.; Revision nr. 61 fig. 39.

Area: Guatemala, Costa Rica; Panama, MAXON nr. 5649, 5651 (W).

128. *Dryopteris leptogrammoides* Ros., Fedde Repert. 9: 68. 1910.

Type from Costa Rica, La Palma, A. et C. BRADE (R!).

A critical species founded on a part of a single leaf. It is near large forms of *D. atrovirens*.

129. *D. firma* (Bak.) C. Chr., Revision nr. 36 fig. 24; Smiths. Misc. Coll. 52: 381.

Area: Jamaica.

130. *D. Pavoniana* (Kl.) C. Chr., Revision nr. 18.

Area: Peru, Bolivia; Ecuador, Mt. Tunguragua, RIMBACH nr. 118 (R, CC).

As shown by the specimen quoted this has a wide-creeping, branched rhizome like *D. firma*. Together with that species and *D. Rosenstockii* it forms a narrow group of species, which are not nearly allied to the South Brazilian species with a wide-creeping rhizome.

131. *D. Rosenstockii* C. Chr. Revision nr. 43 fig. 30.

Area: Ecuador.

132. *D. Santae Catharinae* Ros.; C. Chr. Revision nr. 37 fig. 25.

Area: South Brazil.

133. *D. Jürgensii* (Ros.) C. Chr.; Revision nr. 38 fig. 26.

Area: South Brazil.

134. *D. Mosenii* C. Chr., Revision nr. 33 fig. 27.

Area: Brazil: Minas Geraes.

135. *D. rivularioides* (Fée) C. Chr., Revision nr. 41 fig. 29.

Area: Brazil (Minas Geraes to Rio Grande do Sul), Paraguay, Uruguay, Argentina: Misiones (EKMAN, S).

A series of specimens received from M. B. BERRO, Montevideo, shows that *D. pseudomontana* (Hieron.) C. Chr., Revision nr. 40 fig. 28 is only a form of the variable *D. rivularioides*.

136. *D. scariosa* Ros.; C. Chr., Revision nr. 42.

Area: South Brazil.

137. *Dryopteris tremula* Christ in Lecomte, Notulæ Syst. 1: 234. 1910.

Type from Mexico: state of Michoacan, Morelia, leg. G. ARSÈNE nr. 3106.

I have not seen the type-specimen but numerous specimens from the type-locality collected by ARSÈNE (RB). Unfortunately they are all sterile and without rhizome. Judging from these specimens I cannot with certainty distinguish them from *D. thelypteris*, which has not been found in tropical America; Dr. CHRIST⁶ however, finds that it differs from *D. thelypteris* by its plane fertile segments and minute, remote sori. I, therefore, prefer to let it stand provisionally as a species. It is widely different from all other tropical American species of § *Lastrea* by its non-attenuate lamina and normally furcate veins.

138 *D. siambonensis* (Hieron.) C. Chr., Revision nr. 63 fig. 41.

Area: Argentina.

139. *D. achalensis* (Hieron.) C. Chr., Revision nr. 64.

Area: Argentina.

140. *D. Sprengelii* (Klf.) O. Ktze.; C. Chr. Revision nr. 65 fig. 42; Smiths. Misc. Coll. 52: 387.

Area: West Indian Islands, common; Mexico along the Andes to Ecuador.

Nephrodium Sherringii Jenm. Journ. Bot. 1879: 261, from Jamaica, of which I have now seen an authentical specimen (Kew), I regard as a large, luxuriant variety of *D. Sprengelii*. It agrees with typical *Sprengelii* in colour, texture and pubescence, but it is much larger with pinnæ 12—20 cm long, 3—5 cm broad, the segments of larger pinnæ about 3 cm long, $\frac{1}{2}$ cm broad, deeply lobed with the tertiary veins furcate or even pinnately branched in the lobes.

141. *D. struthiopteroides* C. Chr., Smiths. Misc. Coll. 52: 388.

Area: Guatemala.

142. *D. Mercurii* (A. Br.) Hieron.; C. Chr. Revision nr. 66 fig. 43; Smiths. Misc. Coll. 52: 389.

Area: Mexico along Andes to Ecuador.

It is highly questionable, whether this is specifically distinct from *D. Sprengelii*. I am nearly sure that it is not. Specimens from Panama (MAXON nr. 4696 and 5767) show the pubescence of *D. Mercurii* but are otherwise [not different from *D. Sprengelii*.

143. *Dryopteris conformis* (Sod.) C. Chr. Index 258. 1905. — Fig. 16 b.

Syn. *Nephrodium conforme* Sodiro. Rec. 45. 1883; Cr. vasc. quit. 240. 1893.

Type from Ecuador, leg. SODIRO (Kew!); very typical specimens were collected at Mt. Pifi by A. MILLE (RB).

In texture, habit and size very like *D. Sprengelii*, but lamina throughout shortly pubescent, mostly so on the costæ of both sides but the leaf-tissue of both

surfaces is also minutely hairy and without glands. Stipe long, up to 30 cm to the lowermost pair of glanduliform pinnæ, these in 3—4 very distant pairs and above them 3—4 pairs of auriculiform pinnæ. Larger segments often serrate, linear-oblong or ligulate, 3—4 mm broad with 10—15 pairs of veins that are more distant than in *D. Sprengelii*; basal segments of lower pinnæ not much reduced. Sori very close to the margin; indusium small, hairy, soon falling. Sporangia glabrous.

144. *D. scalaris* (Christ) C. Chr.; Revision nr. 72 fig. 47; Smiths. Misc. Coll. 52: 390.

Area: South Mexico to Costa Rica.

145. *Dryopteris Bonapartii* Rosenstock in Fedde Repert. 7: 303. 1909. — Fig. 16 a.

Type from Ecuador, Mt. Tunguragua, SPRUCE nr. 5254 (RB!).

A most distinct species with long slender pinnæ of thin texture and fully pinnate in the lower half. Evidently a near ally of the preceding species and of *D. Mercurii*, in pubescence and lack of glands resembling *D. conformis*, but basal pinnulæ much reduced, veins more numerous and sori not so close to the margin.

146. *Dryopteris Bradei* Christ, Bull. Soc. bot. Gèneve II. 1: 225. 1909. — Fig. 15 f.

Type from Costa Rica, Irazu, leg. BRADE nr. 245 (R!).

Of this I have seen a single leaf only and I dare not decide with certainty whether it is a good species or not. It is not improbable that it is a thin-leaved form of *D. rudis* or *D. nervosa*.

147. *D. Rusbyi* C. Chr. Smiths. Misc. Coll. 52: 390.

Area: Bolivia.

148. *D. Christensenii* Christ; C. Chr. Revision nr. 70 fig. 46.

Area: Costa Rica, BOLLLEY nr. 67 part. (CC), Candelaria, A. et C. BRADE nr. 418 (R); Panama, Chiriqui, MAXON nr. 5202 bis (W).

149. *D. limbata* (Sw) O. Ktze.; C. Chr. Revision nr. 71; Smiths. Misc. Coll. 52: 390.

Area: St. Kitts, Guadeloupe; Jamaica (?).

D. consanguinea (Fée) C. Chr. (see above nr. 59) may be a small form of this.

150. *D. corazonensis* (Sod.) C. Chr. Revision nr. 57 fig. 36.

Area: Ecuador.

151. *Dryopteris semilunata* (Sod.) C. Chr. Index 291. 1905. — Fig. 16 d.

Syn: *Nephrodium semilunatum* Sodiro Rec. 46. 1883; Cr. vasc. quit. 245. 1893.

Type from Ecuador leg. SODIRO (Kew!).

Stipe to lowest developed pinnæ 50 cm long and bears 3—4 pairs of very distant pairs of glanduliform pinnæ, the lowermost ones not more than 5—6 cm from

the base of the stipe. Pinnæ opposite, coriaceous, like rachis quite glabrous, 10—12 cm long, 2 cm broad, arcuate-ascending, the lower ones at base with a pair of glanduliform free pinnulæ and above them a pair of auriculiform segments, basal segments of medial pinnæ reduced. Segments 5—6 mm broad, acute, falcate, their margins revolute. Veins not very distinct, 10—12-jugate, simple. The specimen seen is sterile; according to SODIRO the sori are near the margin and furnished with a deciduous indusium.

Differs from the allied species (*D. rudis*, *D. piloso-hispida* etc.) by its entirely glabrous leaf and the pale underside.

152. *Dryopteris canelensis* Rosenstock, Fedde, Repert. 7: 302, 1909.

Type from Ecuador, in silva Canelos, SPRUCE (RB!).

A species allied to *D. rudis* but well marked by its short and narrow pinnæ, coriaceous texture, very oblique segments with about 11 pairs of veins and its very tomentose rachis.

153. *D. nervosa* (Kl.) C. Chr., Revision nr. 75.

Area: British Guiana, Costa Rica; Panama, MAXON nr. 4966 (W).

Rhizome creeping. Much like a glabrescent form of *D. rudis* and perhaps it should be united with that species.

154. *D. rudis* (Kze.) C. Chr., Revision nr. 73 fig. 48. Smiths. Misc. Coll. 52: 391.

Area: Mexico along Andes to Ecuador and Bolivia. Jamaica (*P. ctenoides* Jenm.).

A widely spread and rather uniform species, of which I have recently received several specimens, f. inst. from Panama, MAXON nr. 5675 (W) and Bolivia, BUCHTIEN nr. 494 (R). It varies mainly in texture and pubescence; generally the whole lamina is rather densely setose by stiff hairs and the costæ beneath are furnished with some brown scales, still the upper surface can be nearly quite glabrous and such specimens are scarcely to distinguish from *D. nervosa* by any other character than the supramedial sori. This more glabrous and generally smaller form is *D. lasiopteris* (Sod.) C. Chr. Revision nr. 69 fig. 45, which I now do not hesitate to reduce to a synonym of *D. rudis*. About the same form, still with fewer (9) veins is

Dryopteris caeca Rosenstock, Fedde, Repert. 7: 302. 1909, from Ecuador, SPRUCE nr. 5261 (RB).

Aspidium subdecussatum Christ, Bull. L'Herb. Boiss. II. 4: 960. 1904.

Dryopteris subdecussata C. Chr. Index 295. 1905.

Costa Rica, ALFARO nr. 16556 (C) is rather typical *D. rudis*.

Aspidium exsudans var. *myriocarpum* Fourn. Mex. Plant. 1: 94 is also, according to specimens in Herb. Mus. Paris so named by FOURNIER, a synonym of the present species.

Aspidium gleichenioides Christ, Bull. L'Herb. Boiss. II. 4: 960. 1904.

Dryopteris gleichenioides C. Chr. Ind. 268. 1905.

Costa Rica, Forêts du Barba, TONDUZ nr. 1935 (C).

In Smiths. Misc. Coll. 52: 395 I referred this to *D. pterifolia*, it is, however, a rather common form of *D. rudis* with the upperside nearly glabrous.

It can scarcely be doubted that the true *Nephrodium tetragonum* Presl. Rel. Haenk. 1: 85. 1825 is the same as *D. rudis*. Unfortunately the original specimen in Herb. Presl (!) is very defective and it may belong to another related species.

155. *D. Engelii* Hieron.; C. Chr. Revision nr. 74.

Area: Venezuela. Colombia.

Additional specimens from Venezuela, FUNCK et SCHLIM and GOLLMER (B), show that rachis and costæ beneath bear several small scales. Lamina with a pair of reduced pinnæ and 2—3 pairs of glanduliform warts.

My new species *D. Pittieri* C. Chr. Smiths. Misc. Coll. 52: 393 from Colombia, PITTIER nr. 1200 (W) I regard now as a very coriaceous form of *D. Engelii* with rather numerous scales along the costæ beneath.

156. *Dryopteris peruviana* Rosenstock, Fedde, Rept. 7: 298. 1909.

Type from Peru, Cerro de Campana, SPRUCE nr. 4655 (RB!).

Very like *D. rudis* in habit, size and pubescence, different by setose sporangia.

157. *Dryopteris Heineri* C. Chr., Fedde, Rept. 6: 380. 1909; Smiths. Misc. Coll. 52: 392 (erroneously as *D. Heimeri*). — Fig. 16 c.

Type from Brazil, São Paulo, Campinas, A. HEINER nr. 540 (Rg).

The only Brazilian representative of the group of *D. rudis* and a very distinct species with setose sporangia.

158. *D. strigifera* Hieron.; C. Chr. Revision nr. 76.

Area: Colombia.

159. *D. Brausei* Hieron.; C. Chr. Revision nr. 77.

Area: Colombia.

160. *Dryopteris piloso-hispida* (Hook.) C. Chr. comb. nov.

Syn. *Nephrodium piloso-hispidium* Hook. spec. fil. 4: 105. 1862.

Dryopteris pterifolia (Mett.) O. Ktze.; C. Chr. Revision nr. 78 fig. 49 with synonymy; Smiths. Misc. Coll. 52: 395.

Alsophila pilosa Mart. et Gal. Mém. Foug. Mex. 78 tab. 22. 1842.

Area: Mexico along Andes to Bolivia.

The original specimen of *N. piloso-hispidium* Hook. from Ecuador, SPRUCE sine num. (Kew!) is exactly what I previously have named *D. pterifolia* and it is absolutely identical with *Alsophila pilosa* Mart. et Gal. according to authentic

specimens in Herb. Mus. Paris. It is perhaps the largest species of the whole subgenus. The receptacle is clothed with some long hairs which are seen between the sporangia and may be mistaken for an indusium.

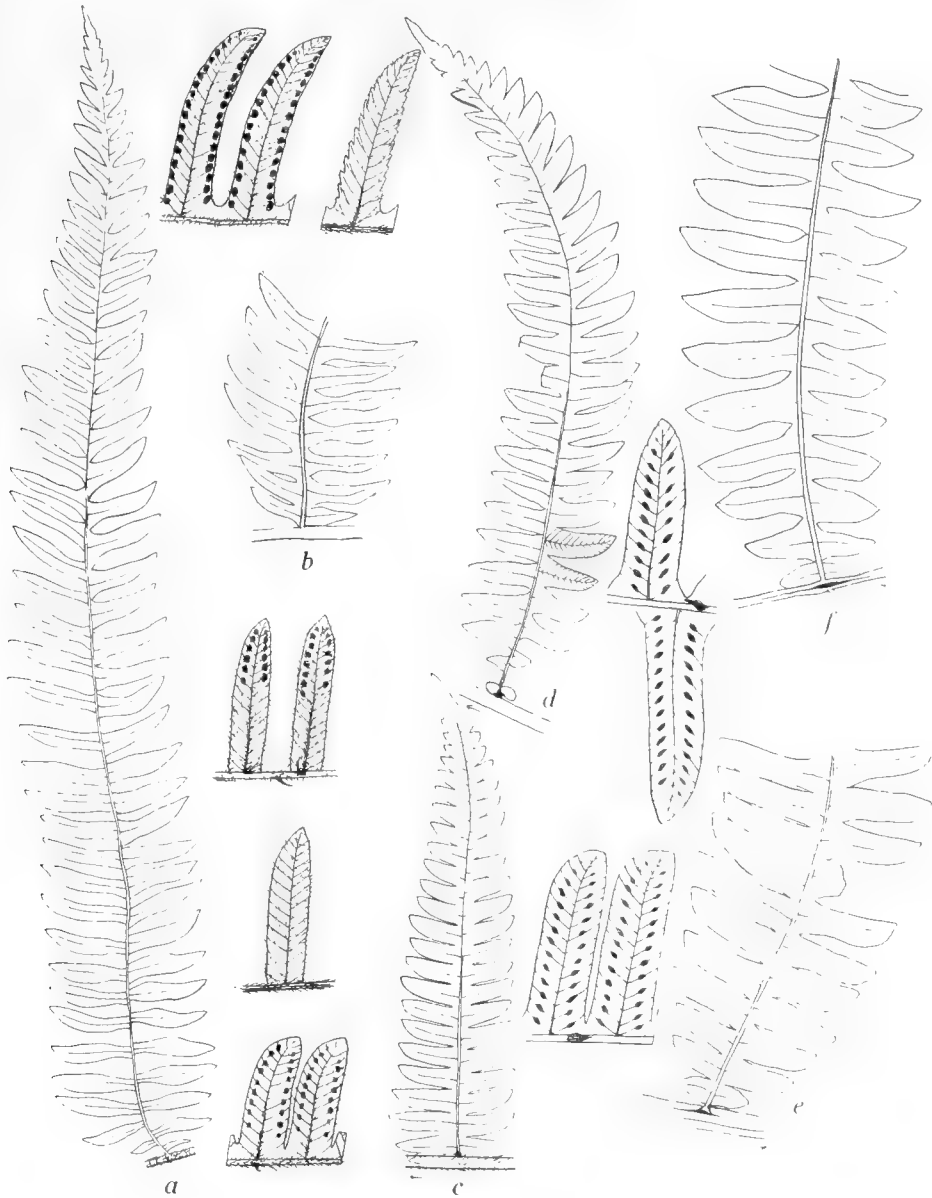


Fig. 16. a. *D. Bonapartii* Ros. (orig.) b. *D. conformis* (Sod.) C. Chr. (orig.) — c. *D. Heineri* C. Chr. (orig.) — d. *D. semilunata* (Sod.) C. Chr. (orig.) — e. *D. euchlora* (Sod.) C. Chr. (orig.) — f. *D. euchlora* var. *inaequans* C. Chr. from Nicaragua. All Pinnæ $\times \frac{4}{5}$, segments $\times 1\frac{1}{2}$, those without sori seen from above.

161. *Dryopteris euchlora* (Sod.) C. Chr. Index 263. 1905. — Fig. 16 e.

Syn. *Polypodium euchlorum* Sodiro, Rec. 58. 1883. Cr. vasc. quit. 290. 1893.

Type from Ecuador, Mt. Pululahua, leg. SODIRO (specim. auth. in Kew!); also Mt. Pichincha, STÜBEL nr. 751 (B).

A very large species with a stipe 60–70 cm long and a lamina of equal length. Rhizome apparently creeping; stipe brownish-stramineous, glabrous and clothed with scattered, adpressed scales, in the upper part bearing 4–5 pairs of tuberculiform, very distant abortive pinnæ. Developed pinnæ numerous, the lowest 1–2 pairs much abbreviated (Type IV), the following alternate at distances of 4–5 cm, the upper ones closer, fresh-green, membranous but thin, very sparsely strigose on the costæ above, otherwise like the straw-coloured rachis quite glabrous, but costæ beneath furnished with small, scattered, deciduous scales. Largest pinnæ up to 20 cm long, 3 3¼ cm broad, the lower ones very contracted at base, incised to a wing 1–2 mm broad. Segments patent, linear, 5 mm broad the apex rounded or subacute and generally faintly toothed, the basal ones of lower pinnæ much reduced, of the upper equal or the anterior one somewhat shortened. Veins 15–18 jugate, simple, not very close. Sori medial, small, exindusiate; sporangia glabrous.

In size, shape of segments, reduction of lamina and contracted base of lower pinnæ allied to *D. piloso-hispida* but very different by thin texture, glabrous frond and medial sori. Together with *D. Glaziovii* it is intermediate between the bipinnatifid species of the group of *D. rudis* and the bipinnate *D. pteroides*.

D. atropurpurea Hieron. (supra nr. 114) I fear should be referred to *D. euchlora*. It has fewer veins, distinctly crenate segments and atropurpureous rachis, otherwise it is not essentially different.

A Central-American fern, fragments of which I have known for a long time, and which was collected recently by MAXON in Panama, I can now with certainty refer to *D. euchlora* as

var. *inaequans* nov. var. — Fig. 16 f.

Agrees perfectly with the type in size and texture, its practically glabrous frond, reduction of lamina and base of lower pinnæ, that are very remote, in shape of segments, number of veins, etc. It differs from the type by its opposite pinnæ and the segments being often crenate (like those of *D. atropurpurea*), but the main difference is the unequal-sided pinnæ, the basiscop side of which is broader than the acroscop one (2½ cm and 2 cm in the lower pinnæ) with 15–16 veins to a side in a basiscop segment, 11–13 in an acroscop one. — Rhizome creeping 1½ cm thick, slightly scaly at the growing apex.

Panama, Canal Zone, humid forests, Chiriqui, 1650 to 1925 m, MAXON nr. 5674 (W).

Nicaragua, Omotépé Island in Lake Nicaragua, U. S. North Pacific Exploring Expedition 1853–56 (W).

By its unequal-sided, very distant, opposite pinnæ this variety looks very diffe-

rent, but I cannot consider it specifically distinct. STÜBEL's nr. 751 from Ecuador shows to some extent the same peculiarities and even in the authenthical specimen of the type one can find a difference of some few millimeters between the length of the segments of the upper and lower side of the lower pinnæ.

162. *D. Glaziovii* Christ; C. Chr. Revision nr. 62 fig. 40,

Area: Brazil, Minas Geraes and Rio.

Since the publication of my Revision I have seen the type-specimen of this species, GLAZIOU nr. 5267 (C) and specimens from Minas Geraes, Itacolumi, 1400 m, SCHWACKE nr. 14109 (C). In these specimens the veins are simple and the basal segments not free; the stipe bears 3—4 pairs of glanduliform pinnæ. Our specimen (H) has furcate veins and the lower segments often quite free, thus belonging to a more divided form that like the preceding species has the basiscop side of the pinnæ enlarged. But the species can be much more divided, fully bipinnate or even tripinnatifid. The bipinnate form with simple or furcate veins is

Gymnogramme patula Fée, Cr. vasc. Brés. 1: 59 tab. 14 fig. 3 1869.

Serra os Orgaos, GLAZIOU nr. 2822 (H); Serra de Ouro Preto, SCHWACKE nr. 15022 (C). FÉE's figure is as a whole excellent, but the lower pinnulæ should be reduced gradually. This form is closely allied to the Andine *D. pteroidea* to which species BAKER in Flor. bras. referred it; it differs by the sori being placed not so close to the margin and not being confluent. Unfortunately I have not seen the base of the lamina, but if I am right in considering it a form of *D. Glaziovii* it very likely has the lower pinnæ abortive and glanduliform.

A still more cut form has the pinnulæ deeply lobed and the veins pinnatifid in the lobes; this is

Gymnogramme expansa Fée, Cr. vasc. Brés. 1: 60 tab. 14 fig. 4. 1869.

Another specimen of GLAZIOU's nr. 2822 (H) agrees perfectly with FÉE's figure; it is sterile.

Comparing Fée's two figures with that of mine we see here a series of forms quite corresponding to those of *D. multiformis*.

While the bipinnatifid forms of the last two species, *D. euchlora* and *D. Glaziovii* certainly are related to *D. piloso-hispida* the species are, on the other hand, also and perhaps more intimately related to the bipinnate *D. pteroidea*. Together with this they belong to a separate little group of large species of rather thin texture, and dark colour. Most species belonging here have a nearly entirely glabrous frond, opposite and sessile pinnæ, which are narrowed towards their base, where a distinct aërophore is to be found. *D. euchlora* stands next to *D. piloso-hispida* and resembles not a little the bipinnatifid form of *D. Glaziovii* while the more cut forms of this latter species are closely allied to the two following species, which are, as it seems, constantly bipinnate. They represent the highest development of cutting in the subgenus, at least as far as American species are concerned.

163. *Dryopteris pteroidea* (Kl.) C. Chr. Index 287. 1905.

Syn. *Polypodium pteroideum* Klotzsch, Linnaea 20: 389. 1847; Hook. spec. fil, 4: 255 tab. 280.

Phegopteris pteroidea Mett. Aspid. u. Pheg. nr. 3. 1858.

Type from Colombia, MORITZ nr. 291 and KARSTEN nr. 40 (B!)

HOOKE's figure represents very well this characteristic species, still the lower pinnulae are generally much more reduced than shown in HOOKE's plate. It varies mainly in size and cutting; I have seen specimens with pinnæ 50 cm long and pinnulae 5—7 cm long by 1 cm broad. The pinnulae vary from being fully entire or faintly crenate to deeply lobed especially at the middle. Texture membranous, colour dark-green, surfaces (the sparsely setose costæ excepted) glabrous rachis slender, sparsely and deciduously pilose. Veins generally once furcate bearing the small, exindusiate sori just within the margin; in lobed pinnulae the veins are pinnately branched. — I have not seen rhizome and stipe; I believe that the rhizome is creeping and METTENIUS says (l.c.) that the lower pinnæ are abbreviated or abortive.

Colombia, KARSTEN nr. 40, MORITZ nr. 291, LINDIG nr. 157, STÜBEL nr. 478 a et 687 (B), SCHLIM nr. 3681 (C).

Venezuela, FUNCK et SCHLIM nr. 491 (C).

Ecuador, SPRUCE nr. 5260, STÜBEL nr. 894 (B).

164. *Dryopteris cochaensis* n. sp.

Syn. *Dryopteris biserialis* Hieron. Hedwigia 46: 343. 1907, non C. Chr.

Type from Colombia: Cocha, leg. STÜBEL nr. 231 (B!)

D. pteroideae magnitudine, habitu, textura, colore valde similis, differt: rachi rachillisque dense setosis, pinnis supra ubique minute pubescentibus, soris fere medialibus.

I think this can safely be distinguished from *D. pteroidea* by the characters pointed out above. It is certainly very closely related to the former species, very much resembling large forms of it. Pinnæ 35—45 cm long; pinnulae 4 cm long by $\frac{1}{2}$ cm broad, broadly sessile (in *D. pteroidea* generally subauriculate), the lower ones gradually reduced, central ones the largest and more or less lobed especially in the middle. Veins furcate or pinnate in the lobes, black and pellucid, bearing the round or oblong exindusiate sori just above the furcation at a considerable distance from the edge.

Prof. HIERONYMUS identified the only specimen of this new species with *D. biserialis* (Bak.) C. Chr. BAKER's species however is a member of the subgenus *Ctenitis* and, of course, totally different from our species. This is a new instance of the impossibility of determining specimens with certainty after Syn. Fil.

165. *Dryopteris Ruiziana* (Kl.) C. Chr. comb. nov.

Syn. *Polypodium Ruizianum* Klotzsch, Linnaea 20: 385. 1847.

Type from Peru, ad Panatahua, leg. Ruiz nr. 70 (B!).

This was founded on a fragment, the upper part of a leaf, but I have no doubt that it is a very distinct species allied to *D. multiformis* and that its proper position is here.

Rachis stramineous, trisulcate above with light brown scales in the furrows, scarcely pubescent. Pinnæ opposite, 15 cm long, $2\frac{1}{2}$ cm broad, coriaceous, glabrous above (the sparsely setose costa excepted), margins, costæ and costulæ beneath rather densely soft-hairy by very long, thin, whitish, pluricellular, patent hairs and with several light-brown thin scales. Segments rather close, linear-falcate, obtuse, the basal ones reduced and auricled at the inner side. Veins simple, 15—17-jugate. Sori about medial, exindusiate.

166. *D. lanipes* C. Chr. Smiths. Misc. Coll. 52: 394.

Area: Guatemala.

167. *D. horrens* Hieron.; C. Chr. Revision nr. 80.

Area: Ecuador.

168. *D. mertensioides* C. Chr. Revision nr. 81 fig. 50.

Area: Costa Rica; Guatemala: San Miguel Uspantán, Depart. Quiché, 6000 ft., HEYDE et LUX ed. DONN. SMITH nr. 3243 (W).

The specimens quoted have indusiate sori.

169. *D. cheilanthoides* (Kze.) C. Chr. Revision nr. 82 fig. 51;
Smiths. Misc. Coll. 52: 396.

Area: Southern Brazil. Jamaica. Mexico along Andes to Peru.

The Andine forms of this most distinct species do not agree perfectly with the Brazilian type, still I can not make them a separate species. The differences are only small; perhaps the most constant is the presence of an auricle on the inner side of the lower basal prolonged segments, which overlaps the rachis above; in the Brazilian form this auricle is very small or absent. As to pubescence and glandulosity the Andine forms are somewhat variable. They have been described under several names, but I cannot see any clear difference between the forms, and I refer them all to a single variety:

var. *resinoso-foetida* (Hook.).

Syn. *Nephrodium resinoso-foetidum* Hk. spec. fil. 4: 105. 1862.

(Ecuador, SPRUCE nr. 5300, Kew! RB; nr. 5299 and 5301 quoted by HOOKER I refer to *D. pachyrachis*).

Lastrea grossa Presl, Epim. 41. 1851. nomen. (Mexico, LEIBOLD, hb. Presl!)

Aspidium Kunzei Fée, 10 mém. 37 tab. 41 fig. A, B.

Dryopteris oochlamys C. Chr. Ind. 280. 1905.

Aspidium decrescens Kze.; Mett. *Aspid.* nr. 202. 1858.

Dryopteris decrescens O. Ktze.; C. Chr. *Ind.* 261. (Venezuela, FUNCK & SCHLIM nr. 1229, L!)

Nephrodium atomiferum Sod. *Rec.* 48. 1883. (Ecuador, SODIRO, Kew!)

Nephrodium Sprengelii var. *persicinum* Jenm. *Journ. Bot.* 1879: 261. (Jamaica!)

Dryopteris cheilanthoides var. *eglandulosa* C. Chr. *Bull. L'Herb. Boiss.* II. 7: 262. 1907.

Dryopteris supina var. *Biolleyi* Christ, *Bull. L'Herb. Boiss.* II 7: 262. 1907; C. Chr. *Revision* 308.

170. *Dryopteris multiformis* n. sp. — Fig. 17.

Ecuador, in præruptis montis Pichincha, 3000 m., leg. A. MILLE (RB!).

Rhizomate oblique erecto, parce squamoso. Stipitibus subfasciculatis, strictis, griseo-stramineis basi fusciscentibus, sulcatis glabris, squamis brunneis sparse onustis, 15—25 cm longis. Lamina ovato-lanceolata, ad apicem pinnatifidum sensim decrescente, versus basin subito attenuata, 40—70 cm longa, 10—30 cm lata, firmo-herbacea vel membranacea, opaca, griseo-viridi, valde variabili: bipinnatifida-bipinnata-tripinnatifida, subglabra vel dense pubescente, rachi costis costulisque utrinque pilis albidis tenuissimis, longis, pluricellularibus, patentibus plus minusve hirtis, marginibus ciliatis, faciebus glabris, costis subtus squamis parvis nonnullis instructis. Pinnis semper oppositis, sessilibus, infimis 2—3 paribus valde reductis, auriculiformibus, sequentibus 2—3 paribus parum abbreviatis, reflexis, medialibus maximis, recte patentibus, inter se 3—4 cm remotis, oblongis vel lanceolatis, acuminatis, plus minusve partitis, segmento vel lacinia basali posteriore semper aucto, falcato, basi auriculam rachin tegentem ferente. In eadem planta formæ sequentes repertæ sunt: 1) forma bipinnatifida, dense pilosa (fig. 17 a—b); pinnis 6—8 cm longis, 1½—2 cm latis, ad alam 2 mm latam pinnatifidis; lacinis acutis, subfalcatis, marginibus integris revolutis; venis simplicibus, 6—7-jugis. 2) forma bipinnatifida major subglabra (fig. 17 c—d); pinnis ad 20 cm longis, 3 cm latis, ad alam 3 mm latam pinnatifidis; lacinis acutis, subfalcatis, marginibus subintegris, vix revolutis; venis simplicibus, 10—12-jugis. 3) forma bipinnata vel tripinnatifida subglabra (fig. 17 e); pinnis ad 20 cm longis medio ad 6 cm latis, versus basin attenuatis, a basi ad medium pinnatis, a medio ad apicem pinnatifidis; pinnulis ad costam late adnatis, subremotis, basi utrinque auriculatis, auriculis ovatis acutis, sursum integris vel plus minusve profunde lobatis; pinnula basali anteriore reducta, venis in pinnulis maximis ad 16-jugis, plerumque furcatis vel subpinnatis. — Soris medialibus exindusiatis, sporangiis glabris.

This remarkable species is certainly closely allied to *D. cheilanthoides*; the second form described above resembles very much the large Mexican form of *D.*

cheilanthoides described as *Aspidium Kunzei* Fée (*Lastrea grossa* Pr.), but it differs by its medial, exindusiate sori, fewer veins and eglandulose underside. The bipin-

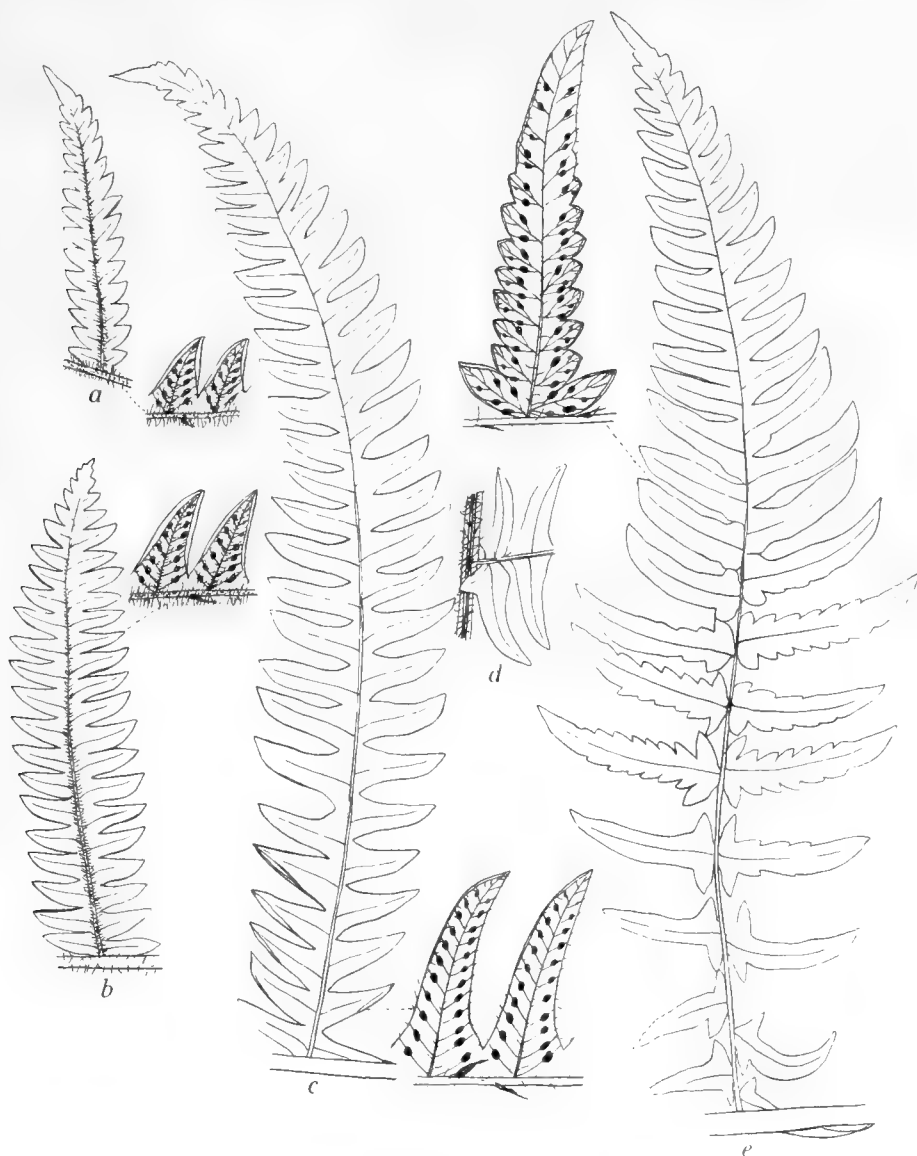


Fig. 17. *D. multiformis* n. sp. The middle pinna of four different leaves of the same plant, $\times \frac{1}{5}$ with segments $\times 1\frac{1}{2}$ (orig.).

nate form resembles in habit not a little *D. pterioidea* differing by the not reduced, auricled lower basal segment, pubescence, position of sori. The two species are, I believe, very remotely related.

Unknown species of *Lastrea*.

The following species, of which I have seen no specimen, can with approximate certainty be referred to the present subgenus.

1. *Aspidium Fischeri* Mett. *Aspid* nr. 192. 1858; *Dryopteris Fischeri* C. Chr. Ind. 266. — Brazil. — Closely allied to *D. ptarmica*.
2. *Phegopteris leptoptera* Fée, Gen. Fil. 244. 1850—52. — San Domingo, DE TUSSAC. According to the description this has setose sporangia. Most likely it is identical with *D. concinna*.
3. *Aspidium sanctoides* Fée, Gen. 292. — Guadeloupe, DUCHASSAING. — In Ind. Fil. referred to *D. opposita*. FÉE compares it to *D. sancta*.
4. *Aspidium confluens* Fée, Gen. 293. — South America, PAMPLIN nr. 75. Fully indeterminable from the description alone.
5. *Aspidium pachychlamys* Fée, 11 mém. 77 tab. 21 fig. 2. 1866. — Guadeloupe, L'HERMINIER. — A form of *D. opposita*?
6. *Nephrodium rigescens* Sodiro, Cr. vasc. quit. 239. 1893; *Dryopteris rigescens* C. Chr. Ind. 289. — Ecuador, Corazon. — Near *D. cheilanthoides*?
7. *Nephrodium elegantulum* Sodiro, Cr. vasc. quit. 243. 1893; *Dryopteris elegantula* C. Chr. Ind. 263. — Ecuador, valle de Pallatanga. — ? —
8. *Nephrodium longipilosum* Sodiro, Sertula Florae Ecuad. series II. 26. 1908 — Ecuador, Corazon. — May be a very hairy form of *D. oligocarpa*.
9. *Nephrodium cinereum* Sodiro. l. c. — Ecuador, Corazon, a variety β *intermedium* from Esmeraldas. A short-hairy species like *D. concinna*, but indusium persistent.
10. *Nephrodium basiattenuatum* Jenman, Gard. Chron. March 17th. 1894; Bull. Bot. Dept. Jamaica n. s. 3: 20. 1896; *Dryopteris basiattenuata* C. Chr. Ind. 254. — Jamaica, Mount Moses. — Probably near *D. sancta*. — According to Miss SLOSON this is apparently not in JENMAN's type-herbarium in New York Bot. Garden, and it was also not found in Kew.
11. *Nephrodium crenulaeum* Jenman, Bull. Bot. Depart. Jamaica n. s. 3: 68. 1896; *Dryopteris crenulaeae* C. Chr. Ind. 259. — Jamaica. — ? — Not in JENMAN's type-herbarium and not in Kew.
12. *Dryopteris Hassleri* Christ, in Bull. l'Herb. Boiss. II. 7: 922. 1907. — Paraguay, HASSLER nr. 9056 a.
13. *Dryopteris Rojasii* Christ in Fedde, Repert. 6: 349. 1909. — Paraguay, HASSLER nr. 10514 a.
14. *Lastrea Cumingiana* Pr. Epim. 37, 1849. — Chile, CUMING.

Subgenus 5. *Glaphyopteris* (Presl) C. Chr.

Biolog. Arbejder tilegnede Eug. Warming p. 80. 1911.

Glaphyopteris Presl, Abhandl. böhm. Ges. Wiss. V. 5: 344, 1848.

A small subgenus of mostly large species with a bipinnatifid lamina and an often very large (1 cm or more long), acute aërophore at the bases of the pinnæ beneath and similar but smaller aërophores at the bases of the midribs of the segments (not found in *D. mapiriensis*)¹.

Segments close, rectangular with broad, bluntly rounded apex and entire margins, rarely the apex is acute. Veins very close and numerous, simple, the basal ones reaching the margins above sinus. Pubescence somewhat variable (see below), the under-surface of most species covered with numerous, sessile, red glands, which are deciduous and therefore not found in older specimens.

By these characters the six species referred to *Glaphyopteris* differ from *Lastrea*, but I fear that the characters mentioned are not sufficient for the segregation of the two proposed subgenera. If one should prefer to treat *Lastrea* as a genus, which would be a very natural treatment, *Glaphyopteris* ought to be referred to it as a subgenus. It is true that the typical species of *Glaphyopteris*, *D. decussata*, is very different from all species referred to *Lastrea*, but it is no doubt intimately related to the three first species mentioned below. These three species show both the characters of *Lastrea* and *Glaphyopteris* besides some others peculiar to them alone; with the same right they could be referred to *Lastrea*, to a proper subgenus or to *Glaphyopteris*. I prefer here to refer them to a proper section of *Glaphyopteris*, which they are perhaps nearest related to. The subgenus thus is divided into two groups each including three species:

1. Group of *D. Thomsonii*. The three species belonging here resemble species of *Lastrea*, especially those related to *D. rudis*, by the shape of the lamina, which is abruptly narrowed downwards with 3—4 pairs of glanduliform warts along the stipe, and further by the shortly and antrorsely setose costæ above; the veins are not so close as in the species of *Glaphyopteris* proper, which they resemble by the presence of aërophores at the base of the midribs beneath.

The pubescence of the costæ beneath (partly also of the rachis) is peculiar and different from all species of *Lastrea* and *Eu-Glaphyopteris*. The costæ beneath are, namely, shortly and often densely cinereo-tomentose by sessile, 2—3-branched hairs, which do not resemble the stalked, branched hairs of *Goniopteris*. Further the sori seem to be indusiate at least in *D. Cañadasii* and *D. macradenia*. The three species belonging here were in my former papers referred to the group of *D. opposita*. ("Revision" nr. 67, 68 and 79).

¹ The most important function of the aërophores or better "pneumatophores" appears to be during the development of the leaf, while the growing parts of this are covered by mucilage. Very likely they are provisions for the aeration of the young parts; in the developed leaf they are shriveled. See BOWER, *Annals of Botany* 24: 427—428, foot-note, 1910.

2. Group of *D. decussata* (*Glaphyopteris* proper). Lamina not narrowed downwards and without glanduliform pinnæ. Still I find along the stipe of a specimen named *Polypodium Percivalii*, which is no doubt a form of *D. decussata*, several long, projecting aërophores which certainly indicate the place of non-developed pinnæ. Rachis and costæ beneath finely downy by short, simple hairs that are deciduous; costæ above and margins very characteristically villous by long, soft hairs. Veins very close and numerous. Sori exindusiate.

Glaphyopteris is probably confined to tropical America. I do not know any Old-World's species which can safely be referred hereto. The Himalayan *D. erubescens* (Wall.) C. Chr. is similar in habit but otherwise quite different.

Key.

1. Lamina with 3—4 pairs of small, tuberculiform, abortive pinnæ along the stipe. Costæ above antrorsely setose, beneath cinereo-tomentose by sessile, branched hairs. Veins 15—20-jugate, not very close.
 2. Under surface not glandular. Indusium present, but rarely seen.
 3. Sori medial or inframedial. Segments broadly obtuse, membranous, shortly and rigidly setose above 171. *D. Cañadasii* (Sod.) C. Chr.
 3. Sorisupramedial. Segments acute, coriaceous, the upperside (costæ and veins excepted) glabrous 172. *D. macradenia* (Sod.) C. Chr.
 2. Under surface densely glandular 173. *D. Thomsonii* (Jenm.) C. Chr.
1. Lamina without auriculiform and glanduliform, abortive pinnæ. Costæ above and margins villous by long, soft hairs; rachis and costæ beneath glabrous or more or less downy by simple hairs. Veins very close.
 2. Veins about 16-jugate. No aërophore at the base of the costules beneath. Texture thin, herbaceous 174. *D. mapiriensis* Ros.
 2. Veins 25—60-jugate. Distinct aërophore at the base of the costules. Texture firm to coriaceous.
 3. Veins 25—40-jugate. Under surface densely glandular. Sori inframedial 175. *D. decussata* (L.) Urb.
 3. Veins 45—60-jugate. Under surface apparently eglandulose. Sori supramedial 176. *D. polyphlebia* n. sp.

171. *Dryopteris Cañadasii* (Sod.) C. Chr.; Revision nr. 68 fig. 44.

Area: Ecuador. — SODIRO describes the species as exindusiate; I have failed to find indusia in authentic specimens.

172. *Dryopteris macradenia* (Sod.) C. Chr.; Revision nr. 79.

Area: Ecuador, SODIRO (authentic specimens in C and Kew!)

Closely related to the preceding species, but the lamina coriaceous, the segments acute and the sori supramedial. Basal segments much reduced.

173. *Dryopteris Thomsonii* (Jenm.) C. Chr.; Smiths. Misc. Coll. 52: 389.

Syn. *Dryopteris Stübelii* Hieron., C. Chr. Revision nr. 67.

Area: Jamaica, Colombia; Ecuador, Llalla, SPRUCE nr. 5651 (Kew, L, RB).

174. *Dryopteris mapiriensis* Rosenstock, Fedde, Repert. 6: 315. 1909. — Fig. 18.

Type from Bolivia: San Antonio prope Mapiri, 800 m, BUCHLIEN nr. 1131 (R!). A most distinct species, different from the allied *D. decussata* by its small size, fewer veins, densely villous rachis and costæ, thin texture and lack of aërophores at the base of the costules.

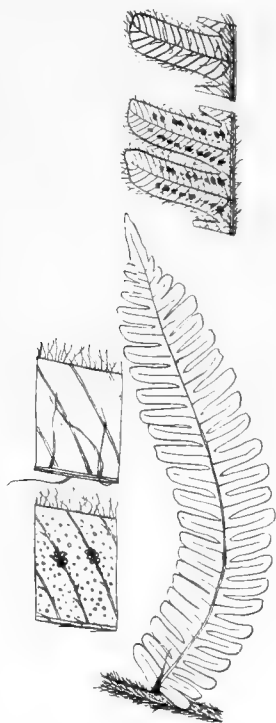


Fig. 18. *D. mapiriensis* Ros. — Pinna $\times \frac{4}{5}$; two segments from the underside and one seen from above, $\times 1\frac{1}{2}$; fragments seen from both surfaces $\times 4$. (orig.)

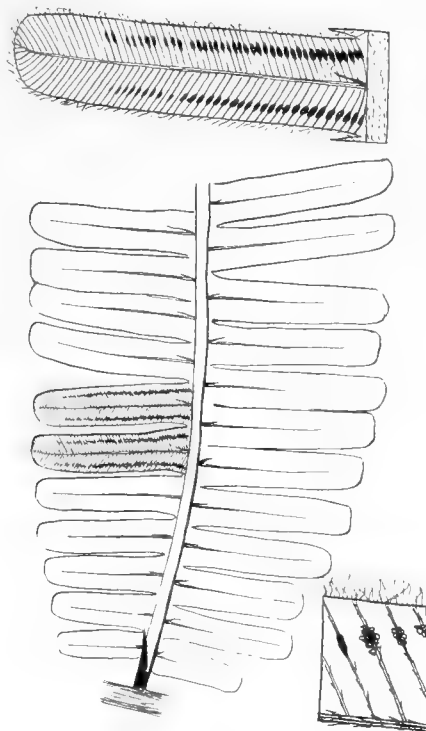


Fig. 19. *D. polyphlebia* n. sp. Portion of pinna, $\times \frac{4}{5}$; segment $\times 1\frac{1}{2}$ and fragment $\times 4$.

175. *Dryopteris decussata* (L.) Urban, Symb. Antill. 4: 19. 1903; C. Chr. Ind. 261.

Syn. *Polypodium decussatum* L. sp. 2: 1093. 1753; Jenm. Bull. Bot. Dept. Jamaica n. s. 4: 130. 1897. (for other synonyms see Ind. Fil.).

Polypodium grammicum Spr. Neu. Entd. 3: 6. 1822!

Gymnogramme microcarpon Fée, 7 mém. 43 tab. 20 fig. 5. 1857.

Polypodium velutinum Sod. Rec. 59. 1883; Cr. vasc. quit. 292. 1893!

Polypodium Percivalii Jenm.; Bak. Ann. of Bot. 5: 456, 1891!

Dryopteris Percivalii C. Chr. Ind. 284. 1905.

LINNAEUS founded this well-known species on "Petiv. fil. 61 t. 2 f. 5", a work to which I have no access, but all later authors quote also PLUMIER tab. 24, which plate illustrates a plant from Martinique and which is our species.

A very large species with fronds up to $3\frac{1}{2}$ m long and pinnæ 30–45 cm long by 4 cm broad. Stipe strong, generally muricate, scaly below and like the rachis downy by short, simple, deciduous hairs; in some (or all?) specimens the stipe bears some long, projecting aërophores. Margins and costæ above densely hairy by long, soft hairs, costæ and costulæ beneath slightly and shortly pubescent becoming glabrous; under-surface covered with red, sessile, deciduous glands. Pinnæ sessile, linear with nearly parallel edges, the aërophore at base thorn-like, 1 cm long. Segments patent, 2 cm long, 3–4 mm broad with an obliquely rounded apex, i. e. the posterior side being bluntly rounded, the anterior one forming about a right angle with the anterior edge, a threadlike aërophore, 2–3 mm long, is found at the base of the costules beneath; basal segments generally somewhat reduced. Veins 25–40-jugate, very close. Sori inframedial or subcostular, exindusiate: receptacles somewhat elevated, oblong; sporangia glabrous, reddish, loose.

The form described here is the typical one, to which belong the following specimens:

West-Indian Islands: Martinique, ISERT (H); SIEBER nr. 189, 349 (B = *P. grammicum* Spr.); L. HAHN nr. 41 (B) — Guadeloupe, L'HERMINIER nr. 173 (B, C); LENORMAND (B); DUCHASSAING (B) — Montserrat, RYAN (H) — Dominica, EGGERS nr. 629 (B); F. E. LLOYD nr. 349 (W) — St. Vincent, EGGERS nr. 6805 (W); H. H. & G. W. SMITH nr. 43 (W) — Grenada, EGGERS nr. 6388 (C, W); BROADWAY (W) — Porto Rico, SENTENIS nr. 1787 (C), 4592 (B, W) — Jamaica, H. SMITH nr. 436 (B); HART nr. 195 (W); CLUTE nr. 163 (W); A. FREDHOLM nr. 3343 (W); UNDERWOOD nr. 1361 (W).

Guiana, LEPRIEUR (B, W) — Mt. Russell, Pomeroon River, Demarara, JENMAN nr. 2080 (Kew = *P. Percivalii* Jenm.),

Venezuela, FUNCK nr. 771 (B); KARSTEN nr. 166 (B).

Costa Rica, La Palma, MAXON nr. 443 (W); WERCKLÉ (C, CC).

Most Andine specimens differ from the type by the more densely pubescent rachis and costæ, more remote and more acute segments and apparently fewer glands on the underside.

A specimen from SODIRO, named by himself *P. velutinum* Sod. belongs to this form, which I therefore call

f. velutina (Sod.)

Ecuador, SODIRO (C) — Peru: San Gavan, LECHLER nr. 2356 (B).

Costa Rica: La Palma, TONDUZ nr. 12639 (C, W) — Carillo, PITTIER et TONDUZ nr. 1165 (C).

var. **brasiliensis** n. var.

A typo differt: stipite spinis acutis instructis; rachi costisque subtus pilis longis patentibus vestitis.

All the Brazilian specimens seen differ from the type by these two characters; otherwise they fully agree.

Brazil: Sta. Catharina, Joinville, O. MÜLLER (R) — SCHWACKE nr. 13320 (C) — Am Rio Comprido bei Ignape, WETTSTEIN u. SCHIFFNER IX. 1901 (Herb. Mus. Wien).

176. **Dryopteris polyphlebia** n. sp. — Fig. 19.

Syn. *Polypodium decussatum* Sod. Cr. vasc. quit. 291. 1893.

Type from Ecuador, leg. SODIRO (B, type specimen, C).

I agree with SODIRO in considering this species distinct from *D. decussata*, which SODIRO described as a new species (*P. velutinum*). It resembles it in most characters but may be distinguished by the following

Very large (I have not seen a complete frond); pinnæ up to 45 cm. long by 7 cm broad with the costa beneath more than 2 mm broad. Pinnæ oblong-elliptical or lanceolate, shortly stalked and narrowed towards the base, in the specimens seen without glands. Segments close with parallel edges, their apex equally rounded to both sides, 3½ cm long by 7 mm broad; veins 50–60 to a side. Sori a little above the middle of the vein, oblong. — Texture coriaceous, fragile, colour grey-green; rachis 8 mm broad, castaneous as young very finely puberulous, soon glabrous.

Besides the specimens from Ecuador I have seen the following from Costa Rica: Carillo, PITTIER nr. 2474 (B, C, W) — Talamanca, TONDUZ nr. 9453 (C, W).

Subgenus 6. **Steiropteris** C. Chr.

Biolog. Arbejder tilegnede Eug. Warming pag. 81. 1911.

Rhizome wide-creeping, ligneous, scaly at the apex; scales firm, castaneous or dull-brown, entire but generally with some few short, acute hairs on the margins. Lamina deltoid or somewhat narrowed below, rarely (in *D. deltoidea*) with several pairs of auriculiform pinnæ, firm to rigidly coriaceous, of a characteristic greyish or brownish colour, more or less pubescent by simple hairs, which partly are short and unicellular, partly long and pluricellular. In most species both kinds of hairs are found intermixed on rachis and costæ beneath, in others only one kind is found; seldom the leaf is quite glabrous. Lamina pinnatifid, or, as a rule, bipinnatifid with sessile or shortly stalked pinnæ, in most species with a distinct acute aërophore at the base. Veins simple, often raised above, those of the basal pair running out to the sinus, more or less connivent and with a cartilagineous

membrane between them; this membrane is in leaves of thick texture often folded and forms on the underside an often hairy keel running from the sinus towards the costa and parallel to the costules. Indusium in most species large, reniform, as a rule persistent. Sporangia glabrous.

A natural group including 13 known species of middle-size or large-grown and partly closely related to each other. The majority of them resemble in habit not a little *D. rudis* and allied species, in venation, indusium and rhizome *D. oligophylla*; the subgenus thus connects the two groups of which the two species named are typical members. From both it can be distinguished by the characteristic carinate fold below the sinus, the grey colour and the pluricellular hairs. From the free-veined species of *Goniopteris* all species are different by the lack of stellate hairs. Glands are absent in all species.

The two last species, *D. glandulosa* and *D. Fendleri*, are somewhat different from the others. Their position in this subgenus is doubtful; see below.

Key.

1. Pinnæ sessile or the lower ones very shortly stalked. All veins free.
 2. Lamina pinnatifid (or pinnate below only) with entire segments
177. *D. Wrightii* (Mett.) O. Ktze.
 2. Lamina bipinnatifid.
 3. Lower pinnæ more or less reduced.
 4. Lamina suddenly narrowed downwards with several pairs of auriculiform, reflexed pinnæ 178. *D. deltoidea* (Sw.) O. Ktze.
 4. Lower pinnæ abbreviated but not auriculiform.
 5. Lamina membranous-papyraceous, pinnæ incised to a narrow wing to costa 179. *D. L'Herminieri* (Kze.) C. Chr.
 5. Lamina rigidly coriaceous; pinnæ incised about halfway to the costa 180. *D. lonchodes* (Eat.) O. Ktze.
 3. Lower pinnæ not reduced, generally reflexed.
 4. Without aërophore. Lamina rather thin.
 5. Rachis and costæ beneath with many short hairs with some few long, pluricellular hairs intermixed. Pinnæ incised to a wing 3—5 mm broad; veins 9—10-jugate
181. *D. densiloba* C. Chr.
 5. Rachis, margins, costæ and veins beneath with many very long, pluricellular hairs, short hairs few. Pinnæ incised nearly to costa; veins 10—11-jugate
182. *D. incana* (Christ) C. Chr.
 4. Aërophore present. Lamina firm, membranous or papyraceous.
 5. Fronds dimorphous. Stipe and rachis very densely clothed

- with woolly, long, pluricellular hairs, under which is a layer of short hairs. 186. *D. valdepilosa* (Bak.) C. Chr.
5. Sterile and fertile fronds similar. Long hairs few or none.
6. Pinnæ incised to a narrow wing to the costa; segments rarely 6 mm broad.
7. Pinnæ with unequal base, i. e. the lower basal segment shorter than the upper
179. *D. L'Herminieri* (Kze.) C. Chr.
7. Basal segments equal and often very reduced in the lower pinnæ. Indusium persistent.
8. Leaf-tissue of both surfaces finely pubescent. Rachis and costæ with several long hairs. Sori medial 183. *D. Leprieurii* (Hk.) O. Ktze.
8. Leaf-tissue glabrous. Costæ beneath glabrous or short-hairy.
9. Veins raised above. Sori submarginal
184. *D. praetervisa* (Kulm.) O. Ktze.
9. Veins scarcely raised above. Sori subcostular
185. *D. densisora* C. Chr.
6. Pinnæ incised to a wing 5—8 mm broad; segments 7—9 mm broad 187. *D. insignis* (Mett.) O. Ktze.
1. Most pinnæ distinctly stalked, scarcely incised halfway down to the costa; segments 6—8 mm broad; lower 2—6 veins on the same side of the costule united along a cartilaginous membrane or keel below the sinus, thus forming 2—4 areoles on each side of the costule. Long, pluricellular hairs none. Lamina upwards suddenly narrowed into a hastate, terminal pinna.
2. Indusium very small, rarely seen; 3—4 pairs of lower veins connivent to the membrane or keel. 188. *D. glandulosa* (Desv.) C. Chr.
2. Indusium persistent, glabrous, 6 pairs of connivent veins
189. *D. Fendleri* (Eat.) O. Ktze.

177. *Dryopteris Wrightii* (Mett.) O. Ktze. Rev. 2: 814, 1891; C. Chr. Ind. 301.

Syn. *Aspidium Wrightii* Mett.; Eaton, Mem. Amer. Acad. n. s. 8: 210. 1860.

Nephrodium Wrightii Hk. sp. 4: 64 tab. 239. 1862; Hk. Bak. Syn. 288;

Jenmann, Bull. Dept. Jam. n. s. 3: 142. 1896.

Type from Cuba orient., Monte Verde, WRIGHT nr. 824 (S), and collected in the same region by MAXON nr. 4482 (W). — JENMANN has found the species in Jamaica, but I have seen no specimens.

A very distinct species, which by authors generally is placed near *D. scolopendria*.

pendrioides, but it agrees in all characters with the other species of § *Steiopteris*, and we here come upon an excellent illustration of the unnatural manner of arranging the species after the degree of cutting. HOOKER's plate cited above is a good figure of the species, and it is unnecessary to give here a new description. Aërophore is not present. It differs from all pinnatifid species of *Dryopteris* by the characters ascribed to § *Steiopteris* and from the other species of this group by its pinnatifid, not bipinnatifid lamina.

178. *Dryopteris deltoidea* (Sw.) O. Ktze. Rev. 2: 812. 1891. C. Chr. Ind. 261.

Syn. *Polypodium deltoideum* Sw. Prod. 133. 1788 — (for other synonyms see Ind. Fil.).

Type from Jamaica; leg. SWARTZ (S! H).

A well-known species, which is the type species of § *Steiopteris*. The pinnae of the lower half of the lamina are reduced to mere lobes, which are reflexed and auricled at the base.

St. Thomas: Le Dru (H).

Porto Rico: EGGERS nr. 814 (B, C), SENTENIS nr. 1001 (S), 2411 (B), 2033, 2256, 2873 (C) — Ventenat (H).
Jamaica: MAXON nr. 915 (H W), 2907 (C, W).

Cuba: WRIGHT nr. 823 (B, S) — PALMER and RILEY nr. 506 et 1028 (H).

Trinidad: Don (L).

179. *Dryopteris L'Herminieri* (Kze.) C. Chr. Ind. 275. 1905.

Syn. *Aspidium L'Herminieri* Kunze; Mett. Aspid. 85 nr. 285. 1858.

Aspidium trichophorum Fée, 11. mém. 81 tab. 23 fig. 2. 1866.

Nephrodium trichophorum Bak. Syn. 265. 1867; Jenman, W. Ind. and Guiana Ferns 215.

Dryopteris trichophora O. Ktze. Rev. 2: 814; C. Chr. Ind. 298.

Aspidium asperulum Fée, 11. mém. 80 tab. 23 fig. 1. 1866.

Dryopteris decumbens C. Chr. Ind. 261. 1905.

Aspidium Capitainei Fée et L'Herm.; Fée, 11. mém. 80 tab. 22 fig. 3. 1866.

Nephrodium Holmei Bak. Ann. Bot. 5: 317. 1891.

Dryopteris Holmei C. Chr. Ind. 271. 1905.

? *Nephrodium clypeolutatatum* Desv. Prod. 258. 1827.

Type from Guadeloupe, leg. L'HERMINIER (B!).

A distinct species resembling in size and general habit *D. mollis* and *D. normalis*, but really a near ally of *D. deltoidea* and *D. Leprieurii*, showing all the characters of § *Steiopteris*:

The wide-creeping rhizome is clothed at the apex with brown scales, which are lighter and thinner than those of *D. deltoidea* and *D. lonchodes*. The stipe is 15—20 cm long, glabrous or finely pubescent. Lamina 2½—4 cm long, 12—15 cm broad, slightly narrowed downwards, membranaceous, grey-green or brownish. Rachis, costae and both sides and costules beneath densely villous by short, uni-

cellular and long, pluricellular hairs, the margins and veins above with solitary long setæ. Aërophore large, acute, hairy (as in *D. deltoidea*). Lower pinnæ more or less reduced, seldom equal to the next pair, sessile or very shortly stalked. Pinnæ alternate, about 8 cm long by 1½ cm broad, the apex acute or acuminate, the base unequal-sided, truncate on the upper, cuneate on the lower side, incised to a narrow wing 1—2 mm broad into falcate or oblique, entire or faintly crenate, acute segments, 4 mm broad. Upper basal segment generally a little longer and almost parallel to rachis, the lower (especially in the lower pinnæ) shorter and oblique. Veins 7—8-jugate, simple, the basal ones running out to the sinus or a little above it with a hyaline, hairy membrane or (in dried specimens) a distinct keel between them. Sori supramedial, furnished with a large, reddish, persistent indusium, which is glabrous or more often pubescent by short hairs and a few long, deciduous setæ.

D. L'Herminieri, with which *A. trichophorum* Fée is absolutely identical, is a fairly constant species, mainly varying in pubescence. *A. Capitainei* Fée is a form with a glabrous indusium and without long, pluricellular hairs on rachis and veins above, but the rachis and costæ beneath are more densely tomentose by the short hairs; to this form belong all specimens from St. Vincent and Grenada, while the specimens from Guadeloupe are partly this form partly typical. *N. Holmei* Bak. is the typical form and *A. asperulum* Fée is intermediate between the two extremes. According to a note in Herb. Berol. by METTENIUS *Nephrodium clypeolulatum* Desv. is this species; if so the name of course has priority. — The species seems to be confined to the Lesser Antilles, from which I have seen the following specimens. METTENIUS mentions a specimen from Jamaica, leg. BREUTEL, which I have not seen, and JENMANN does not know the species as Jamaican. The Brazilian specimens referred hereto by BAKER in Fl. bras. belong to *D. lugubris*.

St. Kitts: BRITTON and COWELL nr. 398 (W).

Montserrat: RYAN (H); HOLME (Kew = *N. Holmei* Bak.).

Guadeloupe: L'HERMINIER (B. hb. FÉE, Paris) - MAZÉ nr. 40 (C), JENMAN (W), Père DUSS nr. 4044, 4405 (W).

Dominica: EGGERS nr. 898 (W); F. E. LLOYD nr. 23 et 754 (W).

Martinique: Père DUSS nr. 1583, 1584 (W) — BORDAZ (C).

[St. Lucia: GRAY according to JENMAN, not seen.].

St. Vincent: JENMAN (W), EGGERS nr. 6883 (C, W), CHECKLEY nr. 10 (C) — Herb. Bot. Gard. Trinidad nr. 6182 (C).

Grenada: SHERRING nr. 222 (W); MURRAY and ELLIOTT (B, W); EGGERS nr. 6102 (C, W), JENMAN (W), BROADWAY nr. 3767 (RB).

180. *Dryopteris lonchodes* (Eat.) O. Ktze. Rev. 2; 813. 1891; C. Chr.

Ind. 275. — Fig. 20.

Syn. *Aspidium lonchodes* Eaton, Mem. Amer. Acad. n. s. 8: 210. 1860.

Nephrodium lonchodes Hk. sp. 4: 99. 1862; Hk. Bak. Syn. 270.

Type from Cuba orient. Monte Verde, WRIGHT nr. 1007, 1008 (B, S, W) — Specimens from the same region (Vicinity of Baracoa) were collected by POLLARD, E. and W. PALMER nr. 239 (W).

Closely allied to the two preceding species, distinguished by its rigid, grey frond, which upwards is suddenly narrowed into a long pinnatifid apex. Scales of rhizome dark-castaneous, rigid. Pubescence mainly as in *D. L'Herminieri*, still the

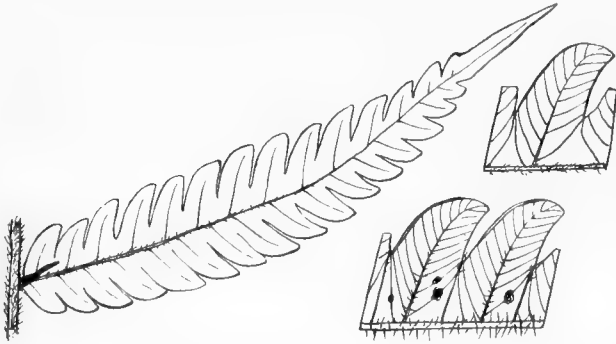


Fig. 20. *D. lonchodes* (Eat.) O. Ktze. Pinna $\times \frac{4}{5}$; two segments from the underside and one from the upperside, $\times 1\frac{1}{2}$.

costæ beneath are lacking the short hairs, and the upperside hairy on the costæ only. Pinnæ sessile with an equal, cuneate base, incised $\frac{1}{2}$ — $\frac{2}{3}$ of the way to the costa into falcate, obtuse or acute lobes. Veins about 10-jugate occasionally forked, raised on the upperside; the basal ones connivent to the sinus; keel distinct. Sori medial, rather small, with a reddish, persistent, slightly setose indusium. — The pinnæ are few, 6—8 to a side, the lower

ones more or less abbreviated; middle ones 10—15 cm long, 2—2 $\frac{1}{4}$ cm broad.

181. *Dryopteris densiloba* C. Chr. Index 261. 1905. — Fig. 21 a.

Syn. *Nephrodium Gardnerianum* Bak. Fl. bras. 1²: 474. 1870; Syn. Fil. 496.

Dryopteris Gardneriana O. Ktze. Rev. 2: 812. 1891.

Dryopteris supralineata Rosenst. in Fedde, Repert. 8: 277. 1910.

Type from Brazil, Organ Mts., GARDNER nr. 190 (Kew!). Other specimens were collected near Rio by GLAZIOU nr. 15760 (B, H) and OHANS (R), and in the state of São Paulo, Serro da mar, WACKET nr. 212 (R = *D. supralineata* Ros.).

D. densiloba is apparently a rare species, closely related to *D. L'Herminieri*, resembling it in size, colour and pubescence. It is of a thinner texture and without aërophore; lower pinnæ not conspicuously reduced, reflexed, the upper ones with a characteristic upcurved apex; pinnæ less incised, the often very close, acute segments being connected by a wing 3—5 mm broad; the keel distinct but not reaching the costa. Veins 9—16 to a side; sori medial or a little above the middle of the vein; indusium persistent, shortly pilose (not glabrous as described by BAKER). — The basal segments of the lower pinnæ are much reduced.

182. *Dryopteris incana* (Christ) C. Chr. Ind. 272. 1905.

Syn. *Aspidium incanum* Christ, Hedwigia 44: 367. 1905.

Type from Amazonas: Puritisa, Juruá Miry, Rio Juruá, E. ULE. Herb. Bras. Amazonas Exp. nr. 5763 (C!).

A critical species of doubtful position; provisionally I place it in this section, because it in colour and pubescence best agrees with the other species. Still it lacks aërophores, and the rhizome is described as erect; the vein-like keel is inconspicuous but present. From the nearest species, *D. Leprieurii*, it differs further by its thin texture, much reduced lower segments of the lower pinnæ, and pubescence, stipe, rachis, costæ on both sides being long-hairy by patent, grey, pluricellular hairs with a few short hairs intermixed. Segments linear, somewhat oblique, acute, connected by a very narrow wing; veins 10—11-jugate; sori small, medial, with a small, short-hairy indusium.

183. *Dryopteris Leprieurii* (Hook.) O. Ktze. Rev. 2: 813. 1891;
C. Chr. Ind. 274. — Fig 21 c.

Syn. *Nephrodium Leprieurii* Hk. sp. 4: 106. 1862; Hk. Bak. Syn. 266; Jenman, W. Ind. and Guiana Ferns 216.

Type from French Guiana, LEPRIEUR (Kew! specim. auth. also in B and W).

The typical form of this species is closely related to *D. L'Herminieri*, differing from it by its non-reduced lower pinnæ and the basal segments of lower pinnæ being considerably reduced; also it is much larger, lamina $1\frac{1}{2}$ — $3\frac{1}{4}$ m long by $2\frac{1}{2}$ —3 dm broad, the opposite pinnæ 12—15 cm long, $2\frac{1}{2}$ —3 cm broad. In pubescence it agrees with *D. L'Herminieri*, still the leaf-tissue of both surfaces is finely pubescent by adpressed hairs. Veins 10—12-jugate, raised above, the basal ones reaching the margin a little above the sinus; keel inconspicuous in some specimens, distinct in others. Sori medial, indusium large, more or less hairy.

JENMAN (l.c.) describes the rhizome as being erect; unfortunately all the specimens from Guiana seen by me want rhizome, but in the Brazilian specimens are found parts of the rhizome, which appears to have been creeping with scattered leaves.

D. Leprieurii is apparently a very variable species, at least if the specimens referred to it really belong to a single species. Even the specimens from Guiana vary not a little, and some of them belong perhaps to *Nephrodium subfuscum* Bak. Syn. Fil. 267. 1967; Jenman, W. Ind. and Guiana Ferns 217; *Dryopteris subfusca* O. Ktze., C. Chr. Ind. 295 from Cayenne, LEPRIEUR, which I cannot distinguish from *D. Leprieurii* from the descriptions alone. JENMAN says that it is intermediate between *N. Leprieurii* and *N. stipulare* (= *D. patens*). The type-specimen of it was not found in the Kew Herbarium.

Besides the authentic specimens of *D. Leprieurii* I have seen some others from Guiana, f. inst.: Demerara, JENMAN (W); Maraval, Bot. Gard. Herb. Trinidad nr. 335 (W). Further I refer here some specimens from Brazil and the central South American Andes, which probably belong to var. β Hook. spec. 4: 106. They differ from the typical form by more numerous veins (16—18 to a side), more deeply incised pinnæ (wing 1— $1\frac{1}{2}$ mm broad), large fronds ($3\frac{1}{4}$ —1 m long,

30—40 cm broad), glabrous leaf-tissue and less pubescent ribs; keel inconspicuous or absent, but then a broad, cartilaginous membrane is seen between the basal veins. Sori medial, indusium glabrous or finely pubescent.

Having not seen two specimens, which are quite identical, and not being able of distinguishing the forms from *D. Leprieurii* by several characters, I prefer to refer all the specimens hereto, although it is very probable that at least some of them belong to a distinct species.

Brazil: Matto Grosso, Santa Anna da Chapada, MALME $\frac{4}{8}$ 1902 (Rg. very beautiful and large specimens with very acuminate pinnæ) — Minas Geraes: Lagoa Santa, WARMING (H).
Bolivia: Mapiri, RUSBY nr. 426 (CC, thin leaved; sori supramedial).
Peru: St. Gavan, LECHLER nr. 2468 (B).

var. *minor* Hieron. Hedwigia 46: 328. 1907.

Ecuador, in valle Pastaza, Stübel nr. 965 (B).

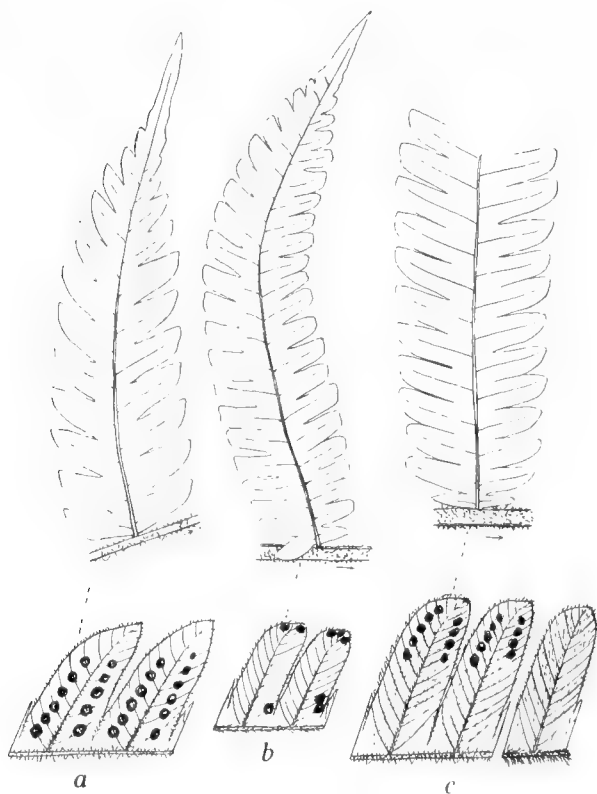


Fig. 21. a. *D. densiloba* C. Chr.; b. *D. praetervisa* (Kuhn) O. Ktze.; c. *D. Leprieurii* (Hook.) O. Ktze.; all from the type-specimen. Pinnæ $\times \frac{4}{5}$, segments $\times 1\frac{1}{2}$.

Type from Venezuela, Tovar, FENDLER nr. 371 (Kew!).

A critical species and perhaps a variety of *D. Leprieurii*. It differs from that species by the perfectly glabrous leaf-tissue, the costæ and costulæ only being sparsely setose by short hairs; rachis rather densely but shortly pubescent. Veins 10—12-jugate, much raised above. Keel distinct beneath but not reaching the costa. Sori supramedial, sometimes close to the margin, in the only specimen seen confined to the lower two anterior veins and to two or three upper veins at the apex of the segment. — Lamina firm, papyraceous, glossy above, very gradually narrowed upwards into a long-acuminated apex. This last character is perhaps the most marked one. Aërophores distinct.

184. *Dryopteris praetervisa*

(Kuhn) O. Ktze. Rev. 2: 813. 1891;
C. Chr. Ind. 285. — Fig. 21 b.

Syn. *Aspidium praetervisum* Kuhn,
Linnaea 36: 111. 1869.

Nephrodium praetervisum Bak.
Syn. Fil. 495. 1874.

185. *Dryopteris densisora* C. Chr. Ind. 261. 1905.Syn. *Aspidium costale* Mett., Kuhn, Linnaea 36: 111. 1869.*Nephrodium costale* Bak. Syn. 495. 1874.

Type from Venezuela, FENDLER nr. 476 (not seen, not found at Kew). Although I have not seen an authentic specimen I do not hesitate to refer to this species several specimens from Costa Rica, which exactly correspond to the original description. Most of these specimens I have previously determined as *D. Leprieurii* var. *palmensis* (Christ) considering them not specifically different from *D. Leprieurii* and using a MS name of Dr. CHRIST for the variety.

D. densisora is very closely allied to *D. Leprieurii* and it is perhaps only a variety of it. It differs by the glabrous surfaces, being sparsely strigose along the costæ only, and especially by the *inframedial* or subcostular sori, which are furnished with a very large, subbullate and subglabrous indusium. Veins about 12-jugate, scarcely raised above. As to all other characters it agrees with *D. Leprieurii*.

Costa Rica: La Palma, TONDUZ nr. 12576 (C, W), WERCKLÉ (C, CC), A. et C. BRADE nr. 347 (R).

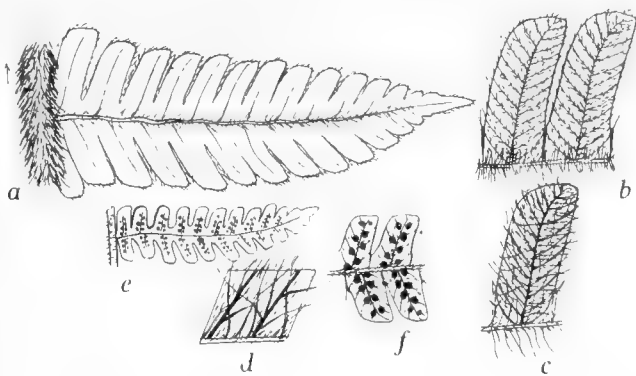
Panama: MAXON nr. 5605 (W).

186. *Dryopteris valdepilosa* (Bak.) C. Chr. Ind. 299. 1905. — Fig 22.Syn. *Nephrodium valdepilosum* Bak. Journ. Bot. 1881: 204.

Type from Colombia, province of Antioquia, KALBREYER nr. 1347 and 1871 (Kew!), and very typical specimens were collected in Costa Rica by J. J. COOPER (W).

A most distinct species, remarkable by its stipe and rachis being very densely clothed with long, soft, patent light brown, pluricellular hairs; similar hairs but shorter and fewer are found on the costæ, costulæ and veins of both sides. These long hairs are (as seen in nr. 1871) deciduous and under them is a rather dense layer of very short, unicellular hairs; costæ above also with many long but antrorse hairs; leaf-tissue of both surfaces

pubescent by minute adpressed hairs. Fronds dimorphous, if the fertile frond of nr. 1347 really belongs to the same plant as the sterile one. Sterile lamina on a stipe 20 cm long, lanceolate, 40—50 cm long, 15—20 cm broad at the middle, membranous, greyish green. Pinnæ close, sessile, lower ones much reflexed, scarcely



D. valdepilosa (Bak.) C. Chr. (KALBREYER 1347). a. pinna of sterile leaf $\times \frac{1}{5}$; b. two segments from the underside and c. one segment seen from above of the same pinna, $\times 1\frac{1}{2}$; d. fragment of the same from the upper side $\times 4$; e. pinna of fertile leaf $\times \frac{1}{5}$; f. segments of the same, $\times 1\frac{1}{2}$.

reduced, central ones about 10 cm long, $2\frac{1}{2}$ cm broad, shortly acuminate, a distinct aërophore at the base beneath, incised to a wing 4–5 mm broad into close oblong segments, which are as broad at the rounded apex as at the base. Veins 10–12, keel distinct. — Fertile lamina much narrower, 20 cm long by 8 cm broad; pinnæ distant, obtuse, $3-3\frac{1}{2}$ cm long, 1 cm broad, incised nearly to costa. Sori near costule, furnished with a persistent, long-ciliated indusium.

D. valdepilosa is intermediate between *D. Leprieurii* and *D. insignis*, abundantly different from both by its dimorphous fronds and woolly pubescence of stipe and rachis.

187. *Dryopteris insignis* (Mett.) O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 272.

Syn. *Aspidium insigne* Mett. Ann. sc. nat. V. 2: 247. 1864.

Nephrodium insigne Bak. Syn. 262. 1867.

Type from Colombia, TRIANA (B, where only a pair of basal pinnæ is to be found).

A true *Steiopteris*, by HIERONYMUS unrightly identified with *D. brachyodus* (Hedwigia 46: 323). It differs from the other species of the subgenus by its entirely glabrous pinnæ, which are only incised $\frac{1}{2}-\frac{2}{3}$; keel very distinct; segments 7–8 mm broad; veins 12–16-jugate, prominent, sori medial, small. Texture firm, membranous, colour grey.

A hairy form of this species is no doubt *Dryopteris lata* Hieron. Hedwigia 46: 327. 1907, from Ecuador, in valle Pastaza, STÜBEL nr. 999 part (B!). It differs by its strigose costæ and veins beneath and the caudate apex of the pinnæ. HIERONYMUS identified it with *Lastrea lata* J. Sm. a name attributed to CUMING pl. philipp. nr. 266, which certainly came from the Philippine Islands. I have seen authentic specimens of CUMING nr. 266 in herb. PRESL and they differ from *D. lata* Hieron. *inter alia* by the stalked pinnæ. I agree with BAKER in considering *L. lata* J. Sm. a form of *D. crassifolia* (Bl.) O. Ktze.

The following two species are as to several characters very different from the species of *Steiopteris* dealt with above, still I think their proper place must be in this subgenus, although it is not improbable that they belong to a proper subgenus. In general aspect they resemble much more species of *Goniopteris* f. inst. *D. nicaraguensis* and *D. megalodus*, and at first I referred them to that subgenus. They differ, however, from *Goniopteris* and agree with *Steiopteris* in the presence of distinct aërophores and in pubescence, stellate hairs being entirely wanting, rachis and costæ beneath are very finely pubescent by minute, simple hairs, which do not differ from the short, unicellular hairs of most species of *Steiopteris*. Long pluricellular hairs of the common kind do not occur, but the costæ and costulæ beneath are more or less furnished with narrow, brown fibrils, which are scale-like hairs consisting of a single row of cells; such fibrils are often found in large

number round the aërophore and at base of the costules and have here fully the aspect of scales. These "scales" are, however, not true scales; under the microscope they are seen to consist of an entangled mass of the long, brown fibrils, probably brought together by mites and thus forming *Acarodomatia*, but not very much resembling those common on the leaves of *Tilia*. — Very remarkable and quite unique is the venation. Below the sinus extends to the costa a cartilaginous membrane, which in dried specimens often is folded and form a keel quite as in the preceding species; the pinnæ being incised scarcely to the middle the lower 4—6 pairs of veins run out to the membrane, and those coming from the same side of the costule are united near the edge of the membrane into a common-vein running along the membrane to the sinus (see fig. 23). — Other peculiarities of the two species are the not deeply cut, stalked pinnæ and the subdistinct terminal pinna.

188. *Dryopteris glandulosa* (Desv.) C. Chr. comb. nov., non O. Ktze.¹⁾

Syn. *Polypodium glandulosum* Desv. Berl. Mag. 5: 317. 1811.

Goniopteris abbreviata Presl, Tent. 183. 1836 (nomen).

Phegopteris abbreviata Mett. Pheg. nr. 45. 1858.

Phegopteris Plumieri J. Sm. Bot. Voy. Herald 228. 1854.

Goniopteris rostrata Fée, 11 mém. 64 tab. 17 fig. 3. 1866 (bona!)

Nephrodium brachyodon auctt. quoad pl. Ind. occ.; Jenman, W. Ind. and Guiana Ferns 235.

Nephrodium dejectum Jenman, Gard. Chron. III. 18: 640. 1895; W. Ind. and Guiana Ferns 241.

Dryopteris dejecta C. Chr. Ind. 261. 1905.

Type from the West-Indies without exact locality (Herb. DESVAUX, Mus. Paris!).

A most distinct species, very well described by METTENIUS and JENMAN (loc. cit.) and well figured by FÉE. It can at once be distinguished from *D. megalodus*, with which it often has been confounded, by its coriaceous texture, yellowish or greyish underside, its falcate lobed, its simple not stellate hairs of the costæ beneath and by the presence of aërophores and scale-like fibrils on the costæ beneath. Further *D. megalodus* and related species have a terminal pinna like the lateral ones, while the terminal pinna of *D. glandulosa* is hastate, i. e. below its broad base and often confluent with it are to be found a pair of short, lateral pinnæ; the next following pair of pinnæ are much larger.

Rhizome 2—3 cm thick, ligneous, short-creeping, naked. Stipe often 60—80 cm long, stramineous, deeply sulcate above, glabrous. Lamina 30—60 long with 8—10 pairs of very remote (6—8 cm), opposite pinnæ, up to 25 cm long by 4 cm broad, distinctly stalked with a prominent dark, acute or obtuse aërophore beneath,

¹⁾ *D. glandulosa* (Bl.) O. Ktze. = *Aspidium glandulosum* Bl. 1828 must subsequently be renamed. I propose for it the new name *Dryopteris malayensis* C. Chr.

the apex caudate, the base of lower pinnæ reduced; lower pinnæ not shortened. Rachis slender, finely downy by minute hairs, soon glabrous. Pinnæ above slightly strigose on the deeply channelled costæ, elsewhere glabrous, the generally flat costæ and costulæ beneath microscopically puberulous, by simple hairs, the leaf-tissue glabrous. Scale-like fibrils of the costæ beneath few and scattered. Pinnæ incised scarcely to the middle into broad, falcate, entire segments. Veins simple, 12—16-jugate, the lower 3—4 forming areoles between the costule and the membrane; sometimes those of the basal pair are truly anastomosing (*nervatio Goniopteridis*). Sori small, medial or somewhat inframedial. I have failed to find an indusium, which JENMAN describes as small and ciliate. Sporangia glabrous.

DESVAUX identified PLUMIER's plate 21 with this species and was no doubt right; *Ph. Plumieri* was based on the same plate. *D. glandulosa* is in its typical form a common fern in the Lesser Antilles. JENMAN records it also from Jamaica and Guiana. *Nephr. dejectum* Jenm. is according to a photograph of JENMAN's original specimen kindly sent me by Miss SLOSSON together with a small fragment of the same not to distinguish from the type. JENMAN describes the rhizome as erect and the sori as indusiate. — I have seen the following specimens:

West-Indian Islands: St. Kitts, BRITTON and COWELL nr. 407 (W); Mrs. ROBINSON (B) — Montserrat, RYAN (H, RB); H. R. HOLME (B) — Guadeloupe, L'HERMINIER (B, C); Père DUSS nr. 4110, 4112 (W) — Dominica, EGGERS nr. 767 (B, C, RB) — Martinique, ISERT (H); BÉLANGER (RB); HAHN nr. 36 (B, RB), nr. 1075 (B); Père DUSS nr. 1656, 1585 (W); SIEBER, Syn. Fil. exs. nr. 168 (B) — (Santa Lucia, t. JENMAN, not seen) — St. Vincent, EGGERS nr. 6741 (W); H. H. and G. W. SMITH nr. 451 (B, C) — Grenada, SHERRING nr. 103 (C, W) — Tobago, EGGERS nr. 5823 (C); BROADWAY (RB) — Trinidad, Bot. Gard. Herb. Trin. nr. 34 (W); BROADWAY nr. 2524 (RB).

Demerara, JENMAN (*N. dejectum* Jenm.).

Nephrodium Grayii, Jenman, W. Ind. and Guiana Ferns 235 1908 from St. Lucia, GRAY nr. 17, is, judging from the description, this species.

In Central America and South-American Andes a plant occurs, which perhaps is specifically different from *D. glandulosa* but which I now regard as a variety of this species. It is

var. **brachyodus** (Kunze).

Syn. *Polypodium brachyodus* Kze. Linnaea 9: 48. 1834.

Phegopteris brachyodus Mett. Pheg. nr. 38. 1858.

Dryopteris brachyodus O. Ktze.; C. Chr. Ind. 255.

Phegopteris Seemannii J. Sm. Bot. Voy. Herald 228 tab. 49. 1854.

Type from Peru: Pompayaco, leg. POEPPIG (not seen).

In pubescence, size, cutting and venation scarcely different from *D. glandulosa* type; it differs mainly by its thinner texture, darker upper side, alternate pinnæ and its less falcate lobes. The costæ and costules beneath are more decidedly fibrillose by red-brown fibrils, which often cover the aërophore. METTENIUS found the difference between this variety and the type in the goniopteroid venation of

the latter (his *Ph. abbreviata*), but this difference does not exist constantly. In the type one can find occasionally the basal veins truly anastomosing as in *D. megalodus*, what never is the case in the variety.

Guatemala: Cubilquitz. Alta Verapaz, v. TÜRCKHEIM ed. DONN. SMITH nr. 8813 (W); BERNOULLI et CARIO nr. 272 (C); vicinity of Cacao, H. S. BARBER nr. 168 and 170 W).

Costa Rica: Vicinity of Turrialba, MAXON nr. 175 (W); forêts de Tsâki, Talamanca, TONDUZ nr. 9476 (C, W); Tuis, PITTIER nr. 11292 (C); WERCKLÉ (C).

Panama: MAXON nr. 5774 (W).

Colombia: Sta. Marta, H. H. SMITH nr. 2690 (C).

In Syn. Fil. 295 *Nephrodium brachyodus* is said to occur also in Malayan Peninsula and Isles. I have seen several specimens from the former locality determined at Kew, but they belong all to a species, which in pubescence and venation is totally different. It is figured by BEDDOME (Ferns brit Ind. Suppl. tab. 379) and is probably an undescribed species belonging to *Cyclosorus*.

189. *Dryopteris Fendleri* (Eaton) O. Ktze. Rev. 2: 812. 1891;
C. Chr. Ind. 264. — Fig. 23.

Syn. *Aspidium Fendleri* Eaton, Mem. Amer. Acad. n. s. 8: 210. 1860.

Nephrodium Fendleri Hook. spec. 4: 82. 1862; Bak. Syn. 295.

Type from Venezuela, Tovar leg. FENDLER nr. 372 (B, and a photograph of EATON's specimen in W!)

Closely related to *D. glandulosa*, with which it agrees in size, cutting and texture, opposite pinnæ, pubescence, presence of aërophores and brown fibrils on costæ beneath and in its hastate, terminal pinna. The main differences are 1) the stramineous costæ and costulæ and light-green surfaces, 2) the supramedial sori, which are furnished with a corrugated, glabrous indusium, and 3) the larger number of veins, about 6 to each side of the costule, running to the cuneate, cartilagineous membrane, which upwards is nearly 1 mm broad.

JENMAN referred *D. Fendleri* to *D. venusta* (Bull. Bot. Dept. Jamaica n. s. 3: 188). *D. venusta* is, however, not at all related to the present species, easily distinguished from it by venation, pinnatifid apex, stellate pubescence and other characters.

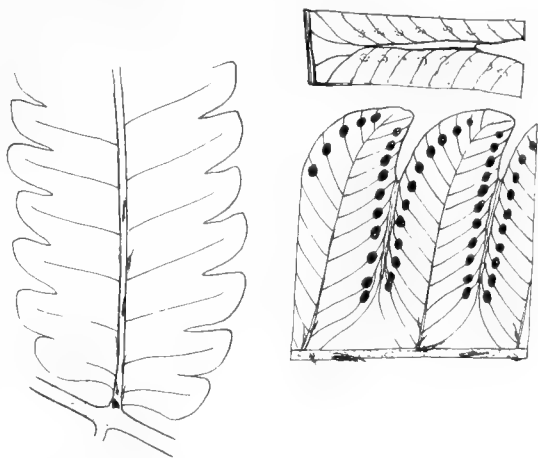


Fig. 23. *D. Fendleri* (Eat.) O. Ktze. Portion of pinna $\times \frac{1}{16}$; segments $\times 1\frac{1}{2}$ and fragment showing the venation below the sinus as seen from the upperside $\times 1\frac{1}{2}$ (orig.).

Subgenus 7. *Cyclosorus* (Link) C. Chr.

Biolog. Arbejder tilegnede Eug. Warming pag. 81. 1911.

The American species of this subgenus are all bipinnatifid and closely related to each other. The rhizome is erect or creeping, clothed with lanceolate or ovate, as a rule hairy and entire scales. Lamina with a broad base or shortly narrowed downwards. Aërophore none; under-surface often glandular, especially on the ribs. Most species rather pubescent by simple, unicellular hairs, only in some few species the longest hairs consist of 2—3 cells, or some of the hairs of the rachis are forked from the very base (cf. inst. in certain forms of *D. oligophylla*). In some species rachis and the costæ beneath bear some few minute, lanceolate scales, which often bear long hairs along the edges. Veins simple, the basal ones either connivent to sinus, where they meet or more often are separated by a cartilagineous membrane that can protrude between the segments as a projecting apophysis, or truly united in the leaf-tissue and sending a branch to the sinus. Sori often large, indusiate. Indusium reniform, generally persistent and pilose, often also glandular. Sporangia glabrous (*D. Martini* excepted, which see).

Cyclosorus is allied to *Lastrea* and *Steiropteris*. Its best characters are venation, non-decrescent lamina and large, persistent indusia. In America it is represented by a dozen species, most of which are very variable and connected by intermediate forms. The subgenus is as a whole the most difficult to deal with and several of the species can be distinguished from each other by certain characters, which can not be explained by words or figures but which are easily observed by the trained eye. Two of the species, *D. mollis* and *D. gongylodes*, are not exclusively American but widely distributed in the Old-World, where the subgenus is richly developed.

Key.

Lamina pinnate only, see *D. pusilla* nr. 54 under § *Lastrea*.

Lamina bipinnatifid:

1. Basal tertiary veins free or connivent to sinus, not normally united in the leaf-tissue.
2. Rhizome erect. Rachis not scaly.
 3. Basal scales broad, thin, yellowish or brown, as a rule glabrous. Lower pinnæ not reduced. Basal segments both prolonged and parallel to rachis 190. *D. patens* (Sw.).
 3. Basal scales narrow-acuminate, ferruginous, glossy, hairy. Lower pinnæ somewhat reduced. Basal segments (at least the posterior one) not conspicuously prolonged.
 4. Pinnæ 4—6 cm long by 1 cm broad, rather firm. 191. *D. Goedenii* Ros.

4. Pinnæ 15—20 cm long by 2—2½ cm broad, thinly herbaceous 199. *D. urens* Ros.
2. Rhizome creeping.
3. Rachis not or very indistinctly scaly. Basal scales ferruginous, narrow-acuminate, as a rule hairy.
4. Lower pinnæ not shortened.
 - 5- Basiscop lower segments of lower pinnæ not or slightly reduced. Pinnæ seldom more than 10—15 cm long, 2 cm broad; veins 6—10-jugate.
 6. Veins not prominent beneath. Lower pinnæ not narrowed towards their base. No scales on rachis and costæ 192. *D. normalis* C. Chr.
 6. Veins more or less prominent beneath. Lower pinnæ more or less narrowed towards their base. Rachis and costæ beneath often with small hairy scales.
 7. Lamina herbaceous or membranous without a distinct terminal pinna. Veins not close.
 8. Underside decidedly pubescent. Cuba and Central America 193. *D. augescens* (Lk.) C. Chr.
 8. Underside practically glabrous. Argentina and Paraguay 194. *D. Berroi* C. Chr.
 7. Lamina coriaceous with a distinct terminal pinna. Veins close, the lower 4 connivent to sinus. Pinnæ generally long and narrow 195. *D. serra* (Sw.) O Ktze.
 5. Basiscop lower segments of lower pinnæ abortive or perfectly obsolete. Pinnæ up to 50 cm long by 3—4 cm broad. Veins 15—20-jugate, the lower 2—4 connivent to sinus 196. *D. oligophylla* Maxon.
4. Lower pinnæ shortened. Leaf throughout much hairy. Lamina firm often coriaceous. Habit of *D. patens*. 198. *D. Bangii* C. Chr.
5. Lamina thinly herbaceous, the hairs burning. Habit of *D. mollis* 199. *D. urens* Ros.
3. Rachis and stipe densely scaly 197. *D. Tuerckheimii* (Donn. Smith) C. Chr.
1. Basal tertiary veins normally united in the leaf-tissue, sending an ex-current branch to the sinus.
2. Sporangia glabrous. Aërophore none. No veins connivent to sinus.
3. Rhizome oblique or short-creeping with narrow, ferruginous scales at the apex. Costæ beneath without scales. Lamina generally narrowed downwards. Pinnæ sessile.
4. Leaf herbaceous. Rachis slender 200. *D. mollis* (Jacq.) Hier.
4. Leaf firm to coriaceous. Rachis strong, quadrangular, very hairy 198. *D. Bangii* C. Chr.

3. Rhizome wide-creeping, nearly naked. Costæ beneath as a rule with small scales. Lamina not narrowed downwards, papyraceous to coriaceous. Pinnæ short-stalked.

201. *D. gongylodes* (Schkuhr) O. Ktze.

2. Sporangia setose. A tuberculiform aërophore at the base of the pinnæ beneath. Above the lowermost anastomosing pair of veins are 3—4 pairs of veins connivent to sinus..... 202. *D. Martini* C. Chr.

190. *Dryopteris patens* (Sw.) O. Ktze., Rev. Gen. Pl. 2: 813. 1891;
C. Chr. Ark. för Bot. 9¹¹: 31 fig. 6.

Syn. *Polypodium patens* Sw. Prod. 133. 1788; Fl. Ind. occ. 1673.

Aspidium patens Sw. Schrad. Journ. 1800²: 34. 1801 et auctt. pro parte.

Polypodium arcuatum Poir. Enc. 5: 528. 1804 (Grenada. Mus. Paris!).

Aspidium stipulare Willd. sp. 5: 239. 1810 (Plum. t. 23).

Nephrodium stipulare Jenman, Bull. Dept. Jam. n. s. 3: 93. 1896.

Dryopteris stipularis Maxon, Bull. Torr. Cl. 33: 198. 1906.

Aspidium macrourum Klf. Flora 1823¹: 365; Mett. Aspid. nr. 219.

Nephrodium macrourum Scott, Gen. ad t. 10. 1834; Bak. Syn. 262.

Nephrodium conspersum Schrad. Gött. gel. Anz. 1824: 869.

Aspidium conspersum Kze. Flora 1839¹: Beibl. 32.

? *Nephrodium polytrichum* Schrad. l. c.

Nephrodium dissimile Schrad. l. c.

Nephrodium albescens Desv. Prod. 258. 1827 (Mus. Paris!).

Lastrea Kohautiana Pr. Tent. 76. 1836!

Lastrea scabriuscula Pr. Tent. 75. 1836; Epim. 35!

Nephrodium schizotis Hook. sp. 4: 107. 1862!

Aspidium abruptum Mart. et Gal. Mem. Ac. Brux. 15: 65. 1842 (f. FOURNIER).

Type from Jamaica, leg. SWARTZ (S!).

In my paper on SWARTZ's type-specimens of ferns (Ark. för Bot. 9¹¹: 28) I have proved that the true *Pol. patens* Sw. is synonymous with *A. stipulare* Willd. and *A. macrourum* Klf. The type-specimens are rather small, while *A. macrourum* is a large-growing form, and *A. stipulare* a form with the upper basal segments very enlarged and deeply lobed, but there is no limit between these forms, which agree exactly in all characters, the size excepted.

The genuine *D. patens* as here understood can be distinguished from related species by these three characters: 1) the erect rhizome, 2) the large, ovate, light-brown, opaque, entire and commonly glabrous scales of the rhizome and stipe; generally they are rather numerous, and 3) by the basal pair of segments, which are much prolonged, acute and both parallel to rachis. Otherwise the species varies very much and it is scarcely possible to give a description, which covers all

its forms. — The fasciculated stipites are deciduously hairy, scaly below, stramineous, the lamina up to 1 m long, firmly herbaceous or even papyraceous, generally dark-green; rachis quadrangular slightly hairy; pinnæ up to 20 cm long, 3–4 cm broad, the lower ones often somewhat reduced and reflexed, strigose on the costæ above, more or less softly pubescent and glandular beneath, especially on costæ and veins; segments subacute, as a rule falcate, the edges often revolute; veins 10 to 16 to a side often somewhat prominent beneath, the lower pair not meeting at sinus but separated by a cartilaginous membrane below the sinus; sori medial or supramedial, the indusium persistent, softly hairy by whitish hairs.

D. patens is distributed over the whole tropical America, common in the West-Indies, Central-America and South Brazil, apparently rather rare in Mexico and the South-American Andes. I have failed to find characters by which forms from different regions may be distinguished from each other, f. inst. most Brazilian specimens agree exactly with the Central-American ones; below I describe four varieties, which however are connected with the type by intermediate forms. It is often impossible to determine with certainty specimens which lack rhizome and lower part of the stipe.

Specimens from Africa and Polynesia are often determined *D. patens*; this species are however no doubt confined to America; the Polynesian plant is partly *D. Harveyi* (Mett.), partly *D. Brackenridgei* (Mett.), the African *D. Bergiana* (Mett.) and *D. Gueintziana* (Mett.).

Below I enumerate some of the specimens examined, mainly such which bear a collector-number.

West-Indian Islands: Trinidad, FENDLER nr. 18 (B, W); Herb. Trin. Bot. Garden nr. 335, 336 (W), 6252 (C) — Grenada, W. E. BROADWAY nr. 1623 (B); EGGERS nr. 6478 (C, W) — St. Vincent, GUILDING nr. 746 (B), H. et G. SMITH nr. 789 (B) — Barbados, (H); EGGERS nr. 7312 (W) — Martinique, SIEBER nr. 354 (B); L. HAHN nr. 35 (B); BÉLANGER nr. 78 et 80 (B); PÈRE DUSS nr. 19, 24, 221 (C), 1578 (B), 4150 (W); ISERT (H) — DOMINICA, IMRAY (B); EGGERS (RB) — Guadeloupe, L'HERMINIER nr. 157 (B); PÈRE DUSS nr. 291 (C), 4030 (C), 4031, 4035, 4038, 4041–43, 4046, 4047, 4344, 4424 (W) — Montserrat, RYAN (H) — Saba (S) — St. Croix, EGGERS (H) — St. Thomas, EGGERS nr. 455 c (B, C); RAUNKJER nr. 13 (CC) — Porto Rico, SENTENIS nr. 1975 (C), 2829, 5868 (W); Mr. et Mrs. HELLER nr. 518a, 927 (W); A. A. HELLER nr. 6343 (W); UNDERWOOD and GRIGGS nr. 273, 632 (W) — San Domingo, EGGERS nr. 1866 a (B); PRENLELOUP nr. 726 (C) — Haïti, WEINLAND nr. 32 (B) — Jamaica, DAY nr. 18 (B); G. L. FISHER nr. 983 (R); UNDERWOOD nr. 2791, 2522, 3235 (W); HART nr. 316 (W); MAXON nr. 830, 932, 1398, 1825, 1928, 2263, 2559 (W) — Cuba, Oriente, WRIGHT nr. 1002 (B, S, W), 2118 (W); EGGERS nr. 4906 (R, W); MAXON nr. 3924, 4101, 4102, 4252, 4280 (W); Prov. Santa Clara, R. COMBS nr. 284 (B); Prov. Habana, BAKER nr. 3804 (W) — Prov. Pinar del Rio, PALMER et RILEY nr. 138, 165, 346, 809 (W), CALDWELL et BAKER nr. 7006 (W); without locality, E. OTTO nr. 81 (B); LINDEN nr. 1396 pt. (B).

Bermuda: Devonshire Marshes, ST. BROWN and N. L. BRITTON nr. 84 (CC).

Florida: South of Miami, Dade Co, A. A. EATON III/1905 (W).

Mexico: Dept. Vera Cruz, Jicaltepec, TLAPACOYO, Colipa et Mirador, LIEBMANN (H); Izhuatlancillo, BOURGEAU nr. 2779 (H); Misantra, SCHIEDE nr. 761 (B); KERBER nr. 435 (W); Córdoba, H. FUNCK nr. 56 (W).

- Guatemala: Dept. Alta Verapaz, near the Finca Sepacuite, COOK et GRIGGS nr. 656 (W), Cubilquitz, v. TÜRKCKHEIM ed. DONN. SMITH nr. 8051 (B, C, W); Dept. Retalhuleu, San Felipe, J. DONN. SMITH nr. 2734 (W); Dept. Santa Rosa, Estanzuela, HEYDE et LUX ed. DONN. SMITH nr. 4424 (B, W); Zamorora, HEYDE et LUX ed. DONN. SMITH nr. 3248 (W); HEYDE nr. 557 (W); Cuajiniquilapa, HEYDE et LUX ed. DONN. SMITH nr. 6289 (W); Vicinity of Cacao, H. S. BARBER nr. 161, 165, 179, 191 (W).
- Honduras: San Pedro Sula, C. THIEME ed. DONN. SMITH nr. 5673 (C, W).
- Nicaragua: Nandaïme, P. LÉVY nr. 3 (H).
- Costa Rica: Navarro, WERCKLÉ (C, CC); Rivière, P. BIOLLEY nr. 44 (C); La Lima, WERCKLÉ (C); Aguacate, C. HOFFMANN nr. 722 (B); Río Ciruelas, TONDUZ nr. 2196 (B, C); Tuis, TONDUZ nr. 11327 (C, W); La Verbena, TONDUZ nr. 8810 (C, W); Juan Viñas, PITTIER nr. 1842 (W); Río General, PITTIER nr. 3340 (W); Buenos Aires, PITTIER nr. 4851 (B, W); San Francisco de Guadeloupe, PITTIER nr. 7154 (W); Río Turrialba, J. DONN. SMITH nr. 6901 (B); Juan Viñas, COOK et DOYLE nr. 179, 233, 243 (W); Río Jirivi, MAXON nr. 130 (W); Meseta central, ALFARO nr. 16848 (C, RB); Sabanilla de Los Granados, ALFARO nr. 16556 bis (RB).
- Panama: MAXON nr. 4781 (W).
- Colombia: Santa Marta, H. H. SMITH nr. 2453 (B, C, Rg) — MORITZ nr. 38, 208 (B), 718 (B, C, H = *A. riparium* Moritz).
- Venezuela: Caracas, LINDEN nr. 199 (B); E. OTTO nr. 424 (B) et alii; La Guayra, ENGEL nr. 20 (B); Tovar, MORITZ nr. 410 (B).
- Ecuador: LEHMANN nr. 5054 (B); Baños, SPRUCE nr. 5303 (H, L, RB), 5304 (H, RB).
- Peru: Lima (W); Ruiz (B); Tarapoto, SPRUCE nr. 4030 (Kew, L = *N. schizotis* Hk. very typical).
- Chile: CUMING (hb. Presl).
- Brazil: Río, GLAZIOU nr. 389 (H), 4669 (B, H), 13353 (H); SELLOW nr. 688 (B); MOSÉN nr. 2700, 2701, 104, 115 (Rg); JELINEK nr. 185 (B); SCHENCK nr. 3783 (C); CASARETTO nr. 2002 (C) — Parahyba, GÖLDI (C) — Minas Geraes, Lagoa Santa, WARMING (H); Ouro Preto, M. GOMES nr. 2663 (C); CLAUSSEN nr. 136 (B, CC) — São Paulo, Río Claro, LÖFGREN nr. 111 (H); Serra de Cubitao, LINDBERG nr. 22 (B); Sororocaba, MOSÉN nr. 3087 (Rg, S); Serra de Caracal, MOSÉN nr. 4621 (Rg); Toledo, ULBRICHT nr. 3 (R); Río Grande, WACKET nr. 93 (R); Alto da Serra, WACKET nr. 201 (R) — Paraná, Icarehý, DUSÉN nr. 6611 (Rg) — Sta. Catharina, Itapocú, SCHWACKE nr. 12961, 12958 (C); Passo Mansa, F. HAERCHEN ed. ROSENSTOCK, Fil. exsicc. austr. bras. nr. 199 (C, R, Rg, W); HAERCHEN nr. 201, 203, 205, 207, 209, 210, 211, 212, 213, 216, 217, 218 (R); San José, GOLTZ nr. 27 (R) — Río Grande do Sul, Sta. Cruz, JÜRGENS ed. ROSENSTOCK, Fil. exsicc. austr. bras. nr. 81 et 85 (B, R, Rg, W); Río Parde, JÜRGENS et STIER nr. 36 et 239 (R); Serra do Melo, JÜRGENS nr. 369 (R); Porto Alegre, STIER nr. 307 (R).
- Argentina: Salto del Encuentro, Río Piray, NIEDERLEIN nr. 1929 (B) — Corrientes, E. PALMER (W).

Varieties of *D. patens*.

var. *dependens* n. var.

Rhizome and scales as in the type, but the leaf much smaller; stipe 20 cm long, lamina 25 cm long by 15 cm broad, herbaceous, light-green; pinnæ opposite, close 8–9 cm long, about 1 cm broad, the basal ones generally reflexed and somewhat abbreviated; upper basal segment of the lower pinnæ broadly ovate, deeply lobed and reflexed over the rachis, the following ones gradually increasing in size to the middle of the pinna. Pubescence thin, underside generally densely glandulose by shining glands. Veins 6–9-jugate, the posterior basal one reaching the margin above the sinus. Sori near the margin, indusia very slightly pubescent or even glabrous.

A characteristic form, which resembles not a little *D. augescens*, but it is thin-leaved and the rhizome and scales different. It seems to be common in Venezuela and similar forms occur in the West Indies, where intermediate forms between the type and the variety are to be found. Such a form is that from St. Kitts, leg. BREUTEL, on which PRESL partly founded his *Lastrea scabriuscula*, which name I retain for a Brazilian form.

Venezuela: La Guaira, ROBINSON et LYON (W); E. OTTO nr. 467 (B); EGGERS nr. 13245 (H); Caracas, MORITZ nr. 79 (B) et alii; Tovar, FENDLER nr. 192 (B); MORITZ nr. 119 et 245 (B).

Colombia: LINDIG nr. 369 (B).

Barbados: HARIOT (C) — Guadeloupe, ISERT (H) — Montserrat, RYAN (H) — San Domingo, Sierra del Palo Quemado, EGGERS nr. 1866 a et c (B) — Haïti, Marmelade, Nash et Taylor nr. 1294 (W) — Jamaica, J. DAY nr. 227 (B) — Cuba, LINDEN nr. 1936 (B).

Guatemala: Dept. Santa Rosa, Zamorora, HEYDE et LUX ed. DONN. SMITH nr. 3246 pt. (W) — Guatemala, Donn. Smith nr. 2463 (B; very small; lamina 10 cm long, 4 cm broad, glabrous).

var. scabriuscula (Pr.).

Syn. *Lastrea scabriuscula* Pr. Epim. 35. 1849 (pt.).

Aspidium nephrodioides Fée, Cr. vasc. Br. 1: 138 tab. 46 fig. 1. 1869.

Not essentially different from var. *dependens*, but much larger; it differs from the type by the thin texture, light colour and sparse pubescence, the upper basal segment of the lower pinnæ is nearly always deeply lobed but not always very prolonged; underside, especially on costæ and costules densely glandular; indusia thin, flat, rather setose.

Rio: MIKAN (herb. PRESL!); GLAZIOU nr. 2359 (H); MOSÉN nr. 2699 (Rg, S); LINDMAN nr. A. 221 (Rg) — Minas Geraes, Ouro Preto, M. GOMES nr. 3008 (C); Tombadouro, SCHWACKE nr. 13543 (C); Lagoa Santa, WARMING nr. 789 (H) — São Paulo, Santos, MOSÉN nr. 3087 (H).

var. deversa (Kze. pt., Mett.).

Syn. *Aspidium deversum* Kze. Linnaea 23: 226, 299. 1850 pt., Mett. Aspid. nr. 209.

Aspidium patens Link, Fil. sp. 100. 1841; ? Raddi, Pl. bras. 1: 32 tab. 48.

Nephrodium deversum Ros. Hedw. 43: 224. 1904.

Dryopteris deversa var. *minor* Hieron. Hedw. 46: 326. 1907.

A. deversum Kze. is partly *D. mollis* partly this variety, which latter was described by METTENIUS under the name *A. deversum* and by him supposed to originate from Cuba; it was described after cultivated specimens, which I have seen (B), but I have not seen a Cuban specimen resembling it, while not a few specimens from South Brazil quite correspond with the original one. It is *A. patens* Link, who quoted the locality: Rio de Janeiro and RADDI's plate. I also believe that this form is RADDI's *A. patens*; RADDI says namely: "Indusiis paucis supra pilis albidis, sed plerumque nudis". — I cannot agree with ROSENSTOCK and HIERNYMIUS in considering this form a species distinct from *D. patens*; the var. *scabriuscula* connects it with the type. It differs mainly from true *D. patens* by its almost quite glabrous leaf and indusia; the underside it rather glandular.

Rio: STÜBEL nr. 1156 (B); REGNELL nr. 252 (Rg) — Minas Geraes, Caldas, MosÉN nr. 2176 (H, Rg, S); Lagoa Santa, WARMING nr. 790 (H) — Sta. Catharina, Blumenau, HAERCHEN (R).
Paraguay: Sierra de Amambay, HASSLER nr. 10411 (RB) — San Salvador, J. D. ANISITS nr. 2700 (Rg).

D. deversa var. *tarapotensis* Ros. Fedde, Repert. 7: 298. 1909 from Peru, Tarapoto, SPRUCE nr. 4066 pt (RB) is a peculiar form with distant pinnæ, the lowermost pair much reduced; it belongs scarcely to the var. *deversa* (Kze.) as here understood, but may be named *D. patens* var. *tarapotensis* (Ros.). It is densely glandulose beneath,

var. *lanosa* n. var.

A large form of the *macroura*-type; characterized by its very dense pubescence of the veins and indusia; the hairs are very long and the indusia are perfectly concealed by a dense mass of long white hairs.

Mexico: L. HAHN nr. 22 (B); BOURGEAT sine num. (B).
Costa Rica: NAVARRO, WERCKLÉ (C).

Aspidium Germani L'Herm.; Fée, 11 mém. 82 tab. 23 fig. 3 from Guadeloupe, L'HERMINIER (Herb. Cosson, Mus. Paris!) is probably a form of *D. patens*. The specimen seen does not agree perfectly with FÉE's figure but appears to be a young plant with very thin leaves and crenate segments. Certainly the general habit of the plant and the scales resemble more *D. normalis* than *D. patens*, but the rhizome is erect. It must provisionally stand among the *species dubiæ*.

191. *Dryopteris Goedenii* Rosenst., Fedde, Repert. 4: 296. 1907.

Type from Brazil, Sta. Catharina, Blumenau, G. GOEDEN (R! also CC).

A rather problematic species, resembling very much small forms of *D. Bangii*, but the rhizome is erect and the lamina not so decidedly narrowed downwards. The scales of the rhizome is like those of *D. mollis*.

192. *Dryopteris normalis* C. Chr. Arkiv. för Bot 9¹¹: 31. 1910.

Syn. *Nephrodium patens* Jenm. Bull. Dept. Jam. n. s. 3: 165. 1896; W. Ind. and Guiana Ferns 240, et auctt. (pro parte).

Aspidium patens Eaton Ferns N. Amer. t. 70 et auctt. (pro parte).

As type of this species I take the Jamaican form, well described by JENMAN and first rightly understood by him. He says (loc. cit.): "this is distinguished absolutely from all its creeping under-ground root stock, upon which the fronds are arranged in a bi-linear series". To this must be remarked that *D. augescens* has a similar rhizome.

D. normalis is in general habit very near small forms of *D. patens*, and specimens without rhizome can not always be determined with certainty. Besides the rhizome the scales show a constant specific character. The scales of the rhizome and the basal part of the stipe are few, mainly confined to the growing apex

of the rhizome; they are narrow-linear, thin, glossy castaneous, ciliated and hairy on the surfaces. A single scale is generally sufficient for determination. Compared with *D. patens* the leaf of *D. normalis* show some peculiarities, which are however not very constant. The lamina is rarely over 4–5 dcm long, gradually narrowed into the pinnatifid apex, pinnæ 10–12 cm long by $1\frac{1}{2}$ –2 cm broad, firmly herbaceous or membranous. Segments acute or rather obtuse, a little oblique but not falcate, the basal ones generally enlarged, especially the upper one, which is parallel to rachis but rarely lobed; the lower one as a rule not very enlarged and forming an angle of 45° with rachis. Upperside glabrous or sparsely pubescent, the costæ softly hairy, underside rather densely and softly pubescent on costæ and veins, and generally glandulose by shining glands, which are most numerous on the veins.

Veins 8–10-jugate, not prominent, the lowest pair meeting at the sinus, where they are not separated by a hyaline membrane. Sori as a rule supramedial; indusium shortly pubescent.

Although *D. normalis* by most authors is believed to be a near relative of *D. patens* I am inclined to believe that it is more closely related to *D. mollis*, which it resembles in the structure of the scales and also in venation; it is, namely, not rare to find truly anastomosing veins as in *D. mollis*, but the lower pinnæ of *D. normalis* are not gradually reduced as in *D. mollis*. To the other side *D. normalis* is through *D. augescens* by intermediate forms connected with *D. serra* and through *D. Feei* with *D. oligophylla*.

JENMAN (W. Ind. & Guiana Ferns 240) says that *D. normalis* is distributed from Florida and Texas to Brazil and in the West-Indies from Bermuda and Bahamas to Trinidad and Dominica. I have examined hundreds of specimens but I have seen none from the Lesser Antilles or South America. As far as I have learned the species is distributed from Porto Rico to Mexico and Guatemala, Texas and Alabama. It is rather variable and below I describe a couple of varieties. Here I enumerate the more important collector-numbers of specimens, which I refer to the type.

Porto Rico: Mr. and Mrs. HELLER nr. 82 (W); G. P. GOLL nr. 120, 896, 897 (W).

Jamaica: MAXON nr. 713, 1005, 1503, 1703, 1782, 2100, 2366, 2536 (W); UNDERWOOD nr. 128, 2568, 2761 (W); FREDHOLM nr. 3346 (W); CLUTE nr. 100 (W); LEVISON nr. 2 (Rg); DAY (B).

San Domingo: L. A. PRENLELOUP nr. 724 (C); M. FUERTES nr. 780 (B).

Cuba: Prov. Habana, BAKER and WILSON nr. 309 (W); H. LEON nr. 142 (W); ABARCA and O'DONOVAN nr. 4095, 5395 (W); van HERMANN nr. 3304 (W); A. H. CURTISS nr. 696 (W); BAKER and O'DONOVAN nr. 4107 (W) — Prov. Santiago, POLLARD and PALMER nr. 96 and 315 (W) — Prov. Pinar del Rio, PALMER and RILEY nr. 106, 294, 624 (W); CALDWELL and BAKER nr. 7107 (W).

Florida: S. M. TRACY nr. 6630, 7632, 9142 (W); F. RUGEL nr. 254 (W); GEO. V. NASH nr. 840, 861 (B, W); UNDERWOOD nr. 181 (W); A. H. CURTISS nr. 3743 (B, W), 4810 (W).

Georgia: R. M. HARPER nr. 1062 (W), 1192 (B, W), 1924 (W).

Alabama: HARPER nr. 131 (W).

Mississippi: S. M. TRACY nr. 8634 (W); J. DONN. SMITH nr. 672 (W).

Louisiana: B. F. BUSH nr. 26 (W); A. B. LANGLOIS nr. 141 (W).

Texas: B. F. BUSH nr. 1466 (W); CURTISS ed. BAENITZ, Herb. Americ. nr. 860 (S); E. PALMER nr. 1433 (W).

While most specimens from the Southern U. S. do not differ materially from the Jamaican type, some of them show a tendency to variation in the direction of *D. augescens*; they are often of a firmer texture, sometimes even coriaceous, and the lower pinnæ are sometimes slightly narrowed towards the auricled base. This latter character is still more pronounced in the two following varieties.

var. **Harperi** n. var.

Lower pair of pinnæ rather reduced and, like the following pair conspicuously narrowed toward the base, the upper basal segment still not very reduced. Pinnæ incised nearly to rachis, the segments oblique, acute, the edges often revolute. The basal anterior vein only runs to sinus, the posterior one reaching the margin about 1 mm above sinus. Sori small, near the edge; indusium subglabrous.

Georgia: Hawkinsville, Pulaski Co., R. M. HARPER nr. 1382 (type, W); rocky bank of Chattahoochee River, Clay Co., HARPER nr. 1788 (W); near Albany, Dougherty Co., POLLARD and MAXON nr. 531 (W).

var. **Lindheimeri** A. Br. pro specie sub *Aspidio*, ms. in Herb. Berol.

Intermediate between typical *D. normalis* and *D. augescens*, and some specimens could as well be referred to the latter. It differs from *D. normalis* type by the firmer texture, the more distant and often opposite pinnæ, the lower ones scarcely reduced but distinctly narrowed towards their base as in the preceding variety and in *D. augescens*; segments acute, often subfalcate; veins generally somewhat prominent beneath, the two lower ones meeting at sinus; sori near to the edge; in some specimens some few small scales are found on the costæ beneath, quite as in *D. augescens*, from which species it differs by its broader pinnæ and by its lamina not being so abruptly narrowed upwards.

Texas: F. LINDHEIMER nr. 742 (B, type, W), 1276 (H); Houston, E. HALL nr. 855 (W); near Kerrville, V. BAILEY nr. 470 (W); near mouth of Pecos River, V. HAVARD (W); San Antonio, Bexar Co., A. A. HELLER nr. 1835 (W); near Laredo, E. PALMER nr. 1432 (W); Edwards Co., R. J. HILL nr. 41 (W).

193. **Dryopteris augescens** (Link) C. Chr. comb. nov.

Syn. *Aspidium augescens* Link, Fil. sp. 103, 1841; Kze. Farrnkr. 134 tab. 59; Mett. Fil. Lips. 91.

Originally described after plants cultivated in the Botanical Garden of Berlin and raised from spores of plants collected by E. OTTO in Cuba; LINK says "Caracas" and METTENIUS "Mexico", but OTTO's original specimens are from Cuba: auf Felsen, Via Tumbadero nr. 89 (B!), with which LINK's original specimens exactly agree like several other specimens from Cuba.

D. augescens is a very critical species, very well described and figured by KUNZE. It is almost exactly intermediate between *D. normalis* and *D. serra*, agreeing with both in the creeping rhizome and the shape and colour of the scales. In general habit it resembles *D. normalis*, from which it differs by 1) the firmer lamina, which rather suddenly narrows upwards into a pinnatifid apex but not having a distinct terminal pinna as *D. serra*, 2) the narrower pinnæ, about 1 cm broad, the lower ones a little narrowed towards the shortly auricled base, 3) the presence of small, hairy scales on the costæ beneath, exactly resembling those of *D. serra*, similar scales are often found on the rachis, 4) the very acute, oblique or patent but not falcate, oblong-triangular segments with somewhat revolute edges, and 5) by the veins being prominent beneath. — From *D. serra*, which it resembles very much in pubescence by hairs and scales, it differs by 1) the shorter pinnæ, which are rarely more than 12 cm long, less acuminate and more deeply cut, 2) the want of a distinct terminal pinna, 3) the lower basal segment of most pinnæ not adnate to rachis, 4) thinner texture and less prominent costæ, 5) the non-falcate segments, and 6) the basal anterior vein only running to sinus, the posterior one generally reaching the margin a little above the sinus.

— Stipes slightly scaly downwards, rather slender, 30—50 cm long; lamina of the same length, fresh- or darkgreen; costæ above more or less pubescent, the upperside elsewhere glabrous; underside not glandulose. Veins about 8 to a side; sori a little supramedial, furnished with persistent, setose indusium.

The following specimens are typical:

Cuba: Prov. Habana, Nazarene, BAKER nr. 1893 (W); Puntos Grandes, H. LEON nr. 165 (W); near Santiago de las Vegas, WILSON nr. 444 (W) — San Antonio de los Baños, BAKER & O'DONOVAN nr. 4132 (W) — Tabureto, E. OTTO nr. 215 (B) — Prov. Pinar del Rio, Guanajay, EARLE et WILSON nr. 1511 (W); near Caiamito, PALMER et RILEY nr. 705 (W) — Isle of Pines, PALMER et RILEY nr. 1120 (W).

Mexico: San Luis Potosí, near Rascon, EDW. PALMER nr. 661 (W); near Tancanhuitz, C. et E. SELER nr. 695 (B); SCHAFFNER sine num. (B) — Jalisco, near Guadalajara, EDW. PALMER nr. 76 (W) — Colima, San Marcos, M E. JONES nr. 529 (W) — Tamaulipas, near Victoria, EDW. PALMER nr. 569 (W).

Costa Rica: Cartago, J. J. COOPER, ed. J. D. S. nr. 6050 (W).

The following specimens from Bahama Islands resemble very much *D. serra* in size and are perhaps belonging to that species; still they agree best with *D. augescens* in most differential characters.

Andros, Nicols, Town, J. and A. NORTHROP nr. 441 (B); Providence, Nassau, J. and A. NORTHROP nr. 173 and 240 (B); RAWSON W. RAWSON (B); EGGERS nr. 4178 (H).

While the Mexican specimens enumerated above are very typical some other specimens are scarcely to distinguish from *D. normalis* var. *Lindheimeri*; these specimens again are intermediate between the type and the larger, common Mexican fern registered in my Index as *D. Feei* C. Chr. It is impossible to me to distin-

guish this *D. Feei* specifically from *D. augescens*, as the two are connected by all intermediate forms. I call the large Mexican form

var. **puberula** (Fée).

Syn. *Aspidium puberulum* Fée, 10. mém, 40. 1865.

Nephrodium puberulum Bak. Syn. 495. 1874.

Dryopteris Feei C. Chr. Ind. 264. 1905.

In structural characters scarcely different from *D. augescens*, but generally larger: stipe up to 70 cm long, stramineous, glabrous, lamina 40—60 cm long: pinnæ 15—20 cm long by 2 cm wide. Veins about 10, prominent beneath, the lower 2—3 meeting at sinus, where as a rule a distinct apophysis is to be found. Undersurface softly villous and sometimes glandulose. The rhizome is very long-creeping. Segments generally very acute and close.

An extremely variable variety; I refer hereto specimens coming very near to typical *augescens*, others very much resembling *D. normalis* and *D. patens*, and others again which are not unlike smaller forms of *D. oligophylla*. Still I have no doubt that all these forms must be referred here and united with *D. augescens*. The most difficult problem to solve is, however, whether the whole series of forms is to refer to a separate species or ought to be united with *D. normalis*. The form from Texas called *D. normalis* var. *Lindheimeri* seems to show that we have only one very variable species, which should be named *D. augescens*, but on the other hand it is unnatural to unite into a single species the typical forms of *D. normalis* and *D. augescens*, and I prefer here to refer the different forms to two species.

It is interesting to note that we here have another example of a series of forms which in their distribution is quite analogous with the series *D. opposita* — *D. panamensis* and *D. Sprengelii* — *D. Mercurii*, all dealt with in my previous papers, and with *D. tetragona* and the intricate forms of it from Mexico — Central America. In all these series the first named species is found in the West-Indies, where it varies only a little, while the second species is mostly developed in Central America, where it varies extraordinarily and where some forms occur, which scarcely can be distinguished from the corresponding island-species.

Aspidium puberulum Fée was described after specimens from Mexico, near Huatusco, SCHAFFNER nr. 247 part., which I have not seen. FOURNIER (Pl. Mex. 1: 95) refer here some specimens of the BOURGEAU collection which I have seen in the museum of Paris. With these specimens agree more or less the following;

Mexico: Lower California, Cape Region, BRANDEGEE (W); Tres Marias Islands, E. W. NELSON nr. 4316 (W); MALTBY nr. 161 (W) — Cuernavaca, BOURGEAU nr. 1318 (H) — Puebla, ARSÈNE nr. 1614, 1690, 1820, 1991, 2006, 2026, 2036, 2145, 2150, 2151 (RB) — Morelia, ARSÈNE (RB, C) — Orizaba, H. E. SEATON nr. 67 (W) — Jalisco, near Guadalajara, ROSE and PAINTER nr. 7416 (W); EDW. PALMER nr. 455 (W); Colima, EDW. PALMER nr. 1229 pt. (W) — Aca-pulco, EDW. PALMER nr. 442 (W) — Tamaulipas, near Victoria, EDW. PALMER nr. 183 and

570 (W) — San Luis Potosí, Rio Verde, EDW. PALMER nr. 32 (W) — Chiapas, near Petapa, COLLINS and DOYLE nr. 94 (W) — Durango, San Ramon, EDW. PALMER nr. 124¹/₂ (W) — Morelos, near Cuernavaca, ROSE and PAINTER nr. 10232 (W).

Guatemala: Depart. Jzabal, Livingston, W. A. KELLERMAN nr. 5082 (W) — Cuajiniquilapa, Depart. Santa Rosa, 2500, HEYDE et LUX ed. J. D. S. nr. 6289 (B, C, W).

var. *geropogon* (Fée).

Syn. *Aspidium geropogon* Fée, 10. mém. app. 1865.

Dryopteris geropogon C. Chr. Ind. 267. 1905.

Differs from var. *puberula* only by the almost glabrous frond and perfectly glabrous indusia.

Mexico: GALEOTTI nr. 6311 (Herb. Paris) — Cuernavaca, BOURGEOU nr. 1319 (H, Herb. Paris) — Lobani, LIEBMANN (H).

194. *Dryopteris Berroi* n. sp. — Fig. 24.

Type from Uruguay, in locis umbrosis ad ripam San Antonio, leg. M. B. BERRO nr. 5243 (CC).

Rhizomate repente paleis brunneis sparse onusto. Stipitibus 4—6 dm longis, stramineis, glabris. Lamina 4—5 dm longa, herbacea vel membranacea, gramineo-viridi, rachi costisque subtus sparse pubescentibus exceptis glabra, bipinnatifida. Pinnis sessilibus, suboppositis, 15—18 cm longis, 1¹/₂—2 cm latis, longe acuminatis, basalibus vix reductis, versus basin parum attenuatis, ad apicem 2—3 mm latam pinnatifidis. Laciniis falcatis, acutis, integris, marginibus revolutis. Venis simplicibus, 10—14-jugis, subtus prominentibus, basalibus ad apophyseum callosam distinctam, saepe acutam excurrentibus. Soris supramedialibus vel margini approximatis, parvis; indusiis persistentibus, reniformibus, glabris vel breviter hirtis.

Intermediate between *D. patens* var. *deversa* and *D. oligophylla*, resembling the former in size, texture, colour and nearly glabrous frond, the latter in the creeping rhizome, prominent veins and the distinct yellowish apophysis, which often resembles a tooth at the sinus. I regard it as a near relative of *D. oligophylla*, like a very reduced form of that species. The lower segments of the lower pinnæ are in large specimens rather reduced but never to the same extent as in *D. oligophylla*; the basal segments of the middle and upper pinnæ are a little prolonged. I have found on the costæ beneath some minute scales, which resemble those of *D. oligophylla*. The rachis and costæ are straw-coloured.

I refer here the following specimens:

Uruguay: BERRO nr. B (C = nr. 5243 of my collection) — Frey Bentos, ARECHA-VALETA nr. 2025 (C).



Fig. 24. *D. Berroi* n. sp. Pinna $\times 1$; and segments $\times 1$ (orig.).

Argentina: Misiones, Loreto, EKMAN nr. 10 (Rg, S); Bonpland, EKMAN nr. 11 (S) — Alta Graecia, Prov. Córdoba, TH. STUCKERT nr. 2605 (C).

Paraguay: Arroyo Fogatingo-mi and San Salvador, J. D. ANISITS nr. 2700 (B, W — a form of thinner texture and veins 8–9-jugate).

195. *Dryopteris serra* (Sw.) O. Ktze. Rev. Gen. Pl. 2: 813. 1891;
C. Chr. Ind. 291 pt.

Syn. *Polypodium serra* Sw. Prod. 132. 1788; Fl. Ind. occ. 1665.

Aspidium serra Sw. Schrad. Journ. 1800²: 33. 1801; Mett. Asp. n. 226
(C. Chr. Ark. för Bot. 9¹¹: 34 fig. 9 et tab. 2 fig. 1, illustrating the type-specimen).

Nephrodium serra Desv.; Bak. Syn. 289; Jenm. Bull. Dept. Jam. n. s. 3: 164.

Type from Jamaica leg. SWARTZ (S!).

In its typical form a most distinct species characterized by its hard, coriaceous texture, long and narrow pinnæ and a distinct terminal pinna. The type-specimen belongs to a small form with a proportionally very long terminal pinna and only 6–8 pairs of lateral pinna, but the species grows much larger. The dimensions of the largest specimen seen from Jamaica are: stipe 130 cm long, over 1 cm thick at base, lamina about 1 m long, about 40 pairs of lateral pinnæ, which are 25 cm long by 8 mm wide, the terminal pinna 17 cm long; this large form does not differ otherwise from the Swartzian type. — Rhizome creeping rather densely clothed by hairy scales. Stipe and upper surface of the light-green lamina glabrous, rachis deciduously and shortly puberulous by thin hairs, which are often furcate, and very sparsely furnished with small scales. The very prominent stramineous costæ and veins beneath rather densely woolly-pubescent and the costæ moreover clothed by rather numerous, very narrow and thin ciliated scales. Pinnæ scarcely reduced towards the base, sessile, the upper basal segment about 1 mm remote from the lower one adnate to rachis, very acuminate towards the apex, regularly and sharply serrated, most deeply at the middle. Teeth triangular, the edges revolute, those of the lower part of the lower pinnæ often nearly obsolete. Veins very close, prominent beneath, stramineous, 7–12-jugate, the lower 2–4 connivent to sinus. Sori a little above the middle of the vein, furnished with a pale or reddish, setose, persistent indusium.

D. serra differs from the related *D. oligophylla* by its much narrower pinnæ, coriaceous texture and the terminal pinnæ, the lower pinnæ are never reduced towards the base in the same manner as in *D. oligophylla*, with which METTENIUS and KUHN united it. The typical form is apparently confined to the larger islands and both the small and the large form are found in all islands.

I have seen the following specimens:

Porto Rico, SENTENIS nr. 5826 (B, C, S, W), 5827 (C, W); UNDERWOOD and GRIGGS nr. 37 (W).

Jamaica, near Troy, UNDERWOOD nr. 2906 (W) — Mt. Diabolo, MAXON nr. 2330 (W) — HART nr. 232 (W) — Hollymount, UNDERWOOD nr. 3460 (W)

San Domingo: WRIGHT, PARRY and BRUMMEL nr. 8 (W); H. VON TÜRKHEIM nr. 2721 (B) — Haïti, near La Barrière Couchant, NASH and TAYLOR nr. 1075 (W) — JAEGER nr. 202 (W).
 Cuba: WRIGHT nr. 923 (S, W) — Oriente, Santa Ana, north of Jaguey, MAXON nr. 4130 (W) — south of Jaguey, MAXON nr. 4169 (W) — Rio Seco, EGGERS nr. 4726 (W) — Mt. Liban, LINDEN nr. 1904 (or 1901) (RB).

In Haïti a form occurs, which is intermediate between *D. serra* and *D. oligophylla*; pinnæ $1\frac{1}{2}$ cm broad, incised about halfway into falcate, very acute segments; in texture and colour it comes nearest to *D. oligophylla*.

Haïti: PICARDA nr. 1030 (C); Rio Bigothon, EGGERS nr. 3308 (C) — San Domingo, Rio Mameges et Rio Bajabonico, EGGERS nr. 2527 (C, CC. H, W).

In Cuba and the Bahamas a form grows, which perhaps belong to *D. serra*, but which I refer to *D. augescens* (see under that species).

196. *Dryopteris oligophylla* Maxon, Contr. U. S. Nat. Herb. 10: 489.

1908. — Fig. 25.

Syn. *Polypodium invisum* Sw. Prod. 133. 1788; Fl. Ind. occ. 1672 (C. Chr. Arkiv för Bot. 9¹¹; 24 fig. 3. 1910).

Aspidium invisum Sw. Schrad. Journ. 1800²: 34. 1801; Mett. Aspid. nr. 211.

Nephrodium Sloanei Bak. Syn. 263. 1874; Jenman, Bull. Dept. Jamaica n. s. 3: 164; W. Ind. and Guiana Ferns 238.

Dryopteris Sloanei O. Ktze. Rev. 2: 813. 1891.

Dryopteris patens subsp. *invisa* C. Chr. Ind. 283.

Aspidium serra var. *incisum* Mett.; Krug, Engl. Jahrb. 24: 118. 1897.

Type from Jamaica, leg. SWARTZ (S!). — In my paper in Arkiv för Bot. 9¹¹: 24—25 I have dealt with the type-specimens of this species.

Rhizome wide-creeping finger thick, clothed with rather thick, dark-brown, hairy scales. Stipites at distances of 5–6 cm, slightly scaly at base, glabrous or finely pubescent, 1 cm thick, often over 1 cm high. Rachis glabrous or more or less pubescent. Lamina 1–2 m long, membranous, chartaceous or coriaceous; pinnæ rather numerous, up to 5 dm long, 3–4 cm broad, the lower ones scarcely reduced, short-stalked, upper ones sessile, long acuminate, the lower 2–3 pairs attenuated towards the base, incised to a narrow wing into subfalcate or oblique entire, acute segments with acute sinuses between, the lower ones of the lower pinnæ gradually reduced, those of the basiscop side abortive and often entirely obsolete. Surfaces eglandulose, glabrous or more or less pubescent, especially on the ribs; minute hairy scales are often to be found on the costæ beneath. Veins 15–20 to a side, simple, prominent beneath; the lower 2–4 run together to a callose membrane, which often forms an apophysis beyond the sinus. Sori rather small, about medial, furnished with large, persistent, glabrous or pubescent indusia.

This very large species is a near relative of *D. serra*, with which METTENIUS united it, but abundantly different from *D. patens*, to which BAKER in Flor. bras.

referred it. From *D. serra* it differs besides by size by the characteristic base of the lower pinnae, the lower basiscop segments being fully obsolete. The species is distributed over the the whole tropical America, but the specimens from different regions do not agree exactly with the West-Indian type. I can distinguish the following forms:

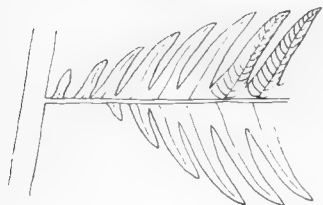


Fig. 25. *D. oligophylla* Maxon.
Basal portion of a lower pinna,
× 4½.

1. *typica*. The common Jamaican form is marked by its very characteristic reddish lamina when dried and by its flesh-coloured indusia, it is often quite glabrous and glossy on the upperside, some specimens are very finely pubescent on the costae and veins beneath; indusia glabrous or minutely pubescent. — With

this form agree most specimens seen from other islands, still some of them, especially those from Porto Rico, are scarcely different from the continental form.

St. Thomas: EGGERS nr. 455d (B, C) — St. Kitts, BRITTON and COWELL nr. 483 (W).

Porto Rico: UNDERWOOD and GRIGGS nr. 75 (W); SENTENIS nr. 411 (C, S, W), 2068 (B, W), 2636 (B, W), 4593 (C, W) — Haïti, PICARDA nr. 1030 (C).

Jamaica: MAXON nr. 1003 (W); O. HANSEN (H); WILSON nr. 682 (B); HART nr. 201 (W); G. L. FISHER nr. 983 (R); UNDERWOOD nr. 3249, 3301 (W).

Cuba: WRIGHT nr. 1003 (S), 3922 (S). Valley of Rio Bayamita, MAXON nr. 3920 (W) — Pinar del Rio, Mts. near Jaco Jaco, BAKER nr. 3804 (W).

2. var. *pallescens* C. Chr. n. var.

Syn. *Aspidium giganteum* Moritz msc. non Bl.

The continental form, differing from the type by the grey-green, coriaceous lamina and the pale or yellowish, more distinctly hairy indusia; rachis and ribs beneath more distinctly hairy and the costae beneath furnished with some small, ciliate, brown scales. It grows larger than the West-Indian form. Some of the smaller forms are scarcely to distinguish from *D. augescens* var. *puberula*.

Mexico: Anganguo, v. CHRISMAR (B).

Costa Rica: NAVARRO, WERCKLÉ (C); La Lima, WERCKLÉ (C, CC) — Rio Corozal, Golfo Dulce, TONDUZ nr. 9978 = DONN. SMITH nr. 7214 (C, W) — Cañas Gardas, PITTIER nr. 10964 (C, W) — Atirro, DONN. SMITH nr. 6900 (W) — Santo Domingo de Golfo Dulce, TONDUZ nr. 10023 (= DONN. SMITH nr. 7215) (C, W) — Rio Turrialba, DONN. SMITH nr. 6901 (W) — Meseta Central de San José, 2000 m, ALFARO nr. 16902 (C, RB) — Veragua, WARSEWICZ nr. 47 (B).

Panama: MAXON nr. 4993, 5763 (W).

Colombia: Santa Marta, H. H. SMITH nr. 2454 (C, Rg) — Ibaqué, STÜBEL nr. 13 (B) — Tolima, SCHMIDTCHEN (B).

Venezuela: MORITZ nr. 424 (B, C, H), 412 (B).

Ecuador: Balao, EGGERS nr. 14660 (W); El Recreo, EGGERS nr. 15037 (W).

3. var. *lutescens* C. Chr. n. var.

Smaller than the preceding variety, as a rule yellowish green; lower pinnae less reduced towards the base.

Brasília: Minas Geraes, Caldas, Mosén nr. 2144, 2145 (H, L, Rg); São Paulo. REGNELL nr. III. 1448 (Rg); WIDGREN (S) — Rio Grande do Sul, JÜRGENS and STIER nr. 182 (R).

4. var. **Kunzeana** (Hook.).

Syn. *Nephrodium Kunzeanum* Hook. sp. 4: 102. 1862.

Dryopteris Kunzeana C. Chr. Ind. 273. 1905.

Aspidium abruptum Kze. Linnaea 9: 93. 1834 (non Blume) etc., see Index.

Closely allied to var. *pallescens* and certainly not specifically distinct from *D. oligophylla*. Rachis and costæ and veins beneath densely but very shortly downy by simple or sometimes forked hairs; scales rather frequent beneath. Sori just within the margin, furnished with a subglabrous indusium.

Peru: POEPPIG (specim. auth. hb. Presl, B) — Tarapoto, SPRUCE nr. 4066 (RB) SCHENKE nr. 47 (R).

Ecuador: Rio Balao, EGGERS nr. 14523 ("8' high" L).

5. var. **aequatorialis** n. var.

Rachis, costæ and veins beneath and indusia densely but shortly hairy. Scales on the costæ beneath rather numerous. Pinnæ numerous, about 30 to a side, rather close, 25 cm long by 2 cm broad.

A most distinct variety, very different from the Jamaican type but connected with it by the var. *pallescens*.

Ecuador: Andes quitenses, SODIRO (C, by SODIRO named *Nephr. patens*) — Santa Jues, Rio Pastaza, Jivaria de Pintuc, STÜBEL nr. 871 (B) — prope Niebli, LEHMANN nr. 5053 (B; "Laub frisch-gelbgrün) — Baños ad fl. Pastaza, SPRUCE nr. 5296 (RB).

Peru: SCHENKE 1909 (R).

Bolivia: M. BANG nr. 2313 (B, W).

197. **Dryopteris Tuerckheimii** (Donn. Smith) C. Chr. Ind. 299. 1905.

Syn. *Nephrodium Tuerckheimii* Donn. Smith, Bot. Gaz. 12: 133 tab. 11. 1887.

Type from Guatemala, Dept. Alta Verapaz, near Coban, leg. H. v. TUECKHEIM, ed. J. DONN. SMITH nr. 704 (W!, B); beautiful specimens were collected at the same locality in 1906—1907 by the same collector, nr. II. 1200 (W) and in the same region, Tamajú, ed. DONN. SMITH nr. 1568 (W).

Perhaps the most distinct species of the whole group, intermediate between *D. patens* and *D. oligophylla*; it resembles the former in the sessile pinnæ, which are not reduced towards the base, the basal segments being enlarged and the upper one often very large and lobed, and by the two basal veins only running to the sinus; it resembles the latter by its creeping, scaly, thick rhizome, by its strong stems and coriaceous texture; from both it differs by the densely scaly stipe, rachis and costæ beneath. The scales are dull-brown, ovate or oblong-acuminate, glabrous and resemble mostly those of the rhizome of *D. patens*. In some specimens the stipe and rachis are entirely covered by such scales. The upper-surface is slightly pubescent, while the underside is densely soft-hairy on the ribs by long,

whitish hairs. Segments falcate with revolute edges. Veins 18--20 to a side. Sori supramedial, furnished with a very densely hairy indusium. Dimensions: Stipe 6--8 dm long, 1 cm thick; lamina 1 m or more long; pinnæ up to 35 cm long, 3½ cm broad, but generally smaller.

198. *Dryopteris Bangii* C. Chr. Vid. Selsk. Skr. VII. 4: 333. 1907 with fig.

Type from Bolivia, near Coroica, A. Miguel Bang nr. 2321 (H!, B). Differs from all forms of *D. patens* and *D. mollis* by its creeping rhizome, its thick quadrangular, very tomentose rachis, its whole hairy appearance and rigid texture, the lamina is narrowed downwards, and the venation is variable, even in the same leaf, the lower veins now being united, now free.

To this species I now refer a number of specimens from Southern Brazil. Among these are some that perfectly agree with the Bolivian type, while others are more thin-leaved with the basal veins rather constantly united. A majority of the specimens were named *Dr. patens* var. *decrescens* by ROSENSTOCK (Hedwigia 46: 114. 1907) and *Aspidium conspersum* Fée, Cr. vasc. Br. 1: 143. 1869 (vix SCHRAEDER) is apparently the same. Some of these specimens are very large, having pinnæ up to 20 cm long by 2 cm wide, in general habit very much resembling *D. patens*, from which species they differ by creeping rhizome, basal scales (which resemble those of *D. mollis*) and the downwards narrowed lamina.

This form can be named var. *patentiformis* Ros. — (The specimens determined by me during these last years are named on the labels *Dryopteris patentiformis* Ros.)

Besides the specimens enumerated by ROSENSTOCK (loc. cit.) I have among others seen the following, all from Brazil:

Minas Geraes, Caldas, MOSÉN nr. 2172 (H, L, Rg, S), 2173 (Rg); Lagoa Santa, WARMING nr. 788 (H); Serra de Ouro Preto, SCHWACKE nr. 10231 (C); Itaculamy, SCHWACKE nr. 14489 (C) — Rio, GLAZIOU nr. 2360 (H); BLANCHET nr. 2635 (H) — Sta. Catharina, Blumenau, HAERCHEN ed. ROSENSTOCK, Fil. austr. bras. exs. nr. 198 (Rg) — Paraná, Serra do Mar, DUSÉN nr. 3725 (C) — Rio Grande do Sul, Rio Pardo, C. JÜRGENS ed. ROSENSTOCK, Fil. austr. bras. exs. nr. 264 (B, C, R, Rg, W).

Another critical form is that named *D. parasitica* var. *procurrens* by ROSENSTOCK (Hedwigia 46: 131. 1907) and distributed by him in his Fil. austr. bras. exs. nr. 356 (R, Rg). It is of a thinner texture, with a more slender rachis and the veins constantly anastomosing. It may be a form of *D. mollis* with creeping rhizome, but I am inclined to consider this and several other critical forms all being forms of a single, very variable species, *D. Bangii*.

199. *Dryopteris urens* Ros. in Fedde, Repert. 4: 5. 1907, C. Chr., Vid. Selsk. Skr. VII. 4: 332. 1907, with fig.

Type from Uruguay, Punta Ballena, ARECHAVALETA (R) and BERRO (CC). Resembles very much some of the thin-leaved forms referred to under the

preceding species, it may perhaps be best known from its soft pubescence by long, whitish, shining hairs, which are said to burn. Under-surface and indusium glandulose, veins free.

200. *Dryopteris mollis* (Jacq.) Hieron. *Hedwigia* 46: 348. 1907.

Syn. *Polypodium molle* Jacq. Coll. 3: 188. 1789; Ic. pl. rar. tab. 640.

Dryopteris parasitica O. Ktze.; C. Chr. Ind. 282 with synonymy.

Nephrodium quadrangulare Fée, Gen. 308. 1850—52.

Aspidium purusense Christ, *Hedwigia* 45: 192. 1906.

Dryopteris Limonensis Christ, Fedde, Repert. 8: 18. 1910.

JACQUIN described his species from specimens cultivated in the gardens of Schoenbrunn and I have seen a leaf therefrom in Herb. Sw. (S). In Ark. för Bot. 9¹¹: 26—28, fig. 4—5. 1910 I have pointed out that *Pol. parasiticum* L. from China can not be identified with *P. molle* Jacq., which is that “*molle*” form occurring in the West-Indies and West Africa and later on described as *Aspidium violascens* Link, characterized by the downwards narrowed lamina.

D. mollis is closely allied to *D. normalis*; in size, texture, pubescence, structure and colour of the scales of the rhizome, sori and other characters the two species are much alike, but the rhizome of *D. mollis* is obliquely erect or short-creeping, not wide-creeping, in the typical forms the lower 2—3 pairs of pinnæ are considerably shortened and the basal pair of veins is truly anastomosing. Still the species varies with regard to the two last named characters. The typical West-Indian form is rather small, thin-leaved and soft-hairy, the lamina gradually attenuate downwards, the basal pair of veins anastomosing; in some continental forms the lamina is not at all narrowed and, as a rule, larger, but otherwise they agree with the type. I have tried to separate such forms as varieties or species but I have failed to find good distinguishing characters, and now I prefer to refer all the different forms to one species, *D. mollis*.

D. mollis is in America distributed from Alabama to Paraguay and Argentina, thus of the same range as *D. patens*. In the Old World very similar forms occur, which probably must be referred to the same species. The West African form, so common in Madeira, is exactly identical with the West-Indian form; the Polyneesian form (*Pol. nymphae* Forst.) is somewhat different but scarcely more so than the American forms differ from each other.

Below I enumerate a part of the specimens examined, especially such which were distributed with numbers. Some of the Costa Rican specimens were determined by CHRIST as *Aspidium prominulum* Christ, Bull. L'Herb. Boiss. 4: 656. 1896; Bull. Soc. bot. Belg. 35: 212 = *Dryopteris prominula* C. Chr. Ind. 286, but they do not at all agree with the description; the species was founded on PITTIER nr. 8198, which I have not seen; judging from the description it belongs to *Goniopteris*. — *Asp. purusense* Christ from Amazonas (HUBER nr. 4459) is a rather common, large form with the upper basal segments enlarged and lobed.

Dryopteris Limonensis Christ, from Costa Rica, A. et C. BRADE nr. 273 is, judging from the description, the typical form of *D. mollis* with the lower pinnæ reduced.

- West-Indian Islands: Trinidad, FENDLER nr. 17 (B, W); hb. Trin. Bot. Gard. nr. 327 (W) — Tobago, EGGERS nr. 104 (C) — Grenada, R. V. SHERRING nr. 18 (W); BROADWAY nr. 3751, 3770 (RB) — St. Vincent, EGGERS nr. 6565 (C); H. H. SMITH nr. 859 (W) — St. Lucia, LEE (W) — Martinique, L. HAHN nr. 466 (B); Père Duss nr. 1586, 4130, 4153 (W), 4424, 4426, 4427 (C), 4616 (W) — Dominica, F. E. LLOYD nr. 690, 721 (W) — St. Thomas, EGGERS nr. 10 (C, H, W) et alii (H) — St. Jan (H) — Porto Rico, UNDERWOOD and GRIGGS nr. 909 (W); GOLL nr. 304 (W); PERCY WILSON nr. 250 (W); SINTENIS nr. 62 (S), 1792 (C, W) — San Domingo, Sierra Palo Quemado, EGGERS nr. 1866 (B); WRIGHT, PARRY et BRUMMEL nr. 31 (W) — Haiti, WEINLAND nr. 73 (B) — Jamaica, MAXON nr. 1715 (= UNDERWOOD nr. 2679), 2364 (W); UNDERWOOD nr. 1541 (W) — Cuba, Oriente, WRIGHT nr. 1001 pt. (B, S, W); Santiago, POLLARD, E. et W. PALMER nr. 63 (W); Pinar del Rio, PALMER et RILEY nr. 135 (H, W), 299, 516 (W).
- U. S. A.: Alabama, Hatchetigbee Bluff, Washington Co., HARPER nr. 130 (W); near Mobile, W. C. DUKES (W) — Louisiana, Lafayette Co., A. B. LANGLOIS (W).
- Mexico: Jalisco, near Guadalajara, PRINGLE nr. 8793 (B, H, S, W) — Tepic, E. PALMER nr. 1939, 1940 (W); J. N. ROSE nr. 3325, 3326 (W); F. H. LAMB nr. 592 (W) — Sinaloa, Rosario, F. H. LAMB nr. 494 (W) — Orizaba, BOURGEOIS nr. 2363 (B, H, S, W).
- Guatemala: Dept. Escuintla, Concepcion, J. DONNELL SMITH nr. 2458, 2738 (W); Cubilquitz, v. TÜRCKHEIM nr. 8357 (C).
- Honduras: San Pedro Sula, C. THIEME ed. DONN. SMITH nr. 5672B (W); Rio Permejo, C. THIEME ed. DONN. SMITH nr. 5698 (W).
- Costa Rica: environs d'Alajuela, PITTIER nr. 542 (C); Terraba, PITTIER nr. 3559 (W); Ile Cocos, PITTIER nr. 12363 (C, W); Ujarra, PITTIER nr. 10566 (C, W) — Navarra, WERCKLÉ (C) — Atirro, DONNELL SMITH nr. 5085 (W) — Cartago, J. J. COOPER ed. DONN. SMITH nr. 6027 (W) — Tuis, TONDUZ nr. 11331 (C); Las Vueltas, TONDUZ nr. 12826 (W), 14582 (C) — San José, COOK and DOYLE nr. 408 (W); ALFARO nr. 16898 (RB); BRADE nr. 416 (R) — Sabanilla de los Graubados, ALFARO nr. 16227 pt. (W) — Rio Sombres, BIDDLEY nr. 1 (C); Tablazo, BIDDLEY nr. 64b (C) — Rio Tiribi, BIDDLEY nr. 100 pt. (C, CC).
- Panama: Boca de Cupe and Cana, R. S. WILLIAMS nr. 851, 852 (W) — Bocas del Toro, HART nr. 51 (W); Chiriqui, HART nr. 251 (W) — E. OTTO nr. 17 (B).
- Colombia: Sta. Marta, H. H. SMITH nr. 1001, 1003 (C) — STÜBEL nr. 372 (B).
- Venezuela: Caracas, MORITZ nr. 114 (B) — Puerto Cabello, KARSTEN nr. 143 (B) — La Guaira, ROBINSON and LYON (W).
- Guiana: Cayenne, LEPRIEUR nr. 26 (B, W) — Cagnai, LEPRIEUR nr. 8 (H) — APPUN nr. 4 (B).
- Ecuador: Balao, EGGERS nr. 14179 (W).
- Peru: Tarapoto, SPRUCE nr. 4039 (RB); rather glandulose throughout = var. *glanduligera* Ros. Fedde, Rep. 7: 304. 1909).
- Galapagos Islands: Charles Island, A. LEE (W — doubtful).
- Brazil: Amazonas, Alto Purus, HUBER nr. 4459 (C) — Bahia, SALZMANN (H); BLANCHET (W) — Pará, J. V. HERNANDEZ nr. F (W) — Rio, MOSÉN nr. 48¹/₂ (Rg, S); REGNELL nr. 251 (Rg); GLAZIOU nr. 10184 (H) — Minas Geraes, Caldas, MOSÉN nr. 2190, 2191 (Rg), 2192 (H, Rg, S), 4615 (Rg); REGNELL nr. 1450 (Rg, W); Casa da Pedra, SILVEIRA nr. 258 (C) — Lagoa Santa, Warming nr. 784 (H) — São Paulo, Campinas, A. HEINER nr. 522 (Rg) — Santos, MOSÉN nr. 3746 (Rg) — Rio Grande do Sul, Rio Grande, LINDMAN nr. A 843 (Rg, W); Excolonia Santo Angelo, LINDMAN nr. A 963 (Rg); Rio Pardo, C. JÜRGENS, Rosenstock, Fil. exsic. austr. bras. nr. 356 (R, Rg) — further numerous specimens from the southern states of Brazil in (R).
- Argentina: Misiones, Posadas, E. L. EKMAN nr. 4, 5, 6, 7 (Rg).
- Paraguay: Paraguari, LINDMAN nr. A 3785 (Rg); ad flum. Riacho Mbopi, LINDMAN nr. 1947 (Rg) — L'Assomption, BALANSA nr. 312 (S).

201. *Dryopteris gongylodes* (Schkuhr) O. Ktze. Rev. Gen. Pl. 2: 811. 1891;
C. Chr. Ind. 268.

Syn. *Aspidium* (*goggilodus*) Schkuhr, Kr. Gew. 1: 193 tab. 33 c. 1809.

Aspidium obtusatum Schkuhr 1806.

Nephrodium unitum Bak. Syn. Fil. 289 et auctt. plur.; Jenman, Bull.
Dept. Jamaica n. s. 3: 189. 1896.

Aspidium Pohlianum Pr. Del. Prag 1: 173. 1822.

Goniopteris cheilocarpa Fée, Gen. 251. 1850—52.

Nephrodium paludosum Liebm. Vid. Selsk. Skr. V. 1: 275. 1849.

(For other synonyms see Index Filicum).

Type from Essequibo.

In Herb. SWARTZ (S) I have seen a specimen no doubt of the type collection, named by SWARTZ *Aspidium obtusatum*, under which name it was figured by SCHKUHR on his plate 33 c. Later on SCHKUHR renamed the species acknowledging that it were not the original *A. obtusatum* Sw. Most pteridologists have called the species *Aspidium* (resp. *Nephrodium*) *unitum* identifying it with *Polypodium unitum* L., but the species of LINNÆUS is said also to be the same as *Nephrodium cucullatum* Bl., and it is, therefore, best to let SCHKUHR's name stand for our species, also because it was applied to the most common American form.

D. gongylodes is a well-known bog-fern, found in almost all tropical and subtropical regions. Its long creeping, black and nearly naked rhizome, its papyraceous to coriaceous lamina, which is not narrowed below, its rather distant and narrow pinnæ with obtusely rounded lobes, its close veins with the two basal ones always truly united, make it easily recognizable from the allied species of this section, of which it is the nomenclaturic type. It varies mainly in pubescence, and one can refer most of the American forms to two varieties, which, however, are connected with intermediate forms:

1. var. *glabra* Mett. As a rule quite hairless but often rather scaly on the costæ and costulæ beneath and most often more or less glandulose by large, capitate glands. These glands are often very numerous on the costulæ and veins and indusia and are here sessile, and in some forms they also are found on the receptacle, intermixed with the sporangia, but here they are stalked. In the common South Brazilian form described as *Aspid. Pohlianum* Pr. (t. sp. orig. Herb. PRESL), which is often exindusiate, these glands are, as a rule, very numerous, but they are also to be found in several specimens from the West Indies and Florida.

2. var. *hirsuta* Mett. Lower surface and indusia rather hairy by short hairs; glands absent or some few are found, mostly among the sporangia; the receptaculum is sometimes also hairy. This form occurs in Southern Brazil, Mexico and in the Lesser Antilles, while the specimens from the Larger Antilles, Florida and Central America mostly belong to var. *glabra*, which is the typical form as figured by SCHKUHR. In size and general habit the two varieties quite agree. They are

rather small, the pinnæ about 10 cm long by 1 cm wide with 6–8 pairs of tertiary veins. A more different form occurs in Southern Brazil and adjacent countries. I call it

var. *longipinna* nov. var.

It is much larger than the type, having pinnæ up to 40 cm long by 30 cm wide with about 15 pairs of veins; further it differs by its thinner texture and scarcely raised veins; it is hairless and, as a rule, also eglandulose.

D. gongylodes is found in America from Florida to Argentina and from Mexico to Panama, while it apparently is unknown from the South American Andes. In the Old World it occurs in about all tropical and subtropical countries, extending to Japan and Corea. The South African form (*Aspid. Ecklonii* Kze.) is nearly quite identical with the American var. *glabra* and similar forms are found in Asia. The Australian and Polynesian form originally described as *Nephrodium propinquum* Rr. B., of which I have seen an authentical specimen (S), corresponds to the var. *hirsuta*, but it may perhaps be considered a different species.

American specimens seen:

- Florida: various localities; CURTISS nr. 3740 (B, W), 6760 (S, W); UNDERWOOD nr. 276 (W); DONNELL SMITH (W); A. P. GARBER nr. 19 (W); MARY C. REYNOLDS nr. 187 (W).
 West-Indian Islands: Guadeloupe, DUSS nr. 4058 (W) — Martinique, DUSS nr. 1587 (W) — Porto Rico, EGGERS nr. 22 (W), 457 (C); SINTENIS nr. 6642 (C, S, W) — Jamaica, near Cinchona, HART (W); UNDERWOOD nr. 156 and 3104 (W); Salt Pond, CLUTE nr. 241 (W); Content Gap, D. WATT nr. 9 (RB) — Cuba, WRIGHT nr. 1100 (S, W); Pinar del Rio, PALMER and RILEY nr. 331 (W).
 Mexico: San Antonio Huatusco, LIEBMANN (H = *Nephrod. paludosum* Liebm. = var. *hirsuta*).
 Costa Rica: Marais de Matina, PITTIER nr. 10266, 10268 (C, W) — Cañas Gardas, PITTIER nr. 11204 (W) — Rio Surubres, BRADE nr. nr. 420 (R).
 Guiana: Surinam, WEIGELT (Herb. Presl) — French G., SAGOT nr. 753 (S).
 Brazil: Rio, GLAZIOU nr. 2362, 2363 (H) — S. Paulo, Santos, MOSÉN nr. 3060 (Rg, S); Campinas, MOSÉN nr. 3938 (H, Rg, S) — Paraná, Icarehý, DUSÉN nr. 6646 (Rg) — S. Catharina, Blumenau, HAERCHEN ed. ROSENSTOCK, Fil. Bras. austr. nr. 88 (Rg, W).
 Paraguay: Guarapi, BALANSA nr. 2193 (H) — Asuncion, ANISITZ (S) — Central Paraguay, TH. MORONG nr. 250 (W).
 Argentina: Misiones, Posadas, EKMAN nr. 9 (Rg) — Jujuy, Quinta, Rob. E. FRIES nr. 255 (Rg).

var. *longipinna*.

- Brazil: Rio Grande do Sul (W) — San Luzia de Rio das Velhas, SCHWACKE (C).
 Uruguay: Puerto del Sauce, BERRO nr. 1250 (CC) — Nueva Palmira, ARECHAVELETA nr. 402 (C).
 Paraguay: Santa Barbara, BALANSA nr. 314^a (S).

Species of doubtful position.

202. *Dryopteris Martini* C. Chr. Index 276. 1905. — Fig. 26.

Syn. *Nephrodium connexum* Kuhn; Bak. Fl. bras. 1²: 489. 1870; Syn. Fil. 502.

Type from Cayenne, leg. MARTIN (B!).

I have seen only an imperfect specimen of this, which shows it being a very distinct species but of a very doubtful position. It is with the greatest doubt only I place it here under *Cyclosorus*; it could as well be referred to *Steiropteris*. Still it does not resemble at all any species of *Steiropteris*, while it in general habit closely resembles certain forms of *D. oligophylla*. It differs from all American species of *Cyclosorus* by the presence of a callose aërophore at the base of the pinnae beneath and by the sporangia being deciduously setose.

Rachis quadrangular, finely puberulous by hamate, deciduous hairs. Pinnæ up to 30 cm long by 3 cm broad, papyraceous; costæ above somewhat strigose, costæ and costulæ beneath setose by spreading, stiff, unicellular hairs; scales and glands none. Margins cut about two-thirds of the way down into falcate, acute segments with the edges somewhat revolute. Veins simple, about 16 to a side, those of the lower 4–5 pairs connivent to sinus, below which is a narrow, cartilagineous and hairy membrane but no keel extending to the costa as in *Steiropteris*, the lowermost pair of veins generally truly united in the leaf-tissue and sending an excurrent branch to the membrane. Sori inframedial, furnished by a subpersistent, reddish, slightly setose indusium. Sporangia with 1–2 short, deciduous hairs. — I have not seen the lower pinnæ and can not say whether the basal lower segments are reduced or obsolete as in *D. oligophylla*.

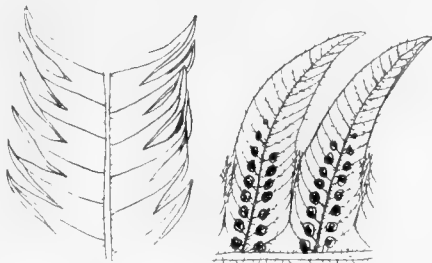


Fig. 26. *D. Martini* C. Chr. Fragment from the middle of a pinna, $\times \frac{1}{15}$; segments $\times 1\frac{1}{10}$ (orig.).

Unknown species of § Cyclosorus.

1. *Aspidium albicaule* Fée, 8 mém. 102. 1857 — Mexico, SCHAFFNER nr. 245.
2. *Aspidium conspersoides* Fée, l. c. 105 — Mexico, Orizaba, SCHAFFNER nr. 335 and 463.
3. *Aspidium pallidum* Fourn. Mex. Pl. 1: 96. 1872; *Dryopteris chlorophylla* C. Chr. Ind. 257 — Mexico, Orizaba, BOURGEAU nr. 2779 bis. These three are all, I believe, forms of what I have called *D. augescens* var. *puberula*.
4. *Aspidium Orizabae* Fée, l. c. 104; *Dryopteris Orizabae* C. Chr. 281 — Mexico, Orizaba, SCHAFFNER nr. 464. Probably *D. oligophylla* var. *pallescens*.
5. *Aspidium pauper* Fée, l. c. 103 — Martinique, Mll. RIVOIRE. A small form of *D. patens*?
6. *Nephrodium oppositum* Fée l. c. 108 — Martinique, Mlle. RIVOIRE. Probably *D. mollis*.

8. *Nephrodium paucijugum* Jenman, Journ. Bot. 1886: 270; *Dryopteris chartacea* C. Chr. Ind. 257 — Jamaica, SHERRING.

On this see MAXON Contr. H. S. Nat. Herb. 10: 489. I believe that the type-specimen of it is at Kew, and I have seen it. It is sterile and undeterminable. It is not improbable that JENMAN was right in considering it a young plant of *D. oligophylla*.

Subgenus 8. *Leptogramma* J. Smith emend. C. Chr.

Biolog. Arbejder tilegnede Eug. Warming pag. 82. 1911.

A small group, in America represented by three species. I confine it to include the species of *Gymnogramme* § *Leptogramma* of Syn. Fil. only having more or less soft-hairy lamina, oblong to linear, exindusiate sori and setose sporangia. The lamina is not or slightly narrowed downwards, the rhizome generally long-creeping, at apex furnished with few hairy scales. The hairs of most species are long, thin, pluricellular. In general habit and venation the species resemble mostly species of *Cyclosorus*, thus the American *D. pilosa* and *D. dasyphylla*, while the third *D. polypodioides* comes very near to species of *Goniopteris*, but I have not found stellate or forked hairs, which constantly are to be found in species of *Goniopteris*. — The typical species of *Leptogramma* is the common *D. africana* (Desv.) C. Chr. (*Gymn. totta* Schlecht.) from Africa and Asia.

Key.

1. Lamina soft-hairy by long pluricellular hairs, herbaceous.
 2. Lamina gradually narrowed towards the apex, several of upper pinnæ confluent at base. Stipe glabrous. Mexico. 203. *D. pilosa* (Mart. et Gal.) C. Chr.
 2. Lamina upwards suddenly narrowed into a pinnatifid apex, lateral pinnæ all free at base; stipe as well as the whole leaf very hairy. 204. *D. dasyphylla* C. Chr.
1. Lamina practically glabrous, membranous. Brazil. 205. *D. polypodioides* (Ratti) C. Chr.

203. *Dryopteris pilosa* (Mart. et Gal.) C. Chr. Ind. 284. 1905.

Syn. *Gymnogramme pilosa* Mart. et Gal. Mém. Acad. Brux. 15: 27 tab. 4 fig. 1. Liebmann, Vid. Selsk. Skr. V. 1: 181 with description.

Type from Mexico: Orizaba, leg. GALEOTTI nr. 6167 and 6268 (Mus. Paris!).

The figure quoted above gives a good illustration of the common form of this characteristic species, which scarcely can be confounded with any other Mexican *Dryopteris*. In general habit it resembles forms of *D. normalis* or of *D. augescens*, but its generally less cut pinnæ, pluricellular hairs, linear to oblong, exindusiate sori and setose sporangia mark it at once. Veins simple, 3–6 to a side, the two basal ones running to sinus. GALEOTTI's and LIEBMANN's specimens belong to a small form having the most pinnæ confluent at base and scarcely incised more than halfway down to the costa, the lower ones often distinctly reduced. This form occurs at altitudes of 2–3000 m, while the species at lower altitudes grows considerably larger. I have seen specimens measuring 30 cm in length (lamina only) and 20 cm in breadth and having the pinnæ incised two-thirds of the way down. Such large forms are exactly intermediate between the typical form and the var. *procurrens*. — *D. pilosa* seems to be confined to Southern Mexico; I have seen the following specimens:

Chinantla, Depart. Puebla, LIEBMANN (H) — Puebla, ARSÈNE (RB, CC) — Jalisco, near Guadalajara, 5000', PRINGLE nr. 2589 (B, S, W), 9346 (W) — Morelos. Sierra de Tepoxtlán 7500', PRINGLE nr. 11265 (B, H, W) — Jalisco, at Rio Blanco, ROSE and PAINTER nr. 7502 (W) — Mexico, SCHMITZ nr. 367 (B).

var. *procurrens* (Fée) Bak. Syn. Fil.

515. 1874. — Fig. 27.

Syn. *Gymnogramme procurrens* Fée, 8. mém. 78. 1857.

Gymnogramme pilosa var. *major* Fourn. Mex. pl. 1: 73. 1872.

Dryopteris pseudo-lotta Christ, Bull. l'herb. Boiss. II. 7: 415. 1907.

Differs from the type by its broad ($2-2\frac{1}{2}$ cm) pinnæ, which are deeply cut, most of them free at base, the lower ones not reduced, often shortly petiolate. Veins 6–10 to a side, often furcate, somewhat prominent beneath.

— Forêt de la deserta Vieja, BOURGEAU nr. 921 (H, S) — Ixtaccihuatl, PURPUS nr. 1603 (W) — Oaxaca, Cerro San Felipe, GONZÁLEZ et CONZATTI nr. 711 (W) — Chiapas, S. Cristobal, G. MUNCH nr. 5 and 23 (C).

204. *Dryopteris dasyphylla* C. Chr. Ind. 260. 1905.

Syn. *Gymnogramma villosa* Link, Hort. Berol. 2: 51. 1833; Fil. sp. 137.

Based on plants cultivated in Hort. Berol. I have seen LINK's type specimens (B) and several others, but all from cultivated plants. In handbooks of ferns it is said to be a Brazilian species, but I have never seen a specimen from Brazil. LINK, however, says (in Fil. sp. 137): "Hab. in provincia Caracas, Brasilia?" It has, I think, never been stated, where the species grows wild.



Fig. 27. *D. pilosa* (M. et. G.) C. Chr. var. *procurrens* (Fée) Bak. Pinna $\times \frac{1}{3}$; segments $\times 1\frac{1}{2}$.

D. dasyphylla is closely related to *D. pilosa* var. *pubescens* and I have been inclined to consider it the same plant; still it is much more hairy, the lamina upwards suddenly narrowed into a pinnatifid apex and with 6—8 pairs of opposite or subopposite, sessile or very short-stalked pinnæ, about 8 cm long by $2\frac{1}{4}$ — $2\frac{1}{2}$ cm wide, the lower ones not reduced but narrowed towards their base, all cut about halfway down into close, broad subacute segments. Veins 6—7 to a side, simple. Sori short, often nearly round, the sporangia setose. Rachis, costæ and veins of both sides with many long, soft, pluricellular hairs.

205. ***Dryopteris polypodioides*** (Raddi) C. Chr. Ind. 285. 1905.

Syn. *Ceterach polypodioides* Raddi, Opusc. Sci. Bologn. 3: 284. 1819; Pl. Bras. 1: 10 tab. 22.

Gymnogramme polypodioides Spr.; Bak. Syn. Fil. 377.

Type from Brazil, Rio, RADDI (not seen). The following specimens seen are all from the mountains of Rio and are very uniform. MOSÉN nr. 2656 (H, Rg, S); GLAZIOU nr. 7252 (B, H); RATHBUN (W); U. S. Explor. Expedition 1838—42 (W); SELLOW (B).

In colour, cutting and shape of pinnæ very similar to *D. monosora* of the subgenus *Goniopteris* and also resembling *D. alsophilacea* of the subgenus *Ctenitis*, still very different by the oblong to linear sori and the setose sporangia. The creeping rhizome is naked and the whole leaf practically glabrous, only the costæ and costulæ are finely downy beneath. Lower pinnæ with a short, cuneate, entire base, acuminate, at both sides equally incised about $\frac{2}{3}$ of the way down to the midrib into broad, obtuse, subfalcate segments; veins about 10 to a side, the lower two running to the sinus.

Subgenus 9. ***Goniopteris*** (Presl) C. Chr.

Biolog. Arbejder tilegnede Eug. Warming pag. 83. 1911.

A large subgenus including about sixty good species and not closely allied to other *Dryopterides*. Its best and most constant character is the presence of unicellular, forked or stellate hairs. Such hairs are to be found always on the scales of the rhizome and lower part of the stipe and in most species they also cover the rachis, especially upwards and above, and the costæ beneath. In some species (*D. nephrodioides* and its relatives) the rachis is very densely but very shortly pulverulent by stellate hairs, in others (*D. sclerophylla*, *D. asterothrix*, *D. reptans* and others) the surfaces (veins and leaf-tissue) bear minute hairs with horizontal branches. The stellate hairs are, as far as I have found, never sessile but consist of a short stalk, which bears at the point 2—6 branches. The shape of the branches

seem to be rather confined within the species and it is to a certain degree of value as specific character. The branches can be short or long, simple or again forked (*D. Eggersii*), erect (*D. asterothrix*), horizontal or recurved (*D. glochidiata*). Besides stellate or forked hairs simple, unicellular, longer hairs occur in most species; some few species are practically glabrous (*D. vivipara*, *D. paucipinnata*), others densely pubescent throughout (*D. curta*, *D. Ghiesbreghtii*). The longer hairs are in most species confined to rachis, costæ and margins; generally the rachis bears as well stellate as simple hairs. Scales are, as a rule, few and mostly confined to rhizome and lower part of the stipe; rarely small, stellate-pubescent scales are also found on rachis and costæ beneath (*D. monosora*, *D. lugubris*). Glands as well as aërophores are always absent.

The pubescence is the main character of the subgenus, but the species show besides other characteristic common features, which can not be described so clearly as to be understood easily by others. Most species are dark-green or greyish-green, membranous or chartaceous, rarely thinly herbaceous or rigidly coriaceous, not much divided, the lamina entire, pinnatifid, pinnate or bipinnatifid. Bipinnate or decompound species I have not seen. Several species, perhaps the majority, are proliferous by buds on the rachis or the rachis is prolonged and rooting at the apex. The species of the section *Eugoniopteris* have impari-pinnate lamina. These two characters, proliferous leaves and impari-pinnate lamina, so common within *Goniopteris*, are unknown or, at best, very rare in all other subgenera.

With regard to venation the species vary not a little, still a certain uniformity can be pointed out. The venation is in correlation to the degree of cutting. In deeply cut pinnæ the veins, which nearly always are simple, are all free with those of the basal pair running to the sinus (f. inst. *D. scabra*), but more often they are connivent to sinus i. e. the lower 1—4 pairs of veins are upcurved and run side by side to the sinus, below which they are very often separated by a cartilaginous membrane. It is often difficult to state whether the veins are connivent or truly anastomosing, i. e. two veins being united into a single ex-current branch (*nervatio Goniopteridis*). In several species both kinds of venation can be found in the same leaf. Seen from the underside the veins often appear to be united, while they, seen from above, are found to be connivent but running very closely side by side. In several species the lower veins (1—3 pairs) are constantly united, while other species (f. inst. *D. nephrodioides*) are very variable in venation, some forms having free, others of the same species anastomosing veins. Within the second action, *Eugoniopteris*, we find an unbroken row from free-veined forms to *D. Ghiesbreghtii* and *D. meniscioides*, the venation of which is perfectly meniscioid.

The species are partly indusiate, partly exindusiate. Large indusia are rare (*D. paucipinnata*, *D. venusta*), in most species the indusia are small and more or less setose by simple or forked hairs, often very small and only seen in the young sori. In some species the receptacle bears long hairs between the sporangia; these

hairs are sometimes mistaken for ciliate indusia. The sporangia of several species are setose by simple or forked hairs. In *D. asplenoides* and others the head of the sporangium is glabrous while its pedicel is furnished with a single, stiff hair.

Goniopteris is a very distinct subgenus, or, I firmly believe, a very natural genus, not nearly related to the other subgenera, *Meniscium* excepted. The relation between *Goniopteris* and *Meniscium* will be explained under the latter. Between the sixty species dealt with below there are, however, two, *D. macrotis* and *D. semihastata*, the position of which as members of *Goniopteris* is rather doubtful. It is possible that these two species belong to *Cyclosorus*. I have not been able to find in the specimens seen stellate hairs, still they agree in colour and texture with *Goniopteris* rather than with *Cyclosorus*. Two other species, *D. glandulosa* and *D. Fendleri*, which resemble very much species as *D. nicaraguensis* and *D. megalodus*, I have referred to *Steiropteris*; they differ from *Goniopteris* in several minute characters.

All the species enumerated below are exclusively American. The subgenus is represented in the Old World by two species only, as far as I know, *D. silvatica* (Pappe et Raws.) C. Chr. from South Africa and *D. prolifera* (Retz.) C. Chr. from Africa, Asia and Australia. The American species can be divided into two rather natural sections:

1. **Asterochlaena.** Lamina upwards gradually narrowed into a pinnatifid apex.
2. **Eugoniopteris.** Lamina terminating in a terminal pinna resembling the lateral ones.

Under the latter group I unite into a small group, *Microdictyon* Fée, those species having meniscioid venation and connecting *Goniopteris* with *Meniscium*. Between the two sections no absolute limit is to be found.

Key.

1. **Asterochlaena.** Lamina entire bipinnatifid, upwards gradually narrowed into a pinnatifid apex.
2. Lamina entire, pinnatifid or with some few free pinnæ below.
3. Lamina without free pinnæ below or casually with a single pair of small free auricles.
4. Lamina lanceolate, crenate or broadly serrulate, scarcely 2 cm broad, shortly cuneate at base; midrib, veins beneath and indusium setose by simple hairs.
5. Lamina entire or crenate. 206. *D. Cumingiana* (Kze.) O. Ktze.
5. Lamina regularly and broadly serrulate or lobed
207. *D. Francoana* (Fourn.) C. Chr.
4. Lamina broadest above the middle, often irregularly pinnatifid, long and gradually narrowed downwards. Costa, veins beneath and indusium stellato-puberulous
210. *D. scolopendrioides* (L.) O. Ktze.

3. Lamina pinnate below but rarely to the middle.
4. Midrib, veins beneath and indusium setose by long, simple hairs. Stellate hairs few.
5. Free pinnæ entire or subentire, not auricled at the upper base.
6. Free pinnæ in 1—2 pairs, auriculiform, $\frac{1}{2}$ —1 cm long
208. *D. Skinneri* (Hk.) O. Ktze.
6. Free pinnæ several, 2—4 cm long.
7. Upper surface (costæ excepted) glabrous. Free pinnæ broadly adnate to rachis. Veins rarely truly united.
209. *D. Levyi* (Fourn.) O. Ktze.
7. Upper surface finely pubescent. Lower free pinnæ sessile with free base. Lower pair of veins often united..... 212. *D. Peripae* (Sod.) C. Chr.
5. Pinnæ lobed, auricled at the upper base.
222. *D. Jamesoni* (Hk.) C. Chr.
4. Midrib and veins beneath stellato-puberulous; indusium small, stellato-pubescent, or none.
5. Free pinnæ small, entire, the lower gradually reduced downwards.
6. Free pinnæ close, numerous, gradually reduced to mere auricles. Leaf coriaceous. Most veins free.
213. *D. dissimulans* Maxon.
6. Free pinnæ few (1—4 pairs) and distant. Leaf membranous or chartaceous. Veins normally anastomosing and forming low meshes along the midrib of the lamina.
211. *D. guadalupensis* (Wikstr.) C. Chr.
5. Free pinnæ large, serrulate, lower ones not reduced.
227. *D. hastata* (Fée) Urb.
2. Lamina pinnate to short of the apex.
3. Pinnæ entire or serrulate or shallowly lobed, lobes as a rule broader than long.
4. Smaller species. Pinnæ seldom over 4 cm long, 1 cm broad.
5. Rachis and veins more or less stellato-puberulous, often also hairy by longer, simple hairs.
6. Pinnæ entire or faintly crenate, chartaceous. Veins normally free. Leaves not terminating in a long, rooting apex. Leaf-tissue nearly glabrous.
7. Pinnæ with a rounded, cordate base, oblong 1—1 $\frac{1}{2}$ cm long, $\frac{1}{2}$ cm broad..... 214. *D. cordata* (Fée) Urb.
7. Pinnæ with a pair of acute, spreading auricles at base, the lower ones gradually reduced.
215. *D. sagittata* (Sw.) C. Chr.
6. Pinnæ serrulate or lobed. Lower veins anastomosing.
7. Surfaces, especially the veins, more or less greyish-pubescent by stellate hairs.
8. Lamina herbaceous or membranous. Pinnæ short-stalked.
9. Sporangia glabrous. Leaves often prolonged into a long rooting tail. 216. *D. reptans* (Gmel.) C. Chr.
9. Sporangia setose by forked hairs. Leaf not proliferous. Surfaces densely pubescent by stellate and simple hairs. 217. *D. asterothrix* (Fée) C. Chr.

8. Lamina coriaceous. Pinnæ sessile.
 218. *D. sclerophylla* (Kze.) C. Chr.
7. Surfaces hairy on the veins only, leaf-tissue glabrous, at least not pubescent by minute, stellate hairs.
8. Costæ beneath stellato-puberulous.
 215. *D. sagittata* var. *tenebrica* (Jenm.).
8. Costæ beneath setose by simple hairs or glabrous.
 9. Pinnæ short-stalked, rather deeply lobed; costæ beneath setose. West-Indian species.
 10. Veins simple, often prominent beneath. Leaf-tissue glabrous; indusium setose.
 219. *D. asplenioides* (Sw.) O. Ktze.
 10. Veins often forked. Leaf-tissue pubescent.
 220. *D. bermudiana* (Bak.) Gilb.
9. Pinnæ sessile. South-American species.
 10. Lamina terminating in a long pinnatifid apex. Rachis and costæ beneath rather setulose; upper surface pubescent.
 11. Pinnæ not auricled at base.
 212. *D. Peripae* (Sod.) C. Chr.
 11. Pinnæ auricled at base, the basal ones reflexed.
 12. Andes. Veins 2—3-jugate. Basal pinnæ reduced. 222. *D. Jamesoni* (Hk.) C. Chr.
 12. Brazil. Veins 5—6-jugate. Basal pinnæ scarcely reduced.
 223. *D. Warmingii* n. sp.
 10. Lamina pinnate to short of the apex, practically glabrous throughout.
5. Stellate hairs none, but rachis, ribs and indusium setose by simple hairs. Pinnæ auricled at the upper base.
 221. *D. semihastata* (Kze.) O. Ktze.
4. Larger species: pinnæ 8—15 cm long, 1½—2½ cm broad, broadly serrate. Rhizome not creeping.
5. Most pinnæ auricled at their upper base, all sessile, the lower ones reflexed. Stellate hairs few or none; upper surface strigose by adpressed hairs.
6. Under surface pubescent by short, non-adpressed hairs. Only the basal pair of pinnæ reflexed. Rachis with few stellate hairs. Pinnæ scarcely more than 5 cm long 223. *D. Warmingii* n. sp.
6. Under surface strigose by adpressed hairs. 2—3 pairs of pinnæ reflexed. Stellate hairs none. Pinnæ 8—15 cm long 224. *D. macrotis* (Hk.) O. Ktze.
5. Pinnæ not auricled. Stellate hairs on rachis and often on costæ beneath. Leaf-tissue generally glabrous.
6. Lower veins as a rule connivent to sinus, rarely united.
7. Most pinnæ sessile, lower ones much reflexed.
 230. *D. paucijuga* (Kl.) C. Chr.
7. Most pinnæ stalked, lower ones scarcely reflexed.
 228. *D. pyramidata* (Fée) Maxon.
6. Basal pair of veins normally united, or at least running side by side to the sinus.

7. Lamina terminating in a long, broad pinnatifid apex equal in length to the lower pinnate portion.
8. Upper surface minutely pubescent. Veins about 3-jugate, those of basal pair the anastomosing.
212. *D. Peripae* (Sod.) C. Chr.
8. Upper surface glabrous. Veins about 6-jugate, the lower 2—3 pairs alternately united.
227. *D. hastata* (Fée) Urb.
7. Lamina pinnate to short of the apex.
8. 2—3 pairs of veins alternately united. Pinnæ about 4 cm broad . . . 264. *D. leucophlebia* (Christ) C. Chr.
8. Only the two basal veins united, the following 2—4 connivent to sinus. Pinnæ rarely 2 cm broad.
9. Lower pinnæ not or slightly reflexed, more or less shortened.
10. Sporangia glabrous. No scales on costæ beneath. Veins 4—5-jugate. Jamaica.
225. *D. serrulata* (Sw.) C. Chr.
10. Sporangia setose by furcate hairs. Small scales on the costæ beneath. Veins 7—8-jugate. Brazil . . . 226. *D. anoptera* (Kze.) C. Chr.
9. Lower 2—3 pairs of pinnæ much reflexed, scarcely shortened . . . 231. *D. refracta* (A. Br.) O. Ktze.
3. Pinnæ incised $\frac{1}{3}$ of the way to the midrib (lobes longer than broad) or to a narrow wing to the costa.
4. Rachis and costæ beneath subglabrous or slightly pubescent by short, stellate hairs and a few longer, simple hairs, not densely pulverulent by stellate hairs or tomentose by simple hairs.
5. Branched hairs with erect or horizontal branches.
6. Rhizome erect or oblique. Pinnæ rarely incised to the middle.
7. Most pinnæ sessile.
8. Lamina terminating in a long, pinnatifid apex. Sporangia setose by branched hairs.
227. *D. hastata* var. *leptocladia* (Fée).
8. Lamina pinnate to short of the apex. Sporangia glabrous.
9. Lower pinnæ not reduced. Veins connivent, not united. Lobes oblique or falcate, close. Stipe in length equal to lamina.
10. Lamina not gradually tapering from base to apex. Pinnæ few (8—10-jugate).
11. Lower pinnæ with a short, cuneate, entire base, not much reflexed.
229. *D. magdalenica* Hieron.
11. Lower pinnæ lobed to the base, much reflexed 230. *D. paucijuga* (Kl.) C. Chr.
10. Lamina gradually tapering from base to apex. Pinnæ numerous (15—25 to a side), incised to or above the middle.
232. *D. gemmulifera* Hieron.

9. Lower 1—3 pairs of pinnæ more or less reduced. Basal veins often united. Lobes patent with open sinuses between. Pinnæ numerous. Jamaica.
10. Indusium small. Sori medial or infra-medial. Veins 6—8-jugate. Pinnæ rarely incised to the middle. 233. *D. usitata* (Jenm.) C. Chr.
10. Indusium large, glabrous. Sori supra-medial. Veins 10—12-jugate. Pinnæ incised above the middle.
 234. *D. venusta* (Hew.) O. Ktze.
7. Most pinnæ short-stalked.
 8. Veins all simple, the lower ones connivent to sinus, rarely united. Leaf-tissue glabrous.
 228. *D. pyramidata* (Fée) Maxon.
 8. Basal veins united and the upper ones often furcate. Both surfaces pubescent.
 220. *D. bermudiana* (Bak.) Gilb.
6. Rhizome creeping. Brazilian species.
 7. Pinnæ scarcely incised to the middle, their base truncate, hairy beneath; veins 6—7-jugate.
 244. *D. Schwackeana* Christ n. sp.
 7. Pinnæ incised above the middle, lower ones narrowed towards the base.
 8. Rachis and costæ beneath without scales. Lower pinnæ sessile, lobed to the base..
 236. *D. scabra* (Pr.) C. Chr.
 8. Rachis and costæ beneath with small scales. Lower pinnæ short-stalked with a short, entire, cuneate base 237. *D. monosora* (Pr.) C. Chr.
 5. Rachis and underside with erect, anchor-shaped hairs. Basal veins united 247. *D. ancyrlothrix* Ros.
4. Rachis and costæ beneath densely pulverulent by short, stellate hairs, which are often intermixed with longer, simple hairs, or, the latter predominating, densely tomentose.
5. Indusium hairy by simple hairs or glabrous. Leaf-tissue of both sides glabrous or hairy by simple hairs.
6. Indusium setose.
 7. Andine species. Rhizome erect (always?). Rachis and costæ beneath without scales.
 8. Rachis (and costæ beneath) without thick, long hairs, but densely stellato-pulverulent.
 9. Veins 10—16-jugate, the basal ones connivent to sinus. Upperside (costæ excepted) glabrous.
 10. Fertile pinnæ and segments not contracted; rachis not gemmiferous. Most pinnæ sessile 238. *D. Eggersii* (Hieron.) C. Chr.
 10. Fertile pinnæ and segments contracted, remote. Rachis gemmiferous. Most pinnæ stalked 239. *D. biformata* Ros.
 9. Veins 6—8-jugate, the basal ones truly anastomosing. Upperside shortly pubescent.
 240. *D. equitans* (Christ) C. Chr.

8. Rachis and costæ beneath besides the mostly once branched stellate hairs clothed with thick hairs, which are often cleft at the point. Most or all pinnæ sessile.
9. Pinnæ 6—7 cm long by $1\frac{1}{2}$ cm broad, sessile, the lower ones reflexed and scarcely narrowed towards their base 241. *D. curta* Christ.
9. Pinnæ 10—14 cm long by $2\frac{1}{2}$ —3 cm broad, the lower ones not reflexed, narrowed towards their base.
10. Grass-green, thin leaved; only the basal pair of veins run to sinus; long hairs of rachis and costæ fewer, reddish.
242. *D. heterotricha* C. Chr. n. sp.
10. Grey-green, firm; 3—4 lower pairs of veins connivent to sinus; long hairs very numerous, thin, rarely reddish; rachis upwards very tomentose. 243. *D. lugubrifformis* Ros.
7. South-Brazilian species. Rhizome creeping.
8. Pinnæ 4—6 cm long by $1\frac{1}{2}$ cm broad, scarcely incised to the middle. Rachis and costæ beneath without scales 244. *D. Schwackeana* Christ n. sp.
8. Pinnæ 15—18 cm long by 2—3 cm broad, incised above the middle. Rachis and costæ beneath with small scales 245. *D. lugubris* (Kze.) C. Chr.
6. Indusium glabrous 234. *D. venusta* (Kze.) C. Chr.
5. Indusium with stellate hairs, rarely absent. Underside throughout with scattered, minute, branched hairs.
6. Lamina rigidly coriaceous, narrowed downwards; pinnæ 4—8 cm long. Basal veins united. 218. *D. sclerophylla* (Kze.) C. Chr.
6. Lamina herbaceous or membranous; lower pinnæ not shortened, 10—25 cm long.
7. Branched hairs with 3—5 cm long, erect or horizontal branches. Sori medial, sporangia glabrous.
8. Veins 10—14-jugate, lower ones often united. No scales on rachis and costæ beneath.
248. *D. nephrodioides* (Kl.) Hieron.
8. Veins 15—16-jugate; scales on rachis and costæ beneath 245. *D. lugubris* var. *quadrangularis* (Fée).
7. Branched hairs of the underside erect, anchor-shaped (i. e. with 2—3 short recurved branches at the top).
8. Sporangia with similar hairs. Veins free, sori inframedial. Brazil 246. *D. glochidiata* (Mett.) n. sp.
8. Sporangia glabrous. Basal pair of veins united. Andes 247. *D. ancyriothrix* Ros.
1. **Eugoniopteris.** Lamina pinnate-bipinnatifid with a terminal pinna, which in most species resembles the lateral ones, in others it is larger and often hastate, i. e. with one or two larger lobes below.
2. Pinnæ without aërophore beneath. Stellate hairs occur always on the scales of the rhizome and in most species also on rachis. Veins free or anastomosing.

3. Veins all free or the lower 1—3 pairs united and sending an excurrent vein towards the sinus; in some species the lowermost pair (rarely the two lower pairs) is meniscioid, i. e. the excurrent branch being interrupted before reaching the next pair of anastomosing veins.
4. Indusium persistent. Lower veins connivent to sinus, rarely truly united. Lower pinnæ mostly with a cuneate base (*D. biformata* excepted).
5. Sori near the margin.
 6. Lamina coriaceous, glabrous; pinnæ scarcely incised $\frac{1}{3}$. Indusium reniform, whitish, glabrous.
 251. *D. paucipinnata* (Donn. Smith) Maxon.
 6. Lamina thinly membranous, hairy on the ribs; pinnæ incised about $\frac{1}{2}$. Indusia reniform, athyroid or asplenoid in the same pinna or even segment.
 254. *D. Schaffneri* (Fée) C. Chr.
 5. Sori medial.
 6. Basal pinnæ not cuneate at base; fertile pinnæ and segments contracted. 239. *D. biformata* Ros.
 6. Basal pinnæ with a long cuneate, entire base; fertile pinnæ and segments not contracted.
 7. Indusium small, setose. Veins 12—14. Andes.
 249. *D. tristis* (Kze.) O. Ktze.
 7. Indusium large, nearly glabrous. Veins 9—11. South Brazil. 253. *D. cuneata* n. sp.
 4. Indusium none or minute, rarely found.
 5. Under surface without minute stellate hairs on costulæ and leaf-tissue.
 6. Lower 2—4 pairs of veins connivent to sinus, only occasionally the lowermost pair is united.
 7. Smaller plant; pinnæ 2—4 to each side, serrulate, densely warted on both surfaces. 258. *D. juruensis* n. sp.
 7. Larger with about 10 pairs of pinnæ, not or inconspicuously warted, more deeply cut.
 8. Lower pinnæ with a long cuneate, entire base, incised about halfway down; 2 pairs of veins connivent to sinus. Sori medial. 249. *D. tristis* (Kze.) O. Ktze.
 8. Lower pinnæ lobed to the very base, scarcely incised to the middle; 3—4 pairs of veins connivent to sinus. Sori inframedial or about medial.
 9. Rachis upwards and costæ beneath densely tomentose; upperside strigose. 243. *D. lugubriiformis* Ros.
 9. Rachis and costæ beneath glabrous or nearly so.
 10. Leaf papyraceous, light-green; costæ and costules prominent, stramineous. Lower pinnæ not reflexed.
 11. Pinnæ 2 cm broad, crenate or slightly lobed; veins 3—5 jugate. Rachis gemmiferous, glabrous. 256. *D. straminea* (Bak.) C. Chr.
 11. Pinnæ 3 cm broad, lobed halfway down, veins 12—18 to a side. Rachis minutely pubescent, not gemmiferous.
 250. *D. nicaraquensis* (Fourn.) C. Chr.

10. Leaf thinner, dark-green; costæ and costules not stramineous. Pinnæ in very distant, opposite pairs, the basal ones reflexed.
252. *D. Fraseri* (Mett.) O. Ktze.
6. Basal pair of veins normally united; in most species the next 2—3 pairs of veins are connivent to sinus and often alternately anastomosing into a flexuose common-branch, which runs to sinus.
7. Pinnæ entire, serrulate or lobed but never cut $\frac{1}{3}$ to the costa. Basal veins upcurved and anastomosing under acute angles, the following pair as a rule also anastomosing.
8. Leaf practically glabrous; in some species the rachis and costæ beneath are minutely puberulous. Sporangia glabrous, or (in *D. obliterata*) deciduously setose.
9. Both surfaces densely warted. Pinnæ 2—4 to a side 258. *D. juruensis* n. sp.
9. Surfaces not warted. Several pairs of pinnæ.
10. Pinnæ entire or very shallowly serrulate. Basal veins spring out from the costule 1 mm above the costa.
255. *D. vivipara* (Raddi) C. Chr.
10. Pinnæ broadly serrate or lobed $\frac{1}{4}$ or a little more. Basal veins spring out from the base of the costule or the posterior one from the costa.
11. Only the lowermost pair of veins anastomosing, or all free. Rachis gemmiferous.
12. Veins 4—6 jugate, the two basal one as a rule not united and not reaching the sinus.
256. *D. straminea* (Bak.) C. Chr.
12. Veins 8—9 jugate, those of the basal pair united 257. *D. Göldii* n. sp.
11. At least 2 pairs of anastomosing veins. Rachis rarely gemmiferous.
12. Terminal pinna small, often abortive. Pinnæ few, wrinkled, 5—8 cm long. Veins 4—5-jugate.
260. *D. nigrescentia* (Jenm.) C. Chr.
13. Terminal pinna confluent with the upper lateral ones.
227. *D. hastata* (Fée) Urb.
13. Terminal pinna distinct.
259. *D. obliterata* (Sw.) C. Chr.
8. Lamina rather densely pubescent throughout beneath. Sporangia densely setose. 261. *D. Rolandii* n. sp.
7. Pinnæ incised to the middle or deeper. Only the basal pair of veins anastomosing under broad angles.
8. Costæ beneath puberulous by stellate hairs.
240. *D. equitans* (Christ) C. Chr.

8. Costæ beneath setose by simple hairs.
262. *D. tetragona* (Sw.) Urban.
5. Under surface with minute stellate hairs throughout, especially on costæ and costules, but also on the leaf-tissue.
6. Terminal pinna stalked, distinct. Pinnæ incised $\frac{1}{3}$ or deeper; as a rule the lowermost pair of veins only are anastomosing under a broad angle.
263. *D. megalodus* (Schkuhr) Urb.
6. Terminal pinnæ confluent with the upper lateral ones, which are adnate to rachis. Pinnæ broadly serrulate (lobes scarcely longer than broad). 2—3 pairs of alternately united veins. 264. *D. leucophlebia* (Christ) C. Chr.
3. *Microdictyon*. 3—10 pairs of meniscioid veins. Pinnæ entire or crenate, seldom shallowly lobed, mostly more than 4 cm broad.
4. Lamina beneath more or less soft-hairy.
5. Veins 6—8-jugate, the 3—4 lower pairs meniscioid. Sporangia when young setose. 265. *D. Poiteana* (Bory) Urban.
5. Veins 10—12-jugate, 8—10 pairs meniscioid. Sporangia glabrous. Lamina densely soft-hairy.
267. *D. Ghiesbreghtii* (Lind.) C. Chr.
4. Lamina glabrous. Meniscioid veins several.
5. Pinnæ narrowed from the middle to base, crenate or serrulate. 266. *D. meniscioides* (Liebm.) C. Chr.
5. Base of pinnæ subcordate. Pinnæ entire. 268. *D. ensiformis* n. sp.
2. Pinnæ with an acute or obtuse aërophore at the base beneath. Stellate hairs none. Several pairs of veins connivent to a cartilagineous membrane below the sinus, the opposite pairs not anastomosing, but two subsequent veins of the same side of the costule generally united near the membrane; see *D. glandulosa* and *D. Fendleri* under § *Steiropteris*.

1. *Asterochlaena* C. Chr.

Biolog. Arbejder tilegnede Eug. Warming 84. 1911.

206. *Dryopteris Cumingiana* (Kze.) O. Ktze. Rev. 2: 812, 1891 C. Chr. Ind. 260.

Syn. *Aspidium Cumingianum* Kze. Farrnkr. 1: 17 tab. 9 fig. 2. 1840.

Nephrodium Cumingianum J. Sm. Bot. Voy. Herl. 237 tab. 50. 1854.

Type from Panama, leg. CUMING nr. 1123 (Kew! a small leaf only).

Apparently a very distinct species, well figured on the plates quoted, still it is possible that it and the following species are forms of one species. *D. Cumingiana* is less hairy, the lamina faintly crenate and 1—2 veins generally joint alternately the excurrent branch from the united basal veins. It appears to be a rare species, which is not found in the large recent collections from Panama.

207. **Dryopteris Francoana** (Fourn.) C. Chr. Biolog. Arb. tilegn. Eug. Warming 84. 1911. — Fig. 28a.

Syn. *Aspidium Francoanum* Fourn. Bull. Soc. Fr. 19: 255. 1872.

Nephrodium stenophyllum Bak. Journ. Bot. 1884: 363.

Nephrodium Harrisoni Bak. Ann. of Bot. 5: 326. 1891.

Dryopteris Harrisoni C. Chr. Ind. 269. 1905.

Polypodium subintegrum Bak. Journ. Bot. 1877: 164.

Nephrodium subintegrum Sodiro, Rec. 54. 1883; Cr. vasc. quit. 263. 1893.

Dryopteris subintegra C. Chr. Ind. 296. 1911.

Type from Nicaragua: Chontales, leg. P. LÉVY nr. 506 (Herb. Cosson, Mus. Paris!).

Differs from the preceding species, its only near relative, by its regularly lobed lamina and by the basal veins being much ascending and not always truly united; also it grows much larger; I have seen leaves measuring 40 cm in length including the stipe, that is equal to lamina in length but generally the leaves are much smaller.

The erect or ascendent rhizome, which is nearly destitute of scales bears several (10—20) fasciculated leaves; young stipes furnished at base with a few small scales with some bi-trifurcate hairs at the margins, glabrous or very sparsely and minutely pubescent by stellate hairs, sulcate above. Lamina lanceolate, 1–1½ cm broad below the middle, tapering gradually to the acuminate point, at base short-cuneate, dark-green, papyraceous or subcoriaceous, ciliate, midrib above strigose by antrorse hairs, midrib and principal veins beneath setose by simple, rigid, patent hairs, otherwise glabrous, the margins incised regularly about 1:3 to the midrib into oblique, broad, obtuse or subacute lobes. Veins about 7-jugate, simple (or rarely furcate), the lower two pairs connivent to sinus, or, the two basal ones truly united and sending a branch to sinus. Sori inframedial, furnished with a sub-persistent, setose indusium. Sporangia glabrous.

D. Francoana is a very constant species. The numerous Central-American specimens seen are all alike and the Ecuadorian one (*P. subintegrum* Bak.) differs only by its somewhat thinner texture and smaller, deciduous indusia. BAKER referred *A. Francoanum* to *N. Skinneri* Hk. (Syn. Fil. 288), but later he described the present species as new. — Specimens seen:

Nicaragua: Chontales, LÉVY nr. 506 (Mus. Paris, Kew).

Costa Rica: Jiménez, DONNELL SMITH nr. 5097 (B, C, W); ALFARO nr. 153 (W, 16521 (C) — Tuis près Turrialba, PITTIER nr. 11236 (C, W) — Vallée de Durui, Talamanca, PITTIER nr. 9406 (C, W) — Chilamate, PITTIER nr. 7501 (W) — Carrillo, PITTIER nr. 1176 (W) — Llanuras de Santa Clara, DONNELL SMITH nr. 6898 (B, W), 6899 (W) — Forêts de Tsâki, TONDUZ nr. 9463 (W) — WERCKLÉ (C).

Ecuador: near S. Miguel, Peripa River, SODIRO (C = *P. subintegrum* Bak.).

208. **Dryopteris Skinneri** (Hook.) O. Ktze. 2: 813. 1891: C. Chr. Ind. 293.

Syn. *Aspidium Skinneri* Hook. Ic. plant. tab. 924. 1854.

Nephrodium Skinneri Moore; Bak. Syn. 287 (excl. syn.).

Type from Guatemala, leg. SKINNER (Kew!). A nearly identical plant was collected in Ecuador, ad fluv. Bombonasa, by SPRUCE nr. 5293 (RB).

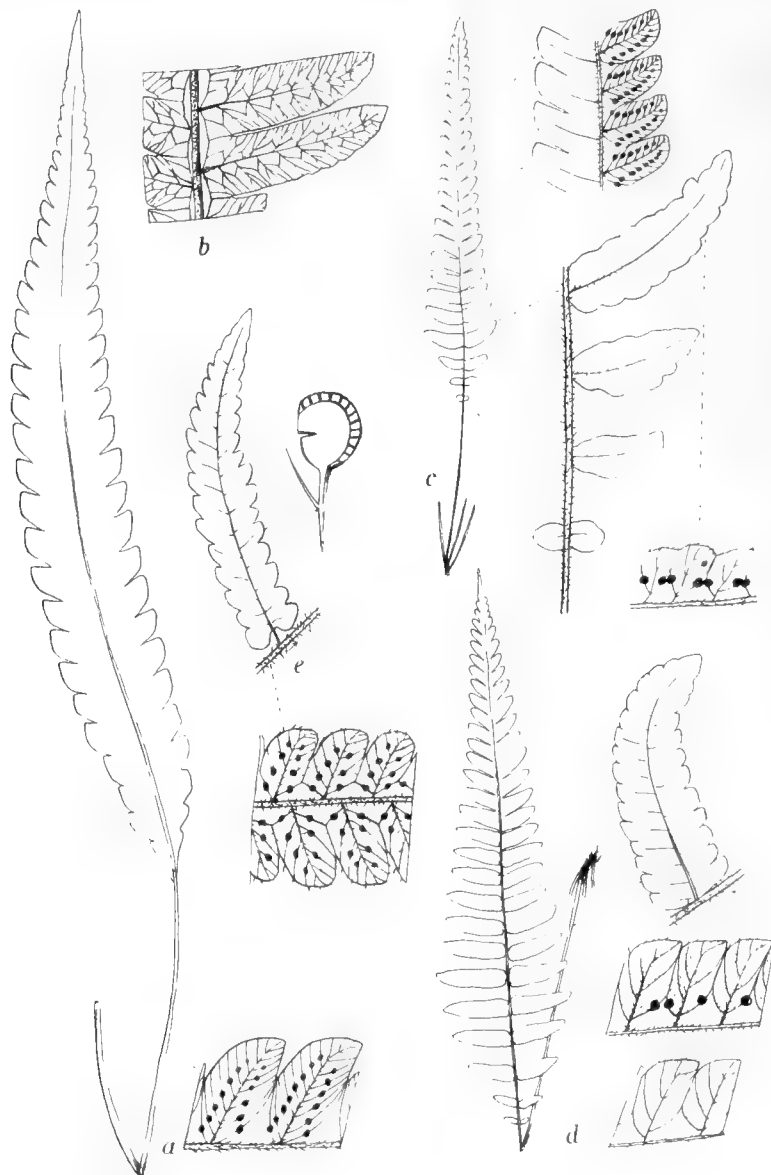


Fig. 28. a. *D. Francoana* (Fourn.) C. Chr. Entire leaf $\times \frac{1}{3}$ and fragment $\times 1\frac{1}{2}$ (orig.). — b. *D. guadalupensis* (Wikstr.) C. Chr.; fragment of a well-developed form, showing the venation, $\times \frac{4}{5}$. c. *D. Levyi* (Fourn.) O. Ktze. Habit of a leaf $\times \frac{1}{10}$ with parts of it, $\times \frac{4}{5}$, and a fragment of the largest pinna, $\times 1\frac{1}{2}$ (orig.). d. *D. Peripae* (Sod.) C. Chr. Habit of a leaf $\times \frac{1}{10}$, a pinna $\times \frac{4}{5}$ and fragments seen from both surfaces, $\times 1\frac{1}{2}$ (orig.). e. *D. asplenioides* (Sw.) O. Ktze. Pinna $\times \frac{1}{5}$, fragment $\times 1\frac{1}{2}$ and sporangium.

HOOKEER's figure of this species is excellent and it is sufficient here to refer to that figure. The species is closely related to *D. Francoana*, agreeing in size, colour, texture and pubescence, still the midrib beneath is more distinctly stellato-puberulous but also setose by patent, rigid, simple hairs; lamina pinnatifid about $\frac{2}{3}$ of the way to the midrib and below it bears about 2 pairs of free, small pinnæ.

209. *Dryopteris Levyi* (Fourn.) O. Ktze. Rev. 2: 813. 1891. C. Chr.

Ind. 275. — Fig. 28 c.

Syn. *Aspidium Levyi* Fourn. Bull. Soc. Fr. 19: 255. 1872.

Nephrodium Levyi Bak. Syn. 502. 1874.

Type from Nicaragua: Chontales, leg. P. LÉVY nr. 463 (Herb. Cosson, Mus. Paris!, also Kew!).

Certainly closely related to *D. Skinneri*, to which species I have referred it previously as a variety, while I now think it is a valid species intermediate between *D. Skinneri* and *D. guadalupensis*. From the latter it differs by venation, pubescence, indusium and by its lamina being broadest below the middle; it agrees with it in size and general habit. It resembles *D. Skinneri* in pubescence, and the upper pinnatifid portion of a leaf is not to distinguish from a leaf of *D. Skinneri*. Still it is a much larger species: stipe 15–20 cm long, lamina 20–30 cm long, 5–8 cm broad below the middle, the larger leaves pinnate in the lower half; basal pair of pinnæ reduced, $\frac{1}{2}$ cm long, the following 3–4 cm long, $\frac{3}{4}$ –1 cm broad, adnate to rachis, obtuse or acute at the point, entire or crenate or the larger ones pinnatifidly cut about $\frac{1}{3}$ to the costa. Veins simple or furcate, in the larger pinnæ about 4 to each side of the lobe, the basal ones connivent to sinus, more rarely truly united. Lamina papyraceous, light-green, somewhat warted beneath. Ribs pubescent by long, simple, and short stellate hairs intermixed. Indusium setose by simple hairs.

210. *Dryopteris scolopendrioides* (L.) O. Ktze. Rev. Gen. Pl. 2: 813. 1891.

Syn. *Polypodium scolopendrioides* L. sp. 2: 1085. 1753 (non ed. II. 1585. 1763).

Aspidium scolopendrioides Mett. Aspid. nr. 235. var. 1. *incisa*. 1858.

Polypodium incisum Sw. Prod. 131, 1788; Fl. Ind. occ. 1640 (vide C. Chr. Ark. för Bot. 9¹¹: 22 tab. 3 fig. 1. 1910).

Aspidium incisum Gris. Fl. br. W. Ind. 694. 1864.

Nephrodium incisum Bak. Syn. Fil. 288. 1867; Jenman, Bull. Dept. Jam. n. s. 3: 141. 1896.

Dryopteris incisa O. Ktze. Rev. 2: 813. 1891; C. Chr. Index 272. 1905.

Polypodium praelongum Poir. Enc. 5: 521. 1804 (t. sp. orig. in Herb. LAMARCK).

Aspidium stenopteris Kze. Farnkr. 2: 48 tab. 120. 1849.

Nephrodium stenopteris Eat. Amer. Journ. Sci. II. 27: 199. 1859; Hk. sp. fil. 4: 64. 1862.

Goniopteris strigosa Fée, 11 mém. 59 tab. 15 fig. 1. 1866.

Nephrodium strigosum Jenm. Bull. Dept. Jam. n. s. 3: 141. 1896.

Type from San Domingo, PLUMIER Fil. tab. 91.

To this species I refer all specimens without free pinnæ but with the lamina narrowed below gradually into an entire or faintly toothed wing, $1\frac{1}{2}$ cm broad. By this character it is always different from *D. guadalupensis* (Wikstr.) with which METTENIUS united it, and I have not found intermediate forms between the two species. In Ark. för Bot. 9¹¹ I have given my reasons for my considering *Pol. incisum* Sw. and *Aspidium stenopteris* identical with the true *D. scolopendrioides* (L.), which was based on PLUMIER tab. 91, but later by LINNAEUS confounded with my *D. guadalupensis* (Wikstr.). The confusion in the nomenclature of these two species was due to SWARTZ, who rightly distinguished the two species but unfortunately used the specific name *scolopendrioides* for the latter species and renamed the former *Pol. incisum*, and most authors have followed SWARTZ in his nomenclature. *D. scolopendrioides* varies mainly in size, especially in breadth, but it is otherwise a rather uniform species, which shows several good distinctive characters. The leaves are densely fasciculate on an erect or shortly oblique rhizome, sometimes 20–30 to a rhizome. The short stem (1–4 cm) is when young clothed with brown or blackish, stellato-pilose scales. Some of the leaves, which are normally sterile, are short and spreading, others up to 4 dcm long, erect and fertile and sometimes ending in a retuse viviparous apex with a rosette of small leaves. The lamina, which tapers gradually from the middle to both ends, is generally linear, ca. 3 cm broad at the middle, and varies from being deeply and broadly serrate to pinnatifid $\frac{1}{3}$ or $\frac{1}{2}$ of the way to the midrib with triangular, acute lobes, which are often very unequal in size, some of them being lengthened, 3–4 cm long. Texture more or less coriaceous, often very rigid. Stem, midrib and veins beneath generally densely stellato-pubescent by short multibranched hairs: upperside and leaf-tissue beneath as a rule glabrous, but the under-surface distinctly verrucose. Veins raised beneath, simple or rarely forked, the basal pair anastomosing. Sori supramedial in a single row — seldom in two rows — furnished with a small stellato-pilose indusium. Sporangia glabrous.

The common Cuban form (*A. stenopteris* Kze.) differs from the type by a longer stem and much lengthened middle segments with furcate veins, which in the largest forms are sometimes found anastomosing, and by its very long decurrent base of the lamina, but I can not consider that form different from the type even as a variety. *Pol. praelongum* Poir. is essentially the same, but its veins are more branched and the sori in the lengthened segments often biserial.

Specimens examined:

Haiti: Port au Prince. PICARDA nr. 385 (C) STENGEL (B).

San Domingo: BALDIS B. Petit Trou, Barabona, v. TIERCKHEIM m. 2843 B.

Jamaica: SWARTZ (S) *Pol. incisum* Sw.

Cuba: Monte Verde, WRIGHT (W. B.) — El Yunque Mt. near Baracoa, UNDERWOOD & EARLE nr. 1268 (W, C); POLLARD & PALMER nr. 163 (W) — Jaguey, EGGERS nr. 4887 (C, RB) Josephina, north of Jaguey, Yateras, 575 m., MAXON nr. 4111, 4140 (W) Monte Libano, 600 m., MAXON nr. 4258 El Guaina, Pinar del Rio, PALMER & RILEY nr. 393 (W, C, H).

211. *Dryopteris guadalupensis* (Wikstr.) C. Chr. Biolog. Arb. tilegn. Eug. Warming 84. 1911 (non O. Ktze.). — Fig. 28 b.

Syn. *Polypodium guadalupense* Wikstr. Vet. Akad. Handl. 1825: 435. 1826.

Polypodium scolopendrioides L. sp. ed. II. 1585. 1763 (non ed. I); Sw. Fl. Ind. occ. 1641 et auctt.

Aspidium scolopendrioides var. 2. *subpinnata* Mett. Aspid. nr. 235. 1858.

Nephrodium scolopendrioides Hk. sp. 4: 65. 1862; Hk. Bak. Syn. 288; Jenman, Bull. Dept. Jam. n. s. 3: 142. 1896.

Dryopteris scolopendrioides C. Chr. Index 291. 1905.

Polypodium domingense Spr. Syst. 4: 51. 1827 (t. spec. orig.).

Goniopteris affinis Fée, Gen. Fil. 250. 1850—52 (t. spec. auth.).

? *Goniopteris ferax* Fée, Gen. Fil. 250. 1850—52.

Aspidium asplenioides var. *portoricense* Kuhn & Christ, et var. *subpinnata* Krug, Engl. Jahrb. 24: 119. 1897.

Dryopteris asplenioides var. β *portoricensis* et γ *subpinnata* Urban. Symb. Antill. 4: 17, 18. 1903.

Type from Guadeloupe, leg. FORSSTRÖM (S!).

A variable species, especially in size and degree of cutting, but fairly constant in several characters, by which it can be distinguished from *D. scolopendrioides*. It resembles that species in its mode of growth, scales and stellate pubescence, but it differs by 1) thinner texture and generally not verrucose under-surface, 2) venation, 3) inframedial sori and 4) by the lamina being fully pinnate below with several pairs of free pinnae, which diminish gradually downwards. Real intermediates between the two species I have not found. The veins are, as a rule, forked or, in the larger form, pinnate in the lobes, always forming a row of very narrow areoles on both sides of the midrib of the lamina and generally also anastomosing in the segments with a single row of meshes along the main-veins or costa (fig. 28 b). The sori are always inframedial, in the smaller forms in a single row, in the larger ones bi- or pluri-serial, furnished with a small stellato-pilose indusium. According to JENMAN the barren fronds are viviparous near the apex. I have seen some fronds from Porto Rico, which are fertile and viviparous at the apex.

To this species I refer a long row of forms, which JENMAN referred to three varieties; these are however connected with so numerous intermediates, that I dare not consider them good varieties, although there is a great difference in habit between the small form figured by PLUKENET tab. 290 fig. 1 (*Pol. scolopendrioides* L. ed. II and Sw.) and the large plants described as *Goniopteris affinis* Fée, var. *portoricensis* Kuhn and var. *littorale* Jenm. In the former the leaves are numerous on very short stems, the fertile ones scarcely more than 20 cm long, 2—2½ cm broad,

at the middle incised a little more than halfway to the midrib with entire, obtuse lobes; the veins once forked and the sori uniserial. This is the common form in Jamaica. In San Domingo the form described as *Pol. domingense* Spr. is apparently very common. Its leaves are longer (30 cm or more) but scarcely broader than in the Jamaican form and on longer stems but otherwise not materially different. In Porto Rico and Guadeloupe the most common forms are the large var. *portoricensis* Kuhn (Porto Rico) and the true *P. guadalupense* Wikstr. (syn. *Goniopteris affinis* Fée) (Guadeloupe). The leaves of these forms are up to 50 cm long on long stems, often more than 10 cm broad at the middle, incised almost to the midrib in broadly linear segments, which are 4—5 cm long, 1 cm broad and often again deeply lobed. Veins pinnate in the lobes and sori in several rows or even in a distinct row on each side of the secondary veins. The middle segments are often considerably and unequally lengthened.

Specimens examined:

- Guadeloupe, FORSSTRÖM S.; L'HERMINIER nr. 128 (B, C = *G. affinis* Fée) — PÈRE DUSS nr. 4059 (CC, H, RB, W), 4389 (C).
 Martinique: LENORMAND (B).
 Porto Rico: SENTENIS nr. 877, 2450, 5452, 5664, 5820, 5840, 5841, 5949, 6108, 6234, 6247 (B and partly C, CC, S, W) — G. P. GOLL nr. 271, 870, 881, 948, 1015 (W) — A. A. HELLER nr. 6174 (W) — Mr. and Mrs. HELLER nr. 354 (W) — UNDERWOOD and GRIGGS nr. 52, 76, 883 (W).
 San Domingo: BERTERO (B = *Pol. domingense* Spr.), BALBIS (B) — WEINLAND nr. 44 (B) — PICARDA nr. 155, 209, 387, all from Haiti (B) — inter Batey et Jamao, EGGERS nr. 2605 (B) — in flumine Mameges, EGGERS nr. 2657 (B) — in monte Isabel de la torre, EGGERS nr. 2733 (B, RB) — La Cumbra, RAUNKJÆR nr. 102 (CC, H) — v. TUECKHEIM nr. 2517, 2644, 2845 (B).
 Jamaica: P. BROWNE (S, ex herb. Linn.) — SWARTZ (S) — Blue Hole, A. FREDHOLM nr. 3192 (W) — near Port Antonio, MAXON nr. 1985, UNDERWOOD nr. 2989, 1703 (W) — near Priestman's river, MAXON nr. 2495 (Rg, W) — Mt. Diabolo, 750 m., MAXON nr. 2259 (W) — WILSON nr. 43 (B) — J. DAY nr. 68 (B).
 Cuba: E. OTTO nr. 38 (B), LENORMAND (B), RUGEL (B) — San Antonio de los Baños, Prov. Habana, v. HERMANN nr. 3357 (W).

The plant described as *Goniopteris gracilis* Moore et Houlst. Gard. Chron. 1856: 301, fig., of which I have seen authentic specimens (B), can, I think, safely be regarded as a form of *D. guadalupensis*, while several other forms, cultivated under the name *G. gracilis*, are to refer to *D. reptans*. The true *G. gracilis* is a form with many free pinnæ and with a proliferous bud in the upper part of the midrib.

212. *Dryopteris Peripae* (Sod.) C. Chr. Index 284. 1905. — Fig. 28 d.

Syn. *Nephrodium Peripae* Sodiro, Rec. 52. 1883; Cr. vasc. quit. 265. 1893.

Type from Ecuador, secus flum. Peripa, leg. SODIRO (Kew!).

A distinct species, perhaps nearest related to *D. Levyi* and *D. guadalupensis*; it resembles also *D. hastata* in its long pinnatifid apex, which is not much shorter than the lower pinnate portion of the lamina, but otherwise it is very different. — The short creeping or decumbent rhizome bears some few stellato-pubescent scales.

Stipes fasciculated, up to 25 cm long, glabrous. Lamina papyraceous, narrow-lanceolate, 30–40 cm long, 4–10 cm broad, pinnate about to the middle, upwards gradually tapering into a long pinnatifid apex. Lowest pinnæ considerably shortened, larger ones sessile and the upper adnate to rachis, distant, opposite, 3–5 cm long, 1½ cm broad, obtuse, shallowly serrulate. Rachis, costæ and veins beneath rather densely setulose by long, rigid hairs and with fewer short, stellate hairs; leaf-tissue of upper-surface minutely pubescent by adpressed hairs of under-surface glabrous, not verrucose. Veins prominent beneath, 3–4 to a side, those of the basal pair much upcurved and generally united before reaching sinus. Sori inframedial, mostly confined to the anterior basal vein. Indusium subpersistent, setose by simple hairs. Sporangia glabrous; receptacle setose.

213. *Dryopteris dissimulans* Maxon et C. Chr. n. sp.

= *D. scolopendrioides* (L.) O. K. × *D. sagittata* (Sw.) C. Chr.?

Type from Cuba: Arroyo de Pedro, ad Jaguey, 600 m., EGGERS nr. 4958 (W!)

Rhizomate erecto, dense radiculoso, squamis brunneis sparse stellato ciliatis onusto. Stipitibus fasciculatis, 3–5 cm longis, strictis, minute stellato-pulverulentis et sparse squamosis. Lamina ad 30 cm longa, supra medium 4 cm lata, versus apicem breviter acuminata, versus basin longe et gradatum attenuata, coriacea, griseo-viridi, ad medium pinnata, supra medium fere ad rachin pinnatifida. Rachitereti, stellato-puberula. Pinnis liberis multijugis sensim reductis, inferioribus 2–3 mm longis et latis, omnibus basi utrinque subauriculatis, adnatis; segmentis supra-medialibus ad 2½ cm longis, a basi, 6 mm lata, versus apicem acutum sensim attenuatis, parum obliquis, sinubus acutis separatis, supra glabris, infra prasertim ad costas venasque breviter stellato-puberula. Venis bi-trifurcatis, ramis versus marginem non raro anastomosantibus. Soris parvis, in segmento biserialibus, fere medialibus. Indusiis parvis, stellato-ciliatis. Sporangii glabris.

In general I am not inclined to consider a form intermediate between two known species a hybrid between them, but in this case I am not far from believing that the present plant is a hybrid between *D. scolopendrioides* and *D. sagittata*. In almost every respect it is exactly intermediate between the two species. In the presence of an indusium and in cutting it resembles the former species, in the shape of the segments and in the large number of downwards gradually dwindling pinnæ, which show a tendency to becoming auricled on both sides at base, it is very like the latter. In venation it is also intermediate. Hereto comes the fact that the sporangia are partly abortive and do not produce spores, and further that EGGERS has gathered both true *D. sagittata* (nr. 4950) and *D. scolopendrioides* (nr. 4887) in the same locality. From *D. guadalupensis* (Wikstr.) *D. dissimulans* differs by coriaceous texture, grey-green colour, somewhat verrucose under-surface, by the large number of free and closely placed pinnæ and by the basal veins rarely being goniopteroid.

214. *Dryopteris cordata* (Fée) Urban, Symb. Antill. 4: 18. 1903; C. Chr. Ind. 258.

Syn. *Phegopteris cordata* Fée, Gen. 244. 1850—52; 6. mém. 13 tab. 6 fig. 3.

Aspidium reptans var. 1. *cordata* Mett. Aspid. nr. 237. 1858.

Polypodium cubanum Bak. Syn. 304. 1867.

Type from Cuba, leg. LINDEN nr. 1873 (not seen).

Rhizome erect, with a few stellato-ciliate scales. Leaves fasciculated; stipe 3—5 cm long, like rachis slightly stellato-puberulous and with some few longer, simple hairs. Lamina up to 30 cm long, 3 cm broad, linear, pinnate from base to short of the apex; larger leaves sometimes terminating in a scaly bud, which produces a rosette of young leaves, grey-green, chartaceous, opaque. Pinnæ numerous (20—30 to a side), close horizontal, distinctly stalked, oblong, $\frac{1}{2}$ — $1\frac{1}{2}$ cm long, $\frac{1}{2}$ cm broad, their base cordate, the apex bluntly rounded, the lower ones often gradually shortened, both surfaces, especially on veins, with not many small stellate hairs. Veins about 8 to each side, generally forked, free. Sori a little above the middle of the vein in a single row on each side of the midrib, exindusiate, but the receptacle with several branched hairs. Sporangia with some bi-trifurcate, deciduous setæ, soon glabrous.

The typical form from Cuba of this species is widely different from most forms of *D. reptans*, with which METTENIUS united it. In Porto Rico a form was collected by (SINTENIS (nr. 6380, 6588) referred hereto by URBAN, which resembles *D. cordata* in shape of the pinnæ; still I am inclined to regard the specimens as young states of *D. reptans*. The pinnæ are shorter, more ovate, scarcely cordate at base, and the sori are distinctly exindusiate.

Cuba: El Yunque, Mt. Baracoa, UNDERWOOD and EARLE nr. 725 (W) — Monte Verde, WRIGHT nr. 1014 (B, S) — Josephina, Yateras, Oriente, MAXON nr. 4117 (W).

Related forms, but doubtfully belonging here:

Bahama Isl.: Andros, JOHN J. and ALICE R. NORTHROP nr. 576 (B), ? 583 (B).

San Domingo: ad flum. Mameges, EGGERS nr. 2656 (B).

215. *Dryopteris sagittata* (Sw.) C. Chr. Ind. 290. 1905.

Syn. *Polypodium sagittatum* Sw. Prod. 132. 1788.

Polypodium hastaeifolium Sw. Schrad. Journ. 1800²: 25. 1801; Hk. et Grev.

Ic. Fil. tab. 203; Bak. Syn. 304; Jenman, Bull. Dept. Jam. n. s. 4: 127.

Polypodium hastatum Sw. Fl. Ind. occ. 1653. 1806.

Aspidium reptans var. 2. *hastaeifolia* Mett. Aspid. nr. 237. 1858.

Aspidium hastifolium Gris. Fl. br. W. Ind. 694. 1864.

Type from Jamaica, leg. SWARTZ (S!).

A distinct species, not closely related to *D. reptans*, with which METTENIUS united it; the plate cited above gives a fair illustration of it. Most pinnæ short stalked and bearing a pair of acute spreading auricles at base, entire or faintly crenulate, the lower ones gradually reduced to mere auricles. Rachis and costæ

beneath rather densely pulverulent by very short-branched stellate hairs, surfaces otherwise glabrous. Leaf greyish-green, chartaceous. Veins simple or forked or subpinnate. Sori apparently exindusiate, about medial. Sporangia glabrous.

Jamaica: MAXON nr. 1176, 1828, 1930 (C), 2544 (W).

Cuba: Arroyo de Pedro, EGGERS nr. 4950 (B, C) — Monte Verde, WRIGHT nr. 812 (S).

var. **tenebrica** (Jenman).

Syn. *Nephrodium tenebricum* Jenm. Journ. Bot. 1882: 326; Bull. Dept. Jam. n. s. 3: 143. 1896; W. Ind. and Guiana Ferns 228.

Dryopteris tenebrica C. Chr. Ind. 297. 1905.

A much larger plant than the type but scarcely different by other characters than the size. "Plant shuttle-cock-like in habit with sometimes as many as 45 developed fronds to one rootstock" (HARRIS in sched.). Stipe 10–12 cm, lamina up to 30 cm long by 8 cm broad. Pinnæ 4–5 cm long, $\frac{3}{4}$ –1 cm broad, serrulate or shallowly lobed. Veins pinnate in the lobes, 2–4 jugate, the basal pair anastomosing and sending a branch to the sinus. JENMAN describes the sori as indusiate, the indusium being minute, soon obliterated.

Jamaica: SHERRING (type, Kew!) — Holly Mount, Mt. Diabolo, HARRIS nr. 8998 (B).

San Domingo: prope Jérémie, WEINLAND nr. 55 (B).

216. *Dryopteris reptans* (Gmel.) C. Chr. Ind. 288. 1905.

Syn. *Polypodium repens* Sw. Prod. 132. 1788 (not 130).

Polypodium reptans Gmel. Syst. Nat. 2²: 1309. 1791; Sw. Fl. Ind. occ. 1655; Bak. Syn. 316.

Aspidium reptans var. 3 *radicans* Mett. Aspid. nr. 237.

Nephrodium asplenioides part. and subsp. *reptans* Jenman, Bull. Bot. Dept. Jam. n. s. 3: 212. 1896; W. Ind. and Guiana Ferns 229.

Dryopteris radicans Maxon, Contr. U. S. Nat. Herb. 10: 490. 1908.

Polypodium repandum Sw. Schrad. Journ. 1800²: 25. 1801; Fl. Ind. occ. 1654; (C. Chr. Arkiv för Bot. 9¹¹: 31 fig. 7, tab. 4 fig. 2. 1910; illustrations of SWARTZ's type-specimens).

Type from Jamaica, leg. SWARTZ (S!).

When excluding the subsp. *sclerophyllum*, which is our *D. asplenioides*, JENMAN's treatment of this extraordinarily variable species under the name *Nephrodium asplenioides* is very good, when the Jamaican forms only are concerned. Other forms occur in Cuba and Central-America, but I have failed to find good characters by which these forms may be distinguished from the typical form described by SWARTZ. I can, however, not agree with METTENIUS in considering *D. cordata*, *D. sagittata* and *D. asplenioides* forms of the same species: they are all in this paper dealt with as good species; also *D. asterothrix*, in my Index referred to *D. reptans* as a subspecies, appears to be a good species. Separating out these four forms as

distinct species there remains a good number of forms, which I refer to a single species, *D. reptans*. Although very different in habit and size they show some common characters: 1) the long radican apex of some leaves, 2) the pubescence of the surfaces, especially the upper one, by forked hairs, 3) the glabrous sporangia; the head of the sporangia appears always to be glabrous, but the pedicel often bears a single hair; the receptacle is as a rule setose by long hairs, which often are longer than the sporangia, 4) the presence of a small indusium consisting of a few cells bearing long simple or forked hairs. — In developed fronds the lower pair of veins, which spring out from the secondary vein 1—2 mm above its base, are goniopteroid and send a branch to the sinus; in young leaves the veins are sometimes all free. The rhizome is erect, clothed at the top with proportionally few glossy, brown, stellato-pubescent scales. Stipites fasciculated, slender, stramineous. Lamina generally herbaceous but frequently chartaceous or even coriaceous, more or less hairy on rachis and ribs by long, simple hairs. Most pinnæ short-stalked, the lower ones not or slightly reduced, the margins cut into shallow rounded lobes or subentire. Veins 2—5 to a side, not very prominent beneath. Sori below the middle of the vein. As to size and shape of leaves and pinnæ nothing can be said, which agrees even with two specimens from the same locality. It seems that leaves from same rhizome but of different age vary very much.

MAXON has recently tried to show that the right name of this species is *D. radicans* (L.). It is illustrated by SLOANE pl. 29 and 30 fig. 1 and by PLUKENET pl. 253 fig. 4, which plates were cited by LINNÆUS under his first description of *Asplenium radicans* (Syst. Nat. ed. X. 2: 1323. 1759), as well as by SWARTZ under his *P. reptans*. If LINNÆUS really founded his *Asplenium radicans* (= *Aspl. radicans* of my Index) on SLOANE's plate, MAXON is certainly right in changing the name, but we have here a case, which exactly corresponds with that of *Asplenium erosum* L., which name MAXON uses for the well-known *A. auritum* Sw. This later case I have dealt with in some detail in my paper on SWARTZ's species of ferns (Arkiv för Bot. 9¹¹: 14—17). It is right that LINNÆUS under the first description of his *Aspl. radicans* quoted the said plates only; but in Spec. plant. ed. II. 1540. 1763, where the name is changed to *Aspl. rhizophyllum*, he immediately after the description cites "Brown. Jam. 92". This additional citation is important, because it proves that LINNÆUS founded his species on dried specimens, not on SLOANE's plates. LINNÆUS bought namely, in the year 1758 a collection of Jamaican plants gathered by P. BROWNE, and described in BROWNE's work on Jamaica. This work from 1758 LINNÆUS did not know during the preparation of the tenth edition of *Systema Naturæ* and therefore it could not, of course, be cited in this edition. It is natural that LINNÆUS tried to determine BROWNE's specimens by aid of SLOANE's and PLUKENET's works, and when finding a plate, which he believed to illustrate the species, he quoted that plate. Knowing BROWNE's work while preparing the second edition of *Species plantarum* he then always cited this work before the citations given in 1759 and always immediately after the description, which proves

that his new species was founded on a specimen from BROWNE, not on the plates quoted. Subsequently *Aspl. radicans* L. (= *A. rhizophyllum* L.) is not the same as *P. reptans* Sw., and the right name of our species must be *D. reptans* (Gmel.).

D. reptans is a common species in Jamaica and Cuba and occurs also but more rarely in some other islands, Florida and Central America. The smallest form occurs in the eastern islands, the largest in Cuba and Florida. I shall here mention some of the more characteristic forms.

1. var. **tenera** (Fée).

Syn. *Goniopteris tenera* Fée, 11 mém. 60 tab. 15 fig. 3. 1866.

A small, thin-leaved form: leaves apparently never radicant. Stipe very slender, shorter than the lamina, this 6—12 cm long, 5 cm broad at the base; pinnæ short-stalked, close, the lower ones reduced or not, about 2½ cm long by 1 cm broad, obtuse at the point, rather deeply lobed, broadest at the truncate base. Veins about 3 to each side. Differs from the next variety by its not radicant leaves and uniform fronds, from *D. asplenioides* by its thin texture, not prominent veins and especially by the leaf tissue of both surfaces being rather densely but minutely pubescent by stellate hairs.

JENMAN's var. *tenera* is probably not the same; I have seen no Jamaican specimens, which exactly corresponds with FÉE's type, although single leaves from rhizomes, which also bear radicant or differently shaped leaves, are very slightly different.

Guadeloupe: L'HERMINIER (type: Herb. Cosson Paris!; B).

Porto Rico: SENTENIS nr. 1770 (B, C, W).

2. var. **eu-reptans** Jenman, loc. cit.

Hereto a good many forms which I have tried in vain to distinguish from each other. Not only are two identical specimens not to be found but the leaves from the same rhizome also are often very different. Very often the sterile and fertile leaves are different, the former being very short-stalked, prostrate, rooting or not, the latter much higher on long stalks and often of a much more rigid texture, radicant or not, the pinnæ generally distant; in other specimens also the short, prostrate leaves are fertile. The pinnæ are very differently shaped, even in leaves of the same rhizome; in the short-stalked leaves they resemble those of var. *tenera*, i. e. short-stalked, obtuse at the apex, rather deeply lobed, 2—4 cm long, ¾—1 cm broad, but not so closely placed. The long-stalked, most often fertile and often radicant leaves are more varying, the pinnæ being sessile or stalked, entire or crenate or lobed, often hastate at base, acute or rounded at the apex, etc. A peculiar form is that described as *Pol. repandum* Sw.; here the short-stalked, prostrate, sterile leaves are radicant, while the fertile leaves are very long-stalked (stipe up to 20 cm high), not radicant, pinnæ very distant, scarcely ½ cm broad, 2 cm long, almost coriaceous, the margins revolute, crenate or almost entire; veins

2—3. Probably such leaves develop only in old plants. In quite young plants the pinnæ are sometimes quite entire, ovate or nearly circular, the veins free: such forms have been referred to *D. cordata*. — Some forms are nearly destitute of stellate hairs.

The following specimens are typical, if such a term may be used here.

- Jamaica: MAXON nr. 2104 (C, H, Rg. W), 2102, 2191 (C, W), 2341, 2557 (W, 2943 (CC, Rg. W); CLUTE 334 (W); UNDERWOOD nr. 1784 (W); and others.
 Haïti: Marmelade, NASH and TAYLOR nr. 1232 (W) — Sto. Domingo: Puerto Plata, EGGERS nr. 1577 (C) — PICARDA nr. 228, 358 (B).
 Cuba: Oriente, Farallones de la Perla, MAXON nr. 4407 (W) — Yateras, MAXON nr. 4429 (W) — Prov. Santiago, El Yunque, POLLARD and PALMER nr. 122 (W) — Prov. Habana, San Antonio de los Baños, ÁBARCA nr. 4531 (W), BAKER nr. 2755 (W) — Prov. Pinar del Río, near El Guama, PALMER and RILEY nr. 126 pt (W) — E. OTTO nr. 85 (B).
 Bahamas: New Providence, RAWSON W. RAWSON (B).
 Venezuela: Caripe, MORITZ nr. 213 (B).

3. var. *angusta* n. var.

Fronds rather uniform, often radicant, very short-stalked, linear, often considerably narrowed downwards. Pinnæ distant, small, rarely more than $1\frac{1}{2}$ cm long, often only $\frac{1}{2}$ cm, scarcely $\frac{1}{2}$ cm broad, obtusely rounded at the apex, the margins entire or slightly crenated, rounded or auricled at base; secondary veins once or twice forked, free or united near the edges.

Very characteristic by the long, very narrow, flaccid leaves, which are often rooting as in *eureptans*. It approaches *D. cordata*, from which it differs by its distant pinnæ, thinner texture, long proliferous apex and pubescence.

- Cuba: Oriente, Monte Verde, WRIGHT nr. 813 (B, type!; C, S) — Bejucal, LIEBMANN (H) — Nazarene, BAKER nr. 1899 (B, W).
 Jamaica: WILSON nr. 54 (B).
 Porto Rico: Utuado, SENTENIS nr. 6380 (C), 6588 (CC; pinnæ nearly circular).
 Guatemala: Dept. Alta Verapaz, Cubilquitz, v. TUERCKHEIM ed. DONN. SMITH nr. 8481 (W) — Pansamá, v. TUERCKHEIM, ed. DONN. SMITH nr. 712 b (W) — Cobán, v. TUERCKHEIM ed. J. D. S. nr. 712 (W).

4. var. *conformis* n. var.

Differs from var. *eureptans* by its mostly uniform leaves, which have the general habit of the short-stalked mostly sterile leaves of *eureptans*, but the stipites are rather long and the lamina often much larger. Long-stalked, differently shaped leaves are not to be found, but often the radicant leaves are somewhat narrower than the non-radicant ones. — All leaves long-stalked, stipe often as long as the lamina or even longer. Lamina mostly lanceolate in outline, often 20 cm or more long but as a rule shorter, 5—8 cm broad, shortly narrowed downwards. Pinnæ very short-stalked the lower ones mostly rather shortened, distant, the largest up to 5 cm long, often cut $\frac{1}{3}$ or more to the costa and cordate at base. — It is less proliferous than *eureptans*, still one or two radicant leaves are to be found in most

specimens. It approaches *D. asterothrix*, from which it differs by its not so densely pubescent fronds, glabrous sporangia and proliferous leaves. From *D. asplenioides* it differs by its obtuse pinnæ, stellate pubescence of the leaf-tissue, scarcely prominent veins and nearly sessile pinnæ; from var. *tenera* by the distant lower pinnæ and proliferous leaves.

Most specimens from Cuba, Central America and Florida belong to this variety, which represents the most developed state of the species; in the largest form it is very different from *eureptans*, but there are numerous intermediate forms between the two varieties.

Florida: Hammock on left bank of Withlacoochee, near Brooksville, cavernous, calcareous rocks, J. DONNELL SMITH (type collection, W) — Dade Co., Castella's hammock, A. A. EATON nr. 262 (W) — Isthacatta, L. M. UNDERWOOD nr. 277 (W) and A. H. CURTISS nr. 5965 W. approaching var. *eureptans*).

Bahamas: Andros, JOHN J. and ALICE R. NORTROP (B).

Cuba: Prov. Habana, Rincon, WILSON nr. 205 (B, W); San Antonio de los Baños, A. H. CURTISS nr. 639 (B, H, W) and VAN HERMANN nr. 3360 (W) — Prov. Pinar del Rio, near El Guama, PALMER and RILEY nr. 126 pt., 230 pt., 403 (W), 248 (CC, H, W) — E. OTTO nr. 62 (B).

Mexico: in speluncis près Orizaba, BOURGEAU nr. 2514 (B, H, S, W); base of calcareous cliffs, near Orizaba, PRINGLE nr. 5594 (W) — Yucatan, SCHOTT nr. 779 (W).

217. *Dryopteris asterothrix* (Fée) C. Chr. comb. nov. — Fig. 29.

Syn. *Goniopteris asterothrix* Fée, Gen. 253. 1850—52.

Phegopteris asterothrix Mett. Pheg. nr. 40. 1858.

Dryopteris reptans **asterothrix* C. Chr. Ind. 288. 1905.

Type from Cuba, leg. LINDEN nr. 1917 (f. FÉE, not seen), nr. 1878 (B!).

This seems to me a very distinct species, although some forms of the protean *D. reptans* resembles it very much. The accompanying figure will give an idea of its habit. The oblique or short-creeping rhizome, which bears some few stellato-pilose scales, bears a small number of leaves, which are of a very thin texture and throughout clothed with small stellate hairs. The stipe is slender, stramineous and like the rachis densely soft-hairy by patent, whitish hairs under which numerous small stellate hairs are to be found. Lamina 15 cm \times 5—6 cm, bipinnatifid, the apex obtuse. Pinnæ opposite or subopposite, 6—10 on each side of rachis, the lower ones short-stalked, the upper ones sessile, at least confluent, 2¹/₂—3 cm long, 1—1¹/₄ cm broad, obtusely rounded at the apex, slightly broader at the base, the lowest pair generally a little shorter and deflexed; and below them a pair of small, reduced pinnæ are often seen; both surfaces densely and shortly stellato-pilose and costæ and veins besides with long, soft, patent hairs. Lobes or segments broad, obtuse, the middle one often a little lengthened. Veins rather indistinct, about 4 to each side of the midrib of the lobe, the lower pair united. Sori small, exindusiate, medial or a little inframedial; receptacle with numerous, long-stalked branched hairs; sporangia very loose, at the bearing 2—3 erect 2—3 branched hairs, which are very deciduous, and therefore the sporangia may be found to be glabrous.

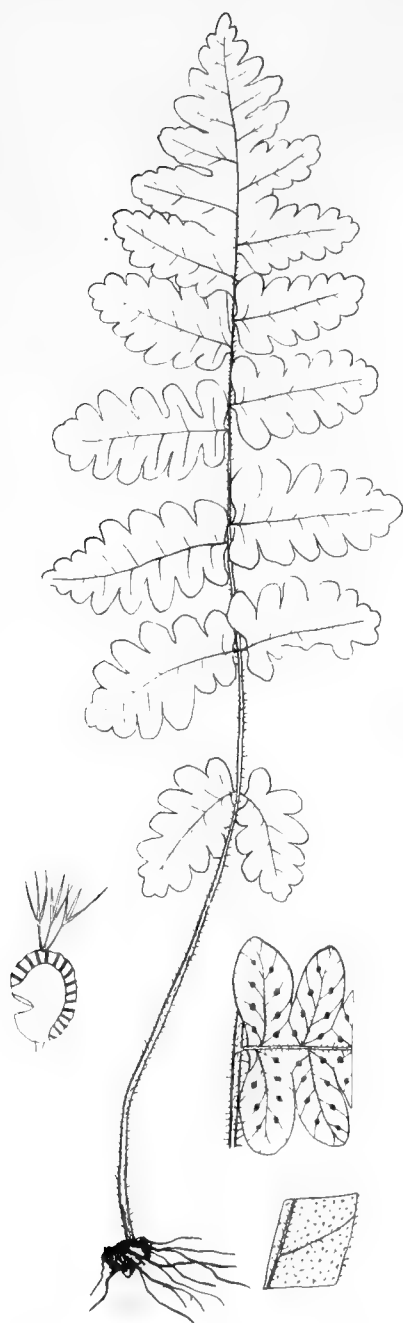


Fig. 29. *D. asterothrix* (Fée) C. Chr. Entire leaf $\times \frac{1}{2}$; base of pinna $\times 1\frac{1}{2}$; fragment $\times 4$; sporangium. (LINDEN 1878).

From *D. reptans* *D. asterothrix* can be distinguished by its size, never rooting apex, its soft-hairy rachis and stipe, setose sporangia and its whole habit. The rhizome bears often under the long-stalked fertile leaves a rosette of small sterile ones. — The following specimens are very uniform; several of them from various herbaria were determined by Dr. Christ as *A. dissidens* Mett., under which name the Guatemalan specimens were distributed by DONNELL SMITH.

Cuba: Prov. Oriente, caverns of Thermopylæ, Monte Libano, 600 m, MAXON nr. 4240 (W).

Jamaica: Mt. Diabolo, UNDERWOOD, nr. 1798 (W) — near Bath, MAXON nr. 1869 (W).

Guatemala: Dept. Alta Verapaz, Xucipac, Cubilquitz, v. TÜRCKHEIM ed. J. D. S. nr. 8355 (B, W).

Costa Rica: WERCKLÉ (C, W).

Venezuela: Tovar, FENDLER nr. 201 (B).

var. **bibrachiata** (Jenman).

Syn. *Nephrodium bibrachiatum* Jenman, Gard. Chr. III 15: 230. 1894; W. Ind. and Guiana Ferns 228.

Dryopteris bibrachiata C. Chr. Ind. 254. 1905.

Not essentially different from the type, but the basal pinnæ the largest (small, reduced pinnæ always absent), on longer petioles (2 mm long), stipe and rachis with fewer or no long soft hairs. JENMAN describes the indusium as distinct; I have failed to find it.

Jamaica: JENMAN (W; type) — John Crow Peak, 5500—6000 ft; MAXON nr. 1317 (= UNDERWOOD nr. 2443) (W) — Mt. Diabolo, MAXON nr. 1940 (W; Rg).

218. *Dryopteris sclerophylla* (Kze) C. Chr.

Biol. Arbejder tilegn. Eug. Warming 84. 1911.

Syn. *Aspidium sclerophyllum* Kze in Spr. Syst. 4: 99, 1827; Linn. 9: 92. 1834.

Aspidium scolopendrioides var. 3. *pinnata* Mett. Aspid. nr. 235. 1858.

Aspidium dissidens Mett. Aspid. nr. 275b. 1858; C. Chr. Ind. 71.

Nephrodium dissidens Hk. sp. 4: 66. 1862. Hk. Bak. Syn. 295.

Dryopteris dissidens O. Ktze. Rev. 2: 812. 1891; Urban, Symb. Ant. 4: 19. 1903.

Nephrodium jamaicense Bak.; Jenm. Journ. Bot. 1877: 264. Bull. Dept. Jam. n. s. 3: 163. 1896.

Dryopteris jamaicensis C. Chr. Index 272. 1905.

Nephrodium asplenioides Bak. Syn. 293 (part?).

Aspidium Sintenisii Kuhn et Christ.; Krug, Engl. Jahrb. 24: 119. 1897.

Dryopteris Sintenisii Urban, Symb. Ant. 4: 19. 1903; C. Chr. Ind. 293.

Type from Cuba leg. POEPPIG (specim. auth. vidi in Herb. Berol. et Herb. PRESL).

A distinct but very misunderstood species, by METTENIUS considered the most developed form of his *A. scolopendrioides*, while Baker (Syn. Fil. 293) very improperly referred it to his *Nephrodium asplenioides*. As the above list of synonyms shows, the species has been described under at least three new names, of which *A. dissidens* was placed in Syn. Fil. under the subgenus *Pleocnemia* (!). It is beyond question that the forms referred by me to *D. sclerophylla* are closely related when not absolutely identical. The main-characters of the species are 1) the coriaceous or papyraceous texture of the lamina, which is of a characteristic grey colour due to the dense pubescence of stellate hairs throughout both surfaces, 2) the thick, undulato-crenate margins of the teeth or lobes, which generally seem to be acute or even mucronate because the margins of the lobes are revolute, 3) the prominent veins, 4) the supramedial or even submarginal sori, and 5) the glabrous sporangia.

The erect or oblique rhizome is at the top densely clothed with red-brown or nearly black, glossy, stellato-pilose scales. The lamina varies in size and degree of cutting but it is always fully pinnate from base to the middle. The basal pair of veins is always truly anastomosing. Sori with a distinct, stellato-pubescent indusium. I can distinguish two forms.

1. (typical *sclerophylla*). Leaves on short stems, long and gradually narrowed below. Pinnæ short (3 cm \times 1 cm) the lower ones gradually smaller almost as in *D. opposita*, the fully developed ones sessile with a subcordate base, the upper ones confluent all broadly serrate, scarcely pinnatifid. Veins simple; sori medial. Scales of rhizome few, brown.

Cuba: POEPPIG (B, hb. PRESL) — Prov. Habana, BAKER and O'DONAVAN nr. 4135 (W); BAKER nr. 1888 (W) — Prov. Pinar del Rio, PALMER and RILEY nr. 230, 227, 391, 533 (W); BAKER nr. 3797 (W) — v. HERMANN nr. 2185, 3256 (W) — Prov. Oriente, WRIGHT nr. 1005 (B, S), 3923 (B, W, S) MAXON nr. 4447 (W) — Prov. Santiago, POLLARD and E. and W. PALMER nr. 41 (W).

2. Leaves on longer stems, ovate or elliptic, shortly narrowed downwards. Pinnæ short-stalked or sessile with cordate base, up to 8 cm long, 1 $\frac{3}{4}$ —2 cm broad, pinnatifid to a narrow wing to the costa. Veins pinnate in the lobes, often furcate and forming costular areoles, very prominent beneath. Sori supramedial or submarginal. Scales of rhizome nearly black, 2 cm long, glossy.

Cuba: Prov. Habana, WILSON nr. 671 (W), CURTISS nr. 592 (H, W), — Prov. Pinar del Rio, PALMER and RILEY nr. 126, 223, 567 (W) — Prov. Oriente, WRIGHT nr. 1001 pt. (S, W). MAXON nr. 4389, 4422 (W), EGGERS nr. 4941 (B).

This form looks very different from the type, but it can not be distinguished even as variety; in some specimens are to be found young leaves, which are fully identical with the typical form. With this large form agree the Porto Rico and Jamaican original specimens of *A. dissidens* Mett. (Syn. *A. Sintenisii* Kuhn) and *N. jamaicense* Bak. The latter differs from the Cuban form only by its lighter scales of the rhizome, more distinctly stalked pinnæ, which are often unequal, at the base, and by less stellato-pilose leaf-tissue and somewhat sided thinner texture. *A. dissidens* is almost quite identical with *jamaicense*. I have not seen the original specimen of it, which was collected in Porto Rico by BALBIS, but a sketch of it is found in Herb. Berol, which is from the hand of METTENIUS and perfectly agrees with the original diagnosis. It is absolutely identical with *A. Sintenisii* from the same island. In this form the veins frequently are united in the lobes, so that they form costular areoles; BAKER, therefore, placed it in the subgenus *Pleocnemia*. The sori are perhaps closer to the margin than in the other forms of the species.

Jamaica: JENMAN (B) HART nr. 347 (W).
Porto Rico: SINTENIS nr. 2136 (B, C, CC, S, W).

219. *Dryopteris asplenioides* (Sw.) O. Ktze. Rev. 2: 812, 1891;
C. Chr. Ind. 253 (part.). — Fig. 28 e.

Syn. *Polypodium asplenioides* Sw. Schrad. Journ. 1800²: 26. 1801; Fl. Ind. occ. 1659.

Aspidium reptans var. 4. *asplenioides* Mett. Aspid. nr. 237. 1858.

Nephrodium asplenioides (Bak. Syn. 293?) part. and 1. sub-sp. *sclerophyllum* Jenman, Bull. Bot. Depart. Jamaica n. s. 3: 211. 1896; W. Ind. and Guiana Ferns 230.

Woodsia pubescens Spr. Nova Acta 10: 233 tab. 16 fig. 5—7 1821!

Type from Jamaica, leg. SWARTZ (S),

I think that this species is distinct from *D. reptans*, although it is difficult to give good characters by which it can be distinguished from not-rooting forms of that species. The main characters are: Leaf pinnate to short of the never rooting apex, often long tapering almost from the base, 3—5 dm long, chartaceous or firmly membranous, fresh-green, often glossy, rachis more or less pubescent by as well long, simple hairs as minute stellate ones. Pinnæ very numerous; often 25 to a side, most of them distinctly stalked, often characteristically falcate, the base generally cordate, the point obtuse or acute, 4—6 cm long, $\frac{3}{4}$ —1 $\frac{1}{2}$ cm broad, the margins subentire, crenate, or, often more or less lobed into rounded, somewhat oblique lobes, often auricled on both sides of the base, ciliate, upperside glabrous, costæ and veins beneath hairy by long, simple hairs and small, stellate hairs, leaf-tissue glabrous. Lower pinnæ not or only slightly reduced. Veins simple, 3-4 jugate, prominent beneath, the basal pair always anastomosing. Sori near the midvein of the lobe, furnished with a small indusium, which is ciliated by simple

or forked hairs. Head of sporangium glabrous, but its pedicel bears a single simple or furcate hair. - None of the specimens referred hereto shows small short-stalked sterile leaves, which are often to be seen in specimens of *D. reptans*; all leaves from the same rhizome are always uniform, the stipes are stramineous, glabrous. 8—15 cm long, slightly scaly below, fasciculated on an erect rhizome, which is rather scaly at the top by glossy, brown, acuminate, stellato-pubescent scales. Larger specimens may be confounded with *D. serrulata*, but I think, that it always can be distinguished from that species by its more decidedly pubescent rachis and costæ and its rounded lobes. From *D. sclerophylla* it differs by its fresh-or-dark-green colour, thinner texture and non stellato-pilose surfaces.

MAXON, believing that *P. asplenioides* Sw. was founded on SLOANE's plate 43 fig. 2, quoted by SWARTZ, reduced SWARTZ's name to a synonym of *D. serrulata*. Granting that SLOANE's plate illustrates our *D. serrulata* rather than *D. asplenioides* I must say again that SWARTZ never founded a species on old figures; his *P. asplenioides* is founded on specimens collected by himself in Jamaica. A specimen from Herb. Sw. (S), which belongs to the present species, agrees very well with SWARTZ's description in Fl. Ind. occ. 1659, but unfortunately it is not labelled with certainty by SWARTZ.

D. asplenioides varies especially in size and texture, which probably is due to age and outer conditions. It is in its true form apparently confined to Jamaica.

Jamaica: MAXON nr. 875, 1406 (= UNDERWOOD nr. 2531), 1507, 1894, 2209, 2258, 2270, 2338, 2866 (W); UNDERWOOD nr. 1800, 320, 2978, 3106 (W); CLUTE nr. 120; HARRIS nr. 7342, 7592 (B); HART nr. 171a, 211 (W).

Haiti: Port au Prince, PICARDA nr. 734 (B) (doubtful).

Cuba: WRIGHT nr. 1801 (S, W).

220. *Dryopteris bermudiana* (Bak.) Gilb. Bull. Torr. Cl. 25: 600. 1898;
C. Chr. Ind. 254.

Syn. *Nephrodium bermudianum* Baker apud Hemsley, Chall. Exp. Bot. 1¹:
86 tab. 13. 1885.

Type from Bermuda (not seen).

Closely related to *D. asplenioides* and the smaller forms difficult to distinguish from that species. Still the pinnae are generally larger (8—10 × 2 cm) and cut more than halfway to the costa. Both surfaces with scattered, whitish, simple or forked hairs, most numerous on the ribs; rachis rather hairy by short stellate hairs and longer, simple ones. Veins 6—7-jugate, the lower pair united or more often running side by side to the sinus, the others very often furcate. Sori medial or supramedial, furnished with a ciliate indusium; hairs of indusium simple, or, rarely forked. Sporangia without hair on the pedicel. The scales of the oblique rhizome are stellato-pilose throughout; they are not so large as shown in the plate quoted, which otherwise illustrate the species very well.

Endemic in Bermuda Islands, GILBERT (W), G. B. Goode (W); W. G. FARLOW (S) - REIN nr. 90 (B).

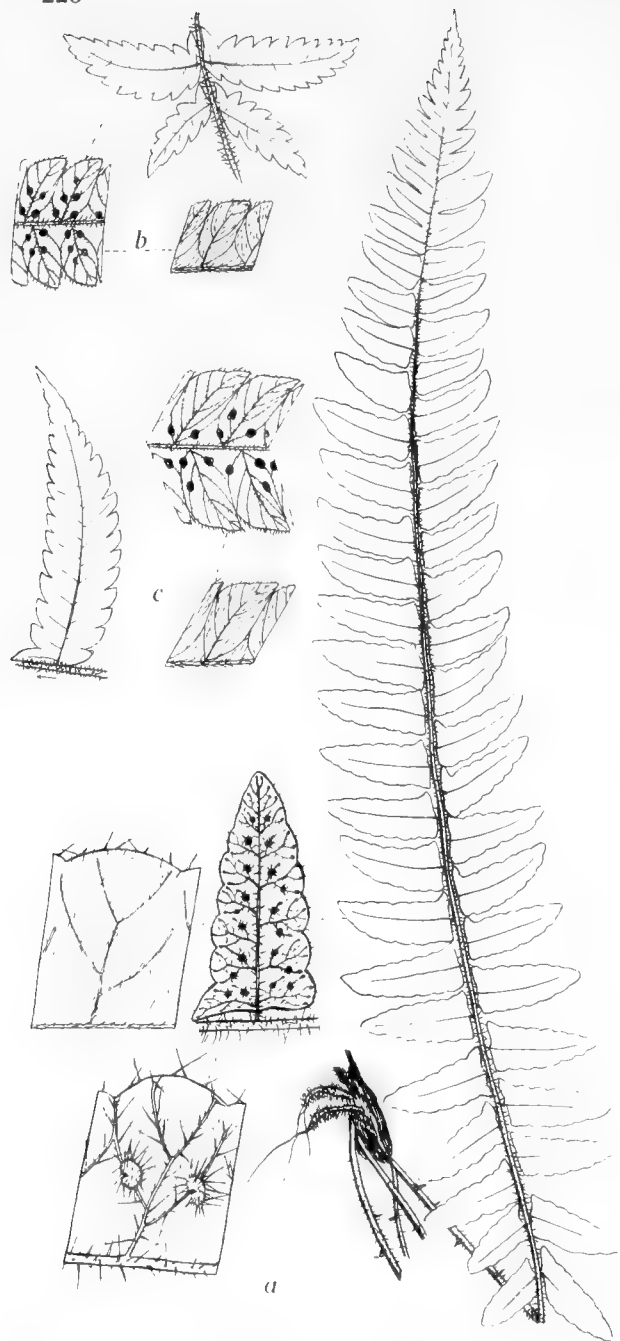


Fig. 30. *a. D. semihastata* (Kze.) O. Ktze., l. POEPPIG. Leaf $\times \frac{1}{15}$; pinna $\times 1\frac{1}{2}$; fragment from the underside with anastomosing veins, $\times 5$. — *b. D. Jamesoni* (Hk.) C. Chr. Two lower pairs of pinna, $\times \frac{1}{15}$; segments from both surfaces $\times 1\frac{1}{2}$ (orig.). — *c. D. Warminгии* n. sp. pinna $\times \frac{1}{15}$ and segment from both surfaces, $\times 1\frac{1}{2}$.

221. *Dryopteris semihastata* (Kze.) O. Ktze. Rev. 2: 291. 1891; C. Chr. Ind. 291. — Fig. 30 a.

Syn. *Aspidium semihastatum* Kze. Linnaea 9: 91. 1834; Mett. Aspid. nr. 179.

Lastrea Poeppigiana Presl, Epim. bot. 40, 1849.

Type from Peru, ad flum. Pampayaco, leg. POEPPIG. A figure of the type-specimen (by METTENIUS) is to be found in (B), and with it agrees perfectly the type-specimens of *L. Poeppigiana* Pr. (hb. PRESL!), which also was collected by POEPPIG "in Cuba"; there is no doubt that PRESL's specimen is of the type-collection, and the locality erroneous; POEPPIG collected in 1829 both in Peru and in Cuba.

D. semihastata resembles much a small form of *D. macrotis*, and together with that species its position within the genus is doubtful. It lacks, namely, stellate hairs, and it is possible that the two species are less cut members of the group of *D. patens*; still in general aspect they resemble other species of the present group in which I place them. The accompanying figure will give an idea of *D. semihastata*; in pubescence, auricled, subentire pinnæ, of which the lower ones are reflexed, and also in venation it agrees with *D. macrotis*, but it is much smaller and with only 2–3 pairs of veins, and the auricles are much shorter.

222. *Dryopteris Jamesoni* (Hook.) C. Chr. comb. nov. — Fig 30 b.

Syn. *Nephrodium Jamesoni* Hook. spec. 4: 66. 1862.

Type from Ecuador, ad ripam fl. Napo, JAMESON nr. 761 (Kew!) a very similar plant was collected in Peru, Tarapoto by SPRUCE nr. 3946 (L).

A small species closely related to *D. semihastata*, which it resembles in size, colour, pubescence and the auricled upper base of the pinnæ, but the lamina is fully pinnate scarcely to the middle, upwards pinnatifid only. Largest leaf seen: stipe 9 cm, lamina 17 cm long, 4 cm broad below the middle. Stipe and rachis rather pilose by long, stiff, simple hairs, between which some very small, stellate hairs can be found. Upper surface densely and adpressedly pubescent, costæ and veins beneath setose by patent setæ. Lowest pair of pinnæ reflexed and considerably reduced. Veins in about 3 pairs in the lobe of the larger pinnæ, the basal ones normally united. Sori inframedial furnished by a setose indusium.

It is possible that *D. Jamesoni* is a small variety of *D. macrotis*, which it resembles closely in pubescence of the surfaces, the auricled pinnæ and other characters. BAKER referred it in Syn. Fil. 293 to *Nephr. molle*, to which it is, of course, not at all related.

223. *Dryopteris Warmingii* n. sp. — Fig. 30 c.

Syn. *Nephrodium molle* var. γ *Jamesoni* Bak. Fl. bras. 1^a: 489. 1870.

Type from Brazil: Minas Geraes, Lagoa Santa, leg. WARMING 1864 (H!) and the same from São Paulo, Cajurú, REGNELL nr. III. 1449 b. (Rg, W).

Rhizomate crasso, ad 5 cm alto, 1 cm crasso, squamis brunneis sparse stellato-ciliatis praedito. Stipitibus fasciculatis. ad 20 cm longis, tenuibus, pilis minutis stellatis puberulis vel subglabris, supra sulcatis. Lamina 15–20 cm longa, ovato-oblonga, sursum in apicem pinnatifidum sensim attenuata, membranacea vel firmo-herbacea, atro-viridi. Rachi dense pilosis; pilis simplicibus longioribus pilis minutis stellatis intermixtis. Pinnis suboppositis, sessilibus, basalibus reflexis vix abbreviatis, medialibus horizontalibus apicibus curvatim ascendentibus, 3–5 cm longis, 1–1½ cm latis, basi superiore distincte auriculatis, acuminatis, grosse lobatis, dense ciliatis, supra pilis simplicibus adpressis subdense strigosis, subtus ubique (praesertim ad costas venasque) pilis simplicibus pubescentibus. Lobis contiguis, falcatis, obtusis, basali anteriore duplo majore, subacuto. Venis simplicibus, 5–6-jugis, basalibus aut anastomosantibus aut ad sinum conniventibus. Soris inframedialibus, parvis; indusiis parvis, dense setosis, mox deciduis. Sporangiiis pilis longioribus maxime in pedicellis sedentibus intermixtis.

I am convinced that this new species is specifically distinct from the Andine *D. Jamesoni*, with which BAKER united it. It is true that the two species agree as to almost all characters, still our new species is larger, has a larger number of veins, the basal pinnæ scarcely reduced, and the lamina is generally fully pinnate at the lower two-thirds.

224. *Dryopteris macrotis* (Hook.) O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 276.

Syn. *Nephrodium macrotis* Hook. spec. 4: 86 tab. 242 B. 1862.

Type from Peru: Tarapoto. SPRUCE nr. 3979 (auth. specimens in RB and L.).

The most developed form of a small group, which includes also the three preceding species, characterized by the strigose upper surface of the lamina tomentose rachis and the reflexed lower pinnæ which like the upper ones are distinctly auricled at the upper base and setose indusia. *D. macrotis* differs from the three other species by its size; stipe and lamina 30–40 cm each, pinnæ 10–15 cm long by $1\frac{1}{2}$ –2 cm broad, the lower 2–3 pairs much reflexed, both surfaces adpressedly strigose, rachis apparently without stellate hairs, in some specimens proliferous like several other species of *Goniopteris*; veins 5–6-jugate, very ascending, free and connivent or not reaching the sinus, the basal ones sometimes truly united. ROSENSTOCK has named the forms with anastomosing veins var. *nephrodioides* in Fedde, Repert. 7: 298. 1909, based on SPRUCE nr. 4658 from Peru Mt. Campana (RB!); the same character is found in specimens of SPRUCE nr. 3979.

225. *Dryopteris serrulata* (Sw.) C. Chr. Ind. 292. 1905; Maxon, Contr. U. S. Nat. Herb. 10: 491. 1908.

Syn. *Polypodium serrulatum* Sw. Schrad. Journ. 1800²: 25. 1801; Fl. Ind. occ. 1663. (C. Chr. Arkiv för Bot. 9¹¹: 34 fig. 8 et t. 5 fig. 1; photograph of the type-specimen).

Aspidium serrulatum Mett. Aspid. nr. 252. 1858.

Nephrodium serrulatum Jenm. Bull. Dept. Jamaica II. 3: 189. 1896.

? *Polypodium Lunanianum* Hew. Mag. Nat. Hist. II. 2: 460. 1838 (t. JENMAN).

? *Polypodium Smithianum* Hew. l. c. 459 (t. Mett. msc.).

Type from Jamaica, leg. SWARTZ (S!)

A well-marked species, distinguished from related species by its pinnæ being broadly serrate only, not pinnatifid, and its inframedial sori. The pinnæ resemble in cutting those of *D. pyramidata*, from which it differs by its lower pinnæ being reduced, by its sessile or subsessile pinnæ and by its few veins. The whole leaf is practically glabrous, the rachis and costæ beneath alone slightly puberulous by minute stellate hairs. Most pinnæ with truncate base, or the lower ones a little narrowed and often auricled. Lobes generally broader than long, often emarginate. Veins 4–5-jugate, the lower 3–4 curved up to the sinus and the basal pair normally united and sending a branch to the sinus. Sori very often confined to the lower veins (still in some specimens all veins are soriferous), distinctly inframedial, often close to the costa. Indusium small, deciduous, furnished with a few furcate hairs. Sporangia glabrous. — Generally the lower 2–3 pairs of pinnæ are gradually shortened, but this is not the case in all specimens. The species varies considerably in size. The largest specimens measure: stipe 30–40 cm, lamina 8 cm,

pinnae 10—13 cm long by $1\frac{1}{2}$ —2 cm broad. The erect rhizome is at the top clothed with many castaneous, acuminate scales.

MAXON (loc. cit.) says that *P. serrulatum* Sw. was founded upon SLOANE's pl. 43 fig. 1 and that *P. asplenioides* Sw., founded upon SLOANE's pl. 43 fig. 2 is the same species. As I have shown in my paper on SWARTZ's species of ferns, SWARTZ has never founded a species upon figures alone but always described them after specimens. The two named Swartzian species were described after specimens collected by SWARTZ himself in Jamaica and MAXON is, therefore, not right in identifying the two species from an examination of the figures quoted only.

D. serrulata seems to be endemic in Jamaica, where a beautiful series of specimens was collected by MAXON (numbers quoted by MAXON loc. cit.), UNDERWOOD, HART and JENMAN (W).

226. *Dryopteris anoptera* (Kze.) C. Chr. Ind. 252. 1905 (excl. syn.). — Fig 31 d.

Syn. *Aspidium anopterum* Kze.; Kuhn, Linnaea 36: 113. 1869 (excl. syn.).

Nephrodium nitidulum Bak. Fl. bras. 1²: 597. 1870; Syn. Fil. 502. (excl. syn.).

Dryopteris nitidula O. Ktze. Rev. 2: 813. 1391.

Goniopteris hastata Fée, Cr. vasc. Br. 1: 107 tab. 33 fig. 2 (non 11 mém. 1866).

Goniopteris bahiensis Fée, l. c. 2: 61. 1872—73.

Type from Brazil: Bahia, leg. MORICAND (B!, RB); RIEDEL (W).

An imperfectly known species, confounded with *Aspidium catacolumbum* Kze. and *A. nitidulum* Kze. figured by ETtingshausen, which no doubt belong to *D. lugubris*. The type-specimen belongs to a species closely related to *D. serrulata*, while other specimens from the same locality, often determined as *D. anoptera*, seem to me to belong to *D. pyramidata*. In size, texture and cutting *D. anoptera*, as understood here, does not at all differ from *D. serrulata*, but it differs by the presence of very small and few scales on the costae beneath, by its more numerous veins, 7—8 to a side, the lower ones truly united, and by its sporangia being setose by bi- or trifurcate hairs. — I have no doubt that *Gon. hastata* Fée (*G. bahiensis* Fée) is this, although the plate shows a plant with a distinct terminal pinna.

227. *Dryopteris hastata* (Fée) Urban, Symb. Antill. 4: 21. 1903; C. Chr. Ind. 269.

Syn. *Goniopteris hastata* Fée, 11 mém. 65. tab. 18 fig. 1. 1866.

Type from Guadeloupe, leg. L'HERMINIER (Herb. Cosson, Paris!; B).

Rhizome short-creeping or decumbent, like the lower part of the stipes with some stellato-pilose scales. Stipe and rachis shortly puberulous by forked and simple hairs, sometimes nearly glabrous. Lamina up to 5 dcm long, but generally much shorter, pinnate in the lower half or two-thirds suddenly narrowed into a long, broad, pinnatifid or lobed apex, herbaceous, strigose on the costae above, slightly puberulous by furcate and simple hairs on costae and veins beneath,

ciliate, dark-green, often glossy. Pinnæ few, 4—8-jugate, 8—10 cm long (or shorter), 2 cm broad, broadest at the middle, the basal ones not or a little shorter, not reflexed, nearly sessile, the upper ones adnate, at least confluent, the uppermost pair shorter and form like a hastate base of the pinnatifid or lobed apex of the lamina, which is 1—2 dm long by 3—4 cm broad and often exceeds in length the pinnate lower part of the lamina. Pinnæ broadly serrate or even crenate only, the teeth broader than long, oblique, bluntly rounded or their outer margin straight. Veins about 6 to each side, the lower 2—3 pairs united alternately into a flexuose branch running to the sinus, or sometimes the lower pair is meniscioid; often some of the following do not reach the excurrent branch but end in the leaf-tissue. Sori inframedial, exindusiate. Sporangia furnished with bi- or trifurcate hairs.

In several characters, especially in venation and pubescence, *D. hastata* resembles *D. obliterated*, to which species BAKER (Icon. plant. t. 1669) referred it, but it differs considerably by the shape of the apex of its lamina and by its setose sporangia. By these two characters connected it is also different from allied species with a pinnatifid apex (*D. serrulata* and others).

Guadeloupe: L'HERMINIER nr. 130 (B, herb. Mus. Paris).

Porto Rico: SENTENIS nr. 5819 (B, C, W), 6268 b, 6668 (B); G. P. GOLL nr. 133, 862 (W); A. A. HELLER nr. 6200 (W); UNDERWOOD and GRIGGS nr. 869 (W).

Tobago: BROADWAY nr. 3432 (B).

Trinidad: HART nr. 561 (W).

var. *leptocladia* (Fée).

Syn. *Goniopteris leptocladia* Fée 11. mém. 63 tab. 16 fig. 3. 1866.

Differs from the type, with which it agrees in pubescence and setose sporangia, by its more numerous pinnæ (10—12 pairs), which are more deeply cut into sub-acute lobes generally longer than broad; the pinnatifid apex is proportionally shorter than that of the type; base of pinnæ as a rule subcordate; only the two, rarely three lower veins united.

Guadeloupe: L'HERMINIER nr. 131 (B, C, authentic specimens); Père DUSS nr. 234 (C), 4115 a (W).

Martinique: Père DUSS nr. 4146 (RB).

Porto Rico: UNDERWOOD and GRIGGS nr. 888 (W; form. approaching the following variety).

var. *sub-auriculata* Kuhn in sched.

Syn *Aspidium asplenioides* f. *exindusiatum* Kuhn; Krug, Engl. Jahrb. 24: 119. 1897.

Dryopteris asplenioides f. *exindusiata* Urb. Symb. Antill. 4: 18. 1903.

Likely a new species, but in pubescence, setose sporangia and long pinnatifid apex agreeing with *D. hastata*, from which it differs by its large number of much smaller pinnæ, 4—8 cm long, 1 cm broad, the lower ones rather reduced, subsessile, the middle ones adnate to rachis, obtuse auricled on both sides of the base, the upper ones confluent, crenate or shallowly and bluntly lobed. Veins

4—5-jugate, the lowermost pair anastomosing. In size, colour and venation it resembles very much *D. asplenioides*, which, however, is indusiate and pinnate to short of the apex with most of the pinnæ stalked; in our variety long simple hairs are few or none on rachis and costæ beneath, which are stellato-puberulous, in *D. asplenioides* long hairs as a rule are more numerous than the stellate ones.

Haiti: Port au Prince, PICARDA nr. 386 (C), 734 (B) — prope Mariani, PICARDA nr. 377 (B) — San Domingo, ad Rio Mameges, EGGERS nr. 2656 (B, C) — v. TUERCKHEIM nr. 2576 (B).

Porto Rico: SENTENIS nr. 5661 (B, C, CC); GOLL nr. 1016 (W).

228. *Dryopteris pyramidata* (Fée) Maxon, Contr. U. S. Nat. Herb. 10: 489. 1908.

Syn. *Goniopteris pyramidata* Fée, 11 mém. 61. tab. 16 fig 2. 1866.

Dryopteris latiuscula Maxon. Contr. U. S. Nat. Herb. 10: 498. 1908.

Nephrodium subcuneatum Bak. Flor. bras. 1²: 487. 1870; Syn. Fil. 503.

Jenman, W. Ind. and Guiana Ferns 234. 1908!

Dryopteris subcuneata O. Ktze. Rev. 2: 813. 1891; C. Chr. Ind. 295.

"*Nephrodium subovatum* Jenman, "Argosy", Demerara". (t. JENMAN).

Type from Guadeloupe, leg. L'HERMINIER (Herb. Cosson, Paris! and authentic specimens in B and C).

A most distinct and uniform species, excellently described by JENMAN and MAXON (loc. cit.). It resembles in size, colour and texture *D. tetragona*, from which it can be distinguished at once by the lamina being gradually narrowed upwards and by its venation. From the allied species without terminal pinnæ it differs by its most pinnæ being distinctly stalked. The lamina is glabrous, the rachis and costæ beneath excepted, which are puberulous by very minute forked or simple hairs. Lower pinnæ narrowed towards the base, generally not reflexed or abbreviated, upper ones with a truncate base, all scarcely incised one third to the costa. Lobes scarcely longer than broad, truncate and often emarginate at the apex. Veins 8—10-jugate the lower 3—4 much curved, connivent to sinus, or the lower pair occasionally united and sending a branch to the sinus; often the anterior basal vein ends in the leaf-tissue, not reaching the opposite posterior one. Sori medial, furnished with a small, ciliate indusium; head of sporangium glabrous, but its pedicel bears normally a stiff hair. — Rhizome obliquely erect or decumbent; stipe 40—60 cm long, lamina 35 cm \times 20 cm; pinnæ 12 \times 2½ cm. Rachis is sometimes gemmiferous in the upper part.

D. latiuscula Maxon is exactly typical and *N. subcuneatum* Bak. is the same. The specimen from S. Domingo referred by MAXON to this species and to which the combination *Dryopteris pyramidata* first was applied, is not so typical, still scarcely different. It has the rachis and costæ beneath clothed with numerous long, whitish hairs, and its pinnæ are incised about halfway to the costa. — *D. pyramidata* seems to be a rather common species in Guiana and the southern West-Indian islands. I have examined the following specimens:

Guadeloupe: L'HERMINIER (B, C), PÈRE DUSS nr. 4054, 4116 (W), MAZÉ nr. 989 (B).
 Martinique: PÈRE DUSS nr. 4757 (W).
 St. Vincent: H. et G. SMITH, nr. 89 (B, W), nr. 785 (W).
 Grenada: R. V. SHERRING (W), BROADWAY nr. 3757, 3761, 3763, 3764, 3768 (RB).
 Trinidad: FENDLER nr. 54 (B), BROADWAY nr. 3292 (RB).
 British Guiana: JENMAN (W) — Lower Orinoco, Eleanor Creek, RUSBY and R. W. SQUIRES nr. 128 (W)
 — French Guiana: LEPRIEUR (B, C, W, Kew = *N. subcuneatum*), SAGOT (B).
 Surinam in savannis: A. KAPPLER, ed. HOHENACKER nr. 1776 (B) — HOSTMANN nr. 15 (B), 465 (B, S).
 San Domingo: WRIGHT, PARRY and BRUMMEL nr. 12 (W).
 Bahia: LUSCHNATH nr. 167 (B), BLANCHET (L).

229. *Dryopteris magdalenica* Hieron. Hedwigia 46: 325 tab. 3 fig 2. 1907.

Type from Colombia, ad Rio Magdalena, leg. STÜBEL nr. 371 (B!).

A small species, founded on a single leaf and perhaps not different from the following, from which it recedes by such small differences as the distinctly cuneate base of the lower pinnæ and the more oblique or subfalcate and not very obtuse segments. The rachis is very slightly stellato-pubescent, and the young sporangia are setose. The rachis is gemmiferous below the apex (not figured by HIERONYMUS). The stipe is short, shorter than the lamina, and the basal pinnæ not much reflexed.

230. *Dryopteris paucijuga* (Kl.) C. Chr.

Biolog. Arbejder tilegn. Eug. Warming 84. 1911. — Fig. 31 c.

Syn. *Aspidium paucijugum* Klotzsch, Linnaea 20: 368. 1847.

Nephrodium deflexum J. Sm. (Hk. Bak. Syn. 292, note under *N. refractum*);

Jenman, W. Ind. and Guiana Ferns 234. 1908.

Dryopteris johnstoni Maxon, Contrib. U. S. Nat. Herb. 10: 489. 1908.

Type from Venezuela: Caripe, MORITZ nr. 205 (B!).

A distinct species, excellently described by JENMAN and MAXON (loc. cit.) It is well-marked by its long stipe, which is considerably longer than the lamina, its sessile mostly opposite pinnæ, the lower ones much reflexed, their base truncate or, in the lower ones, more or less narrowed. Lamina practically glabrous, the rachis upwards setose and slightly stellato-pubescent above, sometimes with a bud in the upper part. Pinnæ scarcely incised to the middle into bluntly rounded or even truncate, a little oblique lobes. Veins 8–10 to each side the lower 2–3 much upcurved and running out to the sinus. Sori medial with a setose indusium. Sporangia with a single long hair on the pedicel.

Venezuela: Island of Margarita, J. R. JOHNSTON nr. 192 (W, type of *D. Johnstoni* Maxon) — Caracas, GOLLMER (B).

Trinidad: St. Ann's Hill, HART nr. 316 (B) — JENMAN (W) — FENDLER nr. 54 (W).

Tobago: EGGERS nr. 5628 (B, W), 5807 (B).

Mexico: Chiapas, Ixtacomitan Mts, 120 m, J. M. ROVIROSA nr. 51 (W).

Honduras: Rio Permejo, Depart. Santa Bárbara, 600', C. THIEME ed. DONN. SMITH nr. 5968 (W).

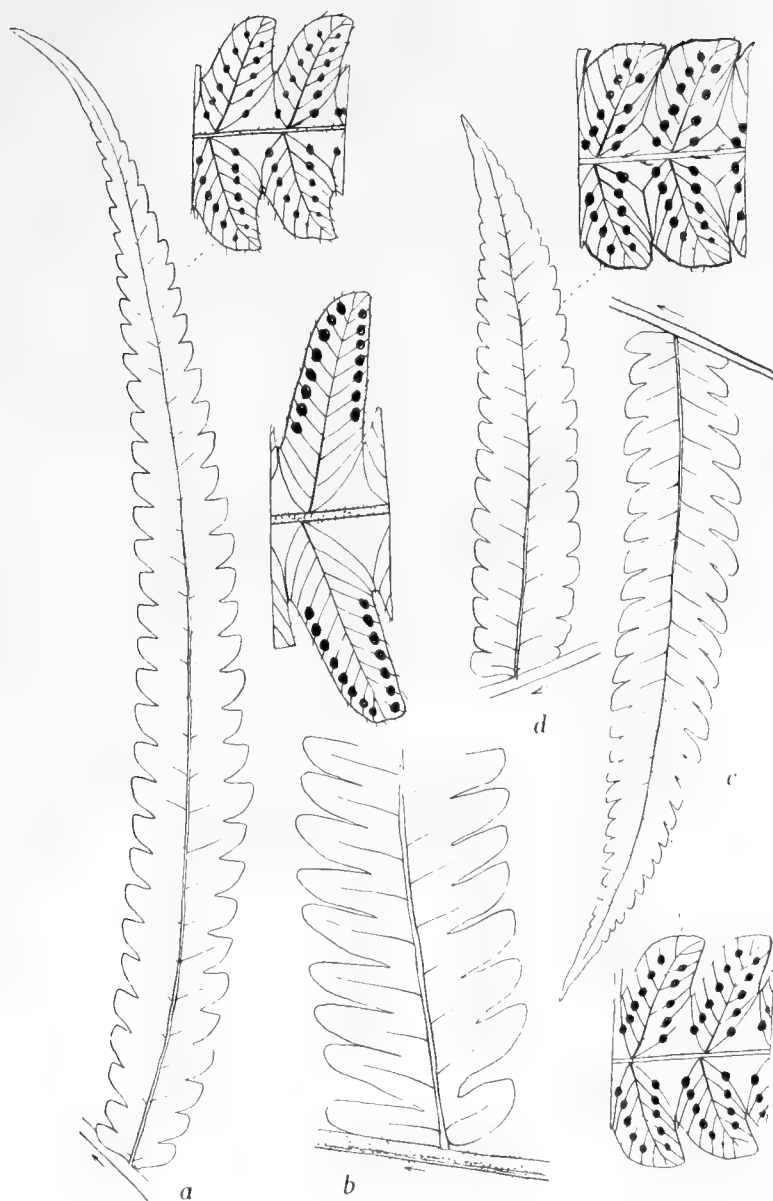


Fig. 31. *a. D. usitata* (Jenm.) C. Chr.; middle pinna (MAXON 1823). *b. D. venusta* (Hew.) O. Ktze., base of middle pinna (MAXON 1954). *c. D. paucijuga* (Kl.) C. Chr. (FENDLER 54) — *d. D. anoptera* (Kze.) C. Chr. (orig.) — All pinnae $\times \frac{1}{12}$, fragments $\times \frac{1}{12}$.

231. *Dryopteris refracta* (Fisch. et Mey.) O. Ktze. Rev. 2: 813.
1891; C. Chr. Ind. 288 pt.

Syn. *Polypodium refractum* Fisch. et Mey.; Kze. Linnaea 23: 283, 321. 1850.

Aspidium refractum A. Br. Ind. sem. ht. Berol. 1856. Mett. Aspid. nr. 238.

Nephrodium refractum Hk. sp. 4: 162 tab. 252. 1862; Bak. Syn. 292.

Type. Founded on cultivated specimens. The species was commonly cultivated in the botanical gardens in the middle of the last century, and I have seen several authentical specimens from various herbaria. It is supposed to be a Brazilian species, but I have seen no specimens from Brazil. Generally it is confounded with *D. riograndensis* (Lindm.), which species is, however, widely different. BAKER (Syn. Fil. 292) supposed *N. deflexum* J. Sm., which is our *D. paucijuga* (Kl.), to be the same, and I am inclined to believe that he was right in this. As a fact *D. refracta* is closely allied to *D. paucijuga*, still there are some differences between the two species, which do not allow me to unite them. In *D. refracta* the lower pair of veins are as a rule truly anastomosing and the 2—3 following veins are connivent to sinus. The peculiar shape of the base of the lower reflexed pinnæ is fairly illustrated on HOOKER's plate 252. Sporangia without setæ on the pedicel.

232. *Dryopteris gemmulifera* Hieron. Hedwigia 46: 326 tab. 4 fig. 3. 1907.

Syn. *Aspidium* (resp. *Nephrodium*) *tetragonum* auctt. plur. quoad pl. andin.

Type from Venezuela: Tovar, leg. MORITZ nr. 204 pro parte (B!).

Closely related to *D. scabra* and *D. paucijuga*, but very characteristic by its lamina narrowing very gradually almost from the very base to apex. It is less cut than *D. scabra*, more than *D. paucijuga*; generally the three lower veins run to sinus. It is also larger: stipe 30—40 cm, lamina up to 60 cm. The rachis and costæ excepted the leaf is entirely glabrous, even not ciliate; the hairs of the rachis and costæ beneath are partly simple and longer, partly very short and stellate.

Colombia: STÜBEL nr. 366, 554, 625 a, 642, 666 (B, for exact localities see HIERONYMUS l. c.); ØRSTED (H) — Santa Marta, H. H. SMITH nr. 994 (C, Rg, W).

233. *Dryopteris usitata* (Jenm.) C. Chr. Index 299. 1905. — Fig. 31 a.

Syn. *Nephrodium usitatum* Jenman, Journ. Bot. 1879: 261. Bull. Dept. Jam. II.
3: 188. 1896; W. Ind. and Guiana Ferns 232.

Type from Jamaica, leg. JENMAN (non vidi).

Scarcely different from *D. venusta*, and probably it is a form of it with narrower pinnæ. Its best distinguishing marks are: the long-acuminate apex of the lamina and pinnæ, its herbaceous texture, its numerous (20—30 to each side) long and narrow pinnæ, which are up to 20 cm long by 1½—2 cm broad, sessile, the lower ones generally shorter, incised 1:3 or more to the costæ into broad, blunt oblique lobes, which are a little longer than broad. Both surfaces naked or very

slightly hairy on rachis and costæ by minute stellate and a few longer, simple hairs. Veins 6—8-jugate, the lower ones connivent to sinus or more often united into a branch to the sinus. Sori about medial; indusium small, soon evanished. — The erect, thick rhizome is clothed with dull-brown ovate scales and a few small fibrils are found on the stem and the lower part of the rachis.

To this species I refer some few specimens from

Jamaica: HART nr. 315 (W) — Mansfield, MAXON nr. 1823 (= UNDERWOOD nr. 2789 (W) — Bath, MAXON nr. 1866 (= UNDERWOOD nr. 2826) (W) — Cascade Portland, D. WATT nr. 227 (RB).

From this species I cannot with certainty distinguish *Nephrodium calcareum* Jenm. Journ. Bot. 1886: 271. Bull. Dept. Jam. II. 3: 162. 1896, Jamaica, SHERRING (Kew, auth. specimen), HART 344 (W). It has the lower pinnæ more distinctly reduced, the most pinnæ are somewhat auricled on both sides and the rachis and costæ beneath more densely stellate-puberulous; the pinnæ are shorter and like the apex of the lamina not so long-acuminate.

234. *Dryopteris venusta* (Hew.) O. Ktze. Rev. 2: 814. 1891.

C. Chr. Ind. 300. — Fig. 31 b.

Syn. *Aspidium venustum* Hew. Mag. Nat. Hist. II. 2: 464. 1838.

Nephrodium venustum Moore, Gard. Chr. 1855: 677 c. fig.; Bak. Syn. 294; Jenman. Bull. Dept. Jamaica II. 3: 188. 1896; W. Ind. and Guiana Ferns 233.

Type from Jamaica (non vidi).

A handsome species, in its most developed form well-marked by its size, submarginal sori and glabrous indusium. It resembles not a little *D. Fendleri*, but it is easily distinguished from that species by its pinnatifid apex. — The thick erect rhizome is clothed with large, brown scales. Stipe strong, quadrangular, up to 50 cm long, slightly scaly like the lower part of rachis. Lamina 7—9 dcm long, gradually narrowed into a pinnatifid apex, downwards a little reduced. Rachis minutely puberulous by stellate hairs. Pinnæ 20 or more to each side, up to 25 cm long by 4 cm broad, incised halfway to the costa or more, sessile, or the lower ones very shortly stalked, acuminate, the lower generally more or less shortened; surfaces glabrous except the costæ beneath, which are minutely stellato-puberulous, the margins ciliate. Texture firm, membranous, colour dark-green; underside often minutely warted. Lobes broad, oblique, subacute or obtuse. Veins about 10-jugate, the basal pair connivent to sinus or frequently united into a branch to sinus. Sori supramedial or submarginal, furnished with a large, persistent, glabrous indusium. Sporangia glabrous.

Known from Jamaica only; I have seen the following specimens: Mt. Diabolo, MAXON nr. 1855, 1933, 1954, 2318 (W).

235. *Dryopteris riograndensis* (Lindm.) C. Chr. Ind. 289. 1905.

Syn. *Polypodium riograndense* Lindm. Ark. för Bot. 1: 230 tab. 3 fig. 6. 1903.

Aspidium refractum Hieron. Engl. Jahrb. 22: 374. 1906 (non A Br).

Nephrodium refractum auctt. plur. quoad pl. brasil.

Dryopteris refracta Ros. Hedwigia 46: 131. 1907.

Type from Brazil: Minas Geraes, Caldas, leg. MOSÉN nr. 2170 (S! Rg).

A very uniform, small species, which unrightly has been identified with *Aspidium refractum* A. Br. It is however certainly a near ally of *D. scabra*, from which it differs by the pinnæ being serrate only and the lower pair of veins being constantly anastomosing. The rachis is above rather densely stellato-puberulous, otherwise the leaf is almost quite glabrous; sometimes the costæ beneath bear some white hairs. The rhizome is decumbent or shortly creeping, a little scaly. Stipes 1–2 dcm long, stramineous like the rachis and costæ. Lamina 12–20 cm long, scarcely narrowed downwards, rather suddenly narrowed into a pinnatifid apex upwards. Pinnæ all sessile, opposite, 5–7 cm long, 1 cm broad, dark-green, herbaceous, the margins only serrate, rarely subentire or lobed. Veins about 3 to a side, the basal ones united or running side by side to the sinus. Sori medial, exindusiate. Sporangia glabrous. — The rachis is often gemmiferous.

Evidently a common species in South Brazil and adjacent countries. I have the following specimens:

Brazil: Minas Geraes, Caldas, MOSÉN nr. 2170, 4636 (Rg) — *Sla. Catharina*, PABST nr. 798 (B) — Rio Grande do Sul: Piratiny, LINDMAN nr. A. 865 (Rg); Cachoeira, MALME nr. 984 (Rg); LINDMAN nr. 1167 a; Silveira Martins, LINDMAN nr. 1167 b (Rg). — Further the specimens quoted by ROSENSTOCK loc. cit. and distributed by him as *Nephrodium refractum*, Fil. austr. bras. exs. nr. 84 and 250 (B, C, Rg, W).

Uruguay: Puerto del tanze, BERRO nr. 1260 (C) — Isla San Gabriel, BERRO nr. 2367 (C, CC).

Argentina: Misiones, Bonpland, EKMAN nr. 8 (Rg, S) — Entrerios, Arroyo Yucharichico, LORENTZ nr. 797 (B).

Paraguay: Coaguzu, BALANSA nr. 310 (CC).

236. *Dryopteris scabra* (Presl) C. Chr. Biolog. Arbejder tilegn. Eug. Warming 84. 1911 — Fig. 32.

Syn. *Polypodium scabrum* Presl, Del. Prag. 1: 169. 1822.

Lastrea scabra Presl, Epim. bot. 41. 1849.

Polypodium tetragonum auctt. quoad plant. austro-brasil.

Nephrodium tetragonum auctt. quoad plant. brasil. (Bak. Fil. bras.;

Syn. Fil. etc.).

Dryopteris pseudotetragona Rosenst. Hedw. 46: 119. 1907.

Type from Brazil, ad Mathias Ramos, leg. POHL (hb. PRESL!).

Rhizome creeping, clothed at the apex with brown, stellato-pilose scales. Stipe greyish-yellow, bisulcate, shortly pubescent, 20 cm long. Lamina ovato-lanceolate, 20–25 cm long, 15 cm broad, dark-green, firmly herbaceous, upwards rather suddenly narrowed into a pinnatifid apex and generally proliferous at the base of the upper pinnæ. Rachis slender, rather densely and shortly hairy by simple and stellate hairs, which latter mostly occur on the upperside of the upper part of the

rachis. Pinnæ opposite or nearly so, sessile 8—9 cm long, 1½—2 cm broad acuminate, the basal pair deflexed and narrowed towards their base, the upper ones divaricating and broadest at the base, ciliate, the costæ above strigose, the upper-side glabrous or very slightly and shortly hairy, costæ and costulæ beneath with patent, simple hairs, the underside otherwise nearly glabrous. Segments oblique or subfalcate, obtuse or subacute, the lower ones of the basal pinnæ short. Veins simple, 10—11-jugate, the lower 2 running to the sinus, where their apices are separated by a pale membrane. Sori medial, with a pilose, persistent indusium. Sporangia glabrous.

The diagnosis above is of the original specimen. With this agree a large number of specimens from Southern Brazil in almost all characters, although I have not seen another specimen, which exactly matches the original one, but it is sure that all these specimens belong to the same species, which is evidently very common from Rio to Rio Grande. Several of the specimens were labelled *Nephrodium tetragonum* Hk. resp. *Dryopteris pseudotetragona* Urb., others *Polypodium tetragonum*. From the andine form of the former as previously understood, now *D. gemmulifera* Hieron., *D. scabra* differs by the short pinnatifid apex of the lamina, from the latter, which it resembles in habit and colour, by its free veins, sessile pinnæ and its pinnatifid apex without a distinct terminal pinna.

D. scabra is a very variable species; the specimens seen can be grouped as follows:
var. **incompleta** (Lindm.)

Syn. *Polypodium tetragonum* var. *incompletum* Lindm. Ark. för Bot. 1: 229 tab. 9 fig. 7. 1903.

Dryopteris pseudotetragona var. *foecunda* Rosenst. Hedwigia 46: 119. 1906.



Fig 32. *D. scabra* (Pr.) C. Chr. leg. POHL. Basal pinnæ, middle pinna and apex of leaf $\times \frac{4}{5}$; segments seen from both surfaces $\times 1\frac{1}{2}$.

Identical with the type in most characters, differing by a longer stem (up to 5 dcm long), the very slender rachis, which is practically quite glabrous (only stellate hairs above), and the almost glabrous leaf. Sori yellow, apparently exindusiate. Rachis as a rule with gemmæ and in some specimens gemmæ also on the costæ of the pinnæ above.

Rio Grande do Sul: Silveira Martins, LINDMAN nr. A 1311 (Rg, W) — Campeste dos Sairaes, C. JÜRGENS, ROSENSTOCK: Fil. austr. bras. exs. nr. 375 (B, R, Rg) — Sta. Cruz, JÜRGENS nr. 200 (R); *ibid.*, ROSENSTOCK: Fil. austr. bras. exs. nr. 86 (B, R) — Porto Alegre, MALME nr. 1441 (Rg). São Paulo: Rio Tietê, GERDER nr. 82 (R).

To this variety I also refer the majority of the specimens by ROSENSTOCK named *D. pseudotetragona* var. *gemmulifera* Hier. with the forma *major* (Hedwigia 46: 119). Some of them are larger and of a more firm texture and a greyish colour, but they do not differ materially probably; they are old stages of the common form.

var. **Caesariana** (Christ).

Syn. *Aspidium Caesarianum* Christ, Denkschr. Akad. Wien 69: 14. (1906) 1907 tab. 3 fig. 1.

Differs from the type and the var. *incompleta* by the less incised pinnæ, segments close, acute; 2 pairs of veins connivent to sinus. Resembles in habit perfectly *D. lugubris* var. *devolvens* from which it differs in pubescence.

São Paulo: Cerqueira Cesar, 500 m, WETTSTEIN & SCHIFFNER (hb. Wien!) — Tietê, GERDER nr. 85 a (R). Rio Grande do Sul: Sta. Cruz, JÜRGENS, ed. ROSENSTOCK: Fil. austr. bras. exs. nr. 87 (R, Rg).

With this variety agrees closely an authentical specimen of *Dryopteris bifrons* Christ in Fedde, Repert. 6: 350. 1909, which was collected by ROJAS in Paraguay, ed. HASSLER nr. 10455 (RB!). The indusium is very small and bears some few long stiff setæ. Another specimen from Paraguay: Cordillera de Altos, FIEBRIG nr. 95 (B) is the same. It is a large form, leaf said to reach 1—1,2 m, and the rachis is richly bulbilliferous, but it does not differ materially from *D. scabra* var. *Caesariana*.

237. **Dryopteris monosora** (Pr.) C. Chr. Biolog. Arbejder tilegn. Eug. Warming 84. 1911. — Fig. 33.

Syn. *Polypodium monosorum* Presl, Tent. 181. 1836 (nomen).

Lastrea monosora Presl, Epim. 36. 1849.

Aspidium monostichum Kze.; Mett. Aspid. nr. 228. 1858.

Dryopteris tristis part. C. Chr. Ind. 298. 1905.

Type from Rio leg. POHL (Herb. PRESL!).

A distinct species, improperly referred to *D. tristis* by BAKER in Flor. bras. and Syn. Fil. It resembles *D. tristis* in the cuneate base of the lower pinnæ, but differs by its creeping rhizome, pinnatifid apex and by the presence of small scales on

rachis and costæ beneath. It is excellently described by METTENIUS and I add here only the following comparative remarks.

The short-creeping rhizome is at the apex rather densely clothed with ovate, brown, stellato-pilose scales. The strong stipes are up to 4 dcm long, and like the rachis and costæ beneath furnished with some small, brown scales. The rachis is minutely pulverulent by stellate hairs, and the costæ of both sides bear a few long hairs, otherwise the lamina is practically glabrous. Lamina 6—7 dcm long, 2—3 dcm broad, upwards rather suddenly narrowed into a pinnatifid apex. Pinnæ 15 cm long, $2\frac{1}{2}$ —3 cm broad, the lower ones shortly stalked and their base distinctly cuneate but shorter than in *D. tristis*, the upper ones sessile with truncate base, their basal segments somewhat abbreviated. Pinnæ incised a little above the middle into close, subfalcate, acute or often submucronate segments, 5 mm broad. Veins simple, 9—10-jugate, the basal ones connivent to sinus. Sori medial, furnished with a small, deciduous, ciliate indusium. — Texture membranous, colour dark- or greyish green, generally brown when dried, surfaces often shining.

The nearest Brazilian relative of *D. monosora* is *D. scabra*, which is smaller and has all the pinnæ sessile, their base not cuneate, entire; further it has fewer veins, of which only the basal pair run to sinus.

D. monosora seems to be restricted to the forests of Rio and São Paulo; it occurs in two forms:

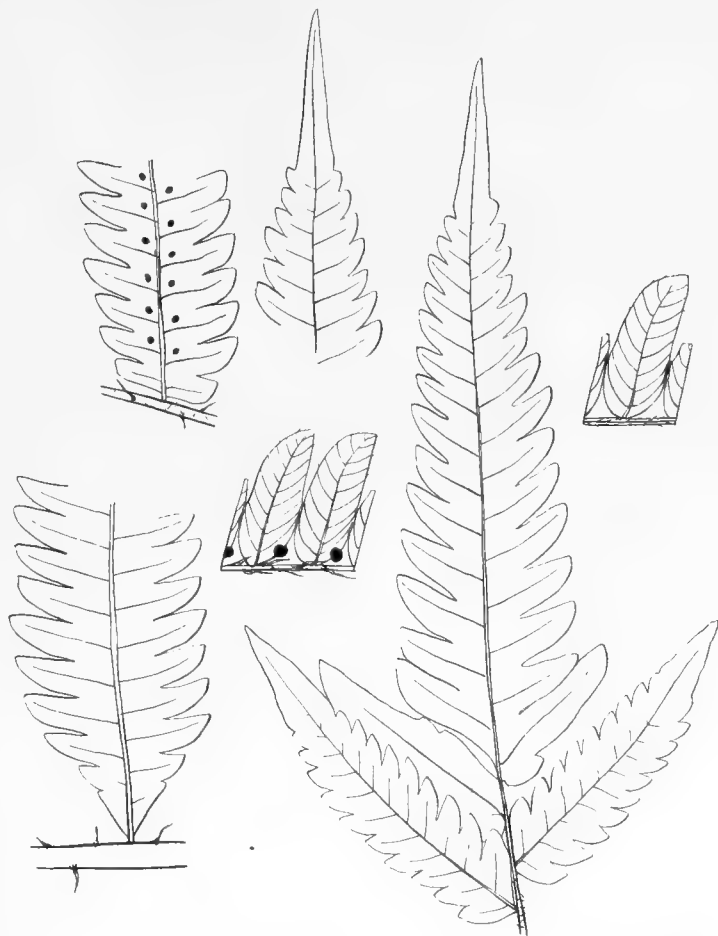


Fig. 33. *D. monosora* (Pr.) C. Chr., leg. POHL. Base of basal pinna, base and apex of a middle pinna and apex of a leaf $\times \frac{4}{5}$; segments seen from both surfaces $\times 1\frac{1}{2}$.

1. f. *typica*.

Rachis without buds; veins 9—10-jugate, the lower 4—5 connivent to sinus; only the anterior basal vein soriferous. Segments generally more acute than in the following form.

Hereto the type-specimen and the following: São Paulo: Rais da Serra, WACKET nr. 224 (R) — Ins. S. Sebastian, CASARETTA nr. 130 (Hb. PRESL; B).

2. var. *Schiffneri* n. var. (*D. Schiffneri* mihi in Herb. plur.).

Rachis often gemmiferous. Veins 13—15-jugate; the lower 3—4 connivent to sinus, about all soriferous. Segments often with rounded apices. Costæ less scaly.

I had described this variety as a new species, but I now consider it the fully developed form of *D. monosora*.

São Paulo: prope Rio Grande ad São Paulo Railway, 800 m., WETTSTEIN et SCHIFFNER nr. 462 (Herb. Mus. Wien) — prope Fazenda bella vista ad flumen Rio Pardo, c. 500 m, WETTSTEIN & SCHIFFNER VII. 1901 (Herb. Wien) — Santos, MOSÉN nr. 3088 (Rg) — Rais da Serra, WACKET nr. 200 (R) — BURCHELL, Cat. pl. bras. nr. 3065 (B) — Dr. BRENNING (B).

238. *Dryopteris Eggersii* (Hieron.) C. Chr. Ind. 263. 1905. — Fig 34 a.

Syn. *Nephrodium Eggersii* Hieron. Engl. Jahrb. 34: 441. 1904.

Aspidium nutans Christ, Bull. L'Herb. Boiss. II. 6: 286. 1906.

Dryopteris nutans Christ, Bull. L'Herb. Boiss. II. 7: 261. 1907.

Nephrodium tetragonum Sod. Cr. vasc. quit. 249. 1893 (pro parte?).

Type from Ecuador, El Recreo, leg. EGGERS nr. 15319 (B!).

A weakly characterized species of the same general aspect as *D. nephrodioides*, *D. lugubris* and allied species. From *D. nephrodioides* it differs by the absence of stellate hairs on the leaf-tissue and by the presence of large, brown scales on the erect rhizome and the basal part of the stems. From *D. lugubris*, which it resembles closely, it differs by its erect rhizome, thinner texture, fewer veins and, especially, by the absence of scales on rachis and costæ beneath. A good character is also the shape of the stellate hairs of rachis; they bear on a stalk 3—4 branches, which are again forked. Lamina 7—8 dcm long with 20—25 pairs of pinnæ, 15 cm long by 2 cm broad. Rachis densely pulverulent by short stellate hairs and upwards furnished with several long, white hairs. Upper surface glabrous except the strigose costæ and ciliate margins, under-surface throughout finely pubescent by simple hairs, the costæ besides densely stellato-pulverulent. Veins 10—12-jugate, simple, the 2—3 lower ones connivent to sinus. Sori medial, the indusium subpersistent, setose by simple hairs. Sporangia glabrous or rarely furnished with a few setæ. — From *D. gemmulifera* it differs by its denser pubescence of stellate hairs, absence of gemmæ and by its lamina being shortly narrowed upwards, not gradually tapering to the point.

D. nutans Christ from Costa Rica is this species; the specimens are larger than the original one, dark-green and less pubescent beneath. *N. tetragonum* Sod.,

represented by an authentic specimen (C), is a large form of *D. Eggersii* with pinnæ 25 cm long.

Ecuador: El Recreo, EGGERS nr. 15319 (B); Llalla, SPRUCE nr. 5668 (RB); Andes quitenses SODIRO (C).

Colombia: SMIDTCHEN (B); SCHLIM (B).

Costa Rica: Navarro, WERCKLÉ (C, W); Llanuras de San Carlos, C. BRADE nr. 483 (R).

239. *Dryopteris biformata* Rosenst., Fedde, Repert. 7: 300. 1909. — Fig 34 c.

Type from Peru: Tarapoto, SPRUCE nr. 4037 (RB!).

A rather doubtful species, marked by its contracted fertile pinnæ and segments; this may be an individual character, and the species can not be said to be well understood from the original specimens alone. The very remote, stalked

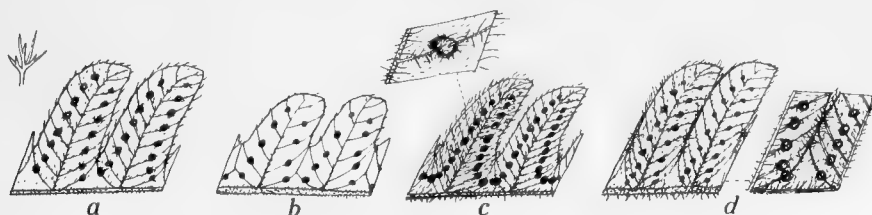


Fig. 34. Segments of a. *D. Eggersii* (Hieron.) C. Chr. (orig. of *A. nutans* Christ, with a hair from the rachis). — b. *D. equitans* (Christ) C. Chr. (orig.). — c. *D. biformata* Ros. (orig., with a fragment $\times 4$). — d. *D. lugubriiformis* Ros. (orig.) — all $\times 1\frac{1}{2}$.

pinnæ give the leaves a peculiar habit; the sterile pinnæ resemble those of *D. tristis*, but they are much more hairy and the lower ones not with a cuneate, entire base. The rachis is gemmiferous. The mature sori form a continuous line. Rachis densely stellato-pulverulent without long hairs.

240. *Dryopteris equitans* (Christ) C. Chr. comb. nov. — Fig 34 b.

Syn. *Nephrodium equitans* Christ, Bull. l'herb. Boiss II, 6: 163. 1906.

Type from Costa Rica: Navarro 1400 m, leg. WERCKLÉ (C!).

Although this species resembles *D. tetragona* in having a rather distinct terminal pinna and the basal pair of veins being truly anastomosing, it no doubt is a near ally of *D. Eggersii* and its proper place is in this section. It differs from *D. tetragona* in its erect rhizome, distinct indusium and densely pulverulent rachis and costæ beneath, from *D. Eggersii* in its anastomosing basal veins and apex of frond, from *D. nephrodioides* var. *Biolleyi* by the absence of stellate hairs on the leaf-tissue and indusium. — The dark-green, membranous pinnæ, which are up to 15 cm long, $1\frac{1}{2}$ –2 cm broad, are scarcely incised to the middle, above very slightly and shortly pubescent, the costæ and costulae beneath pulverulent by stellate hairs but without long hairs. The rachis is pulverulent by very small stellate hairs, which on a short pedicel bear 3–4 very short and generally simple branches. Lobes a little oblique, obtuse with open sinuses between. Veins 6–8-jugate, those

of the lower pair truly anastomosing and sending a branch to the sinus, where the next pair meet. Sori medial, small, furnished with a small, ciliate indusium. Sporangia glabrous. — The terminal pinna is not so distinct as in *Eugoniopteris*; it is broader and deeply lobed at base, but the basal lobes is much shorter than the uppermost pair of lateral pinnæ.

A have seen the type-specimens only.

241. *Dryopteris curta* Christ, Bull. L'Herb. Boiss. II. 7: 263. 1907. (p. p.) — Fig. 35.

Type from Costa Rica: buissons et broussailles de Tuis, 650 m, leg. A. TONDUZ nr. 11323 (Cl, W).

Habit of *D. Eggersii* and *D. scabra* but smaller and its rachis very hairy by hairs of two kinds intermixed: 1) numerous, short stellate hairs with 3—5 short, simple branches on a short stalk, and 2) rather numerous, much longer, patent hairs, which often are cleft at the apex. — Stipe 3—4 dcm long, sulcate, stellato-pulverulent and with scattered small, flat scales. Lamina membranous, 3—4 dcm long, broadest at the base, upwards gradually narrowed into a pinnatifid apex. Pinnæ all sessile, 6—7 cm long, $1\frac{1}{2}$ cm broad, the edges nearly parallel, truncate at base, shortly obtuse at the point, the basal pairs reflexed, the upper ones horizontal, the upperside throughout hairy by numerous adpressed antrorse simple hairs; costæ beneath very hairy like rachis, veins and leaf tissue finely pubescent by simple, erect hairs. Pinnæ incised scarcely more than halfway down to the midrib into close, falcate, obtuse, entire segments. Veins 8—9-jugate, simple or, not rarely, forked, the lower 2—3 connivent to sinus. Sori medial, furnished with a subpersistent, setose indusium. Sporangia glabrous.

In the most complete specimen (W) the rachis bears a small gemma.



Fig. 35. *D. curta* Christ.
Pinna $\times \frac{1}{5}$ and segments
seen from both surfaces
 $\times 1\frac{1}{2}$ (orig.).

242. *Dryopteris heterotricha* n. sp. — Fig. 36.

Syn. *Dryopteris nephrodioides* var. *setulosa* Hieron. Hedwigia 46: 327. 1907.

Type from Ecuador, Mt. Tunguragua, leg. SPRUCE nr. 5298 (H, L, RB), and gathered in the same locality by STÜBEL nr. 849 (B) and in valle Pastaza by the same nr. 931 a (B).

Rhizomate?. Stipitibus ad 1 dcm longis, quadrangularibus, brevissime stellatim pulverulentis, versus basin sparse squamosis, squamis adpressis, ferrugineis, sparse stellato-ciliatis. Lamina 6—7 dcm longa, bipinnatifida, versus apicem pinnatifidum sensim attenuata, herbacea, atroviridi. Rachi pilis stellatis brevissimis dense pulverulenta et pilis cylindricis, crassis, rufis, nitidis apicibus sæpe furcatis onusta,

ad basin pinnarum superiorum interdum bulbifera. Pinnis remotis, alternis, subhorizontalibus, inferioribus breviter petiolulatis, superioribus sessilibus, oblongis, 10–14 cm longis, $2\frac{1}{2}$ cm latis, longe acuminatis, inferioribus vix brevioribus versus basin breviter attenuatis, superioribus basi truncatis, supra ubique pilis adpressis simplicibus deciduis sparse strigosis, ad costas pilis cylindricis rufis dense setosis, infra inter venas glabris, ad costas costulasque pilis similibus pilis stellatis brevissimis intermixtis hirtis, ad alam 4–5 mm pinnatifidis. Laciniis parum obliquis, obtusis vel subacutis, versus apicem leviter crenatis. Venis simplicibus, 10–11-jugis, basalibus duabus ad sinum conniventibus. Soris medialibus. Indusiis subpersistentibus, pilis simplicibus dense setosis. Sporangii glabris.

By its peculiar pubescence different from all allied species with exception of *D. curta* and *D. lugubriformis*. Dr. HIERONYMUS referred STÜBEL's specimens to *D. nephrodioides*, the pubescence and venation of which are totally different.

243. *Dryopteris lugubriformis*

Rosenst. Fedde, Repert. 7: 299. 1909. — Fig. 34 d.

Type-specimen from Peru orient.: Tarpoto, SPRUCE nr. 4749 (RB!).

Closely allied to the preceding species, but leaf firmer, grey-green, throughout more densely hairy, especially above, terminating in a subsimilar terminal pinna, and the lower 2–3 pairs of veins connivent to sinus; by these two last characters it resembles *D. tristis* and allied species, but its proper place seems to be here. Rachis gemmiferous above, sori inframedial, small, clothed with a hirsute, persistent indusium. The long hairs of rachis and costæ are rarely red and far more numerous but thinner than in *D. heterotricha*; rachis upwards especially densely hirsute, the edges densely ciliate.

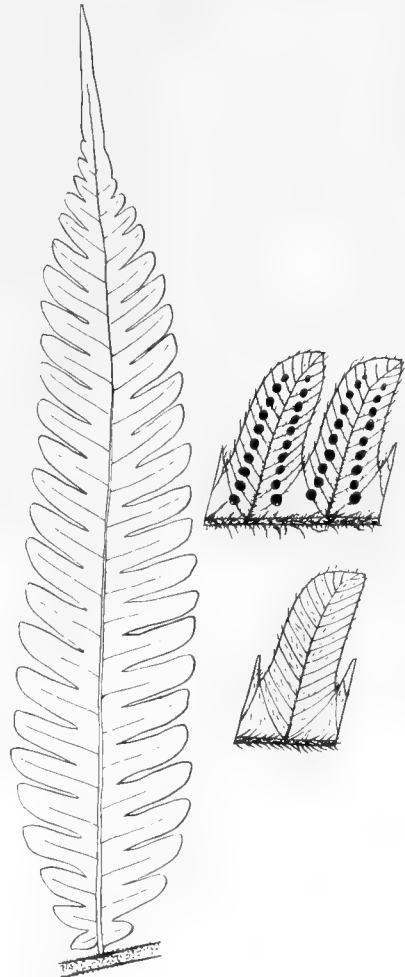


Fig. 36. *D. heterotricha* n. sp. The second pinna from below $\times \frac{4}{5}$ and segments seen from both surfaces, $\times 1\frac{1}{2}$.

244. *Dryopteris Schwackeana* Christ msc. n. sp. — Fig. 37.

Type from Brazil: Minas Geraes, Ouro Preto, locis humidis, leg. SCHWACKE nr. 14892–14893 (C!).

Rhizomate repente, squamis brunneis stellato-ciliatis instructis. Stipitibus quadrangularibus, griseis, ad 20 cm longis, brevissime stellato-pulverulentis et squamis paucis deciduis onustis. Lamina ovato oblonga, 15–25 cm longa, medio ad 10 cm lata, bipinnatifida, in apicem pinnatifidum abrupte attenuata, herbacea, luteo-viridi. Rachi pilis simplicibus longioribus pilis stellatis minutis intermixtis subdense tomentosa. Pinnis 9–10-jugis, alternis, inferioribus reflexis, mediis horizontalibus, sessilibus, oblongis, mediis 6 cm longis, $1\frac{1}{2}$ cm latis, apice abrupte acutis, rarius acuminatis, basi truncatis, inferioribus versus basin parum attenuatis, supra costa excepta strigosa glabris, subtus ad costas costulas venasque pilis mollibus albidis simplicibus et stellatis (his brevioribus) hirtis, vix ad medium pinnatifidis. Laciniis approximatis, obliquis, acutis, basali superiore sæpe longiore. Venis simplicibus, 6–7-jugis, 3–4 ad sinum conniventibus. Soris medialibus, indusiis dense setosis. Sporangiis glabris.



Fig. 37.
D. Schwackeana n. sp. Pinna $\times \frac{4}{5}$ and segments seen from both surfaces. (MOSÉN 4616).

A small species in habit not unlike certain forms of *D. mollis*, but venation and stellate pubescence show clearly that it belongs to the group of *D. lugubris*. By its reflexed lower pinnæ it resembles *D. lugubris* var. *devolvens* and *D. scabra*, from the first it differs by its small size, few veins, herbaceous texture etc., from the latter by 3–4 connivent veins the pinnæ scarcely narrowed towards the base and less incised and in its not gemmuliferous rachis. — Further specimens:

Minas Geraes: Caldas, MOSÉN nr. 4616 (Rg) — Lagoa Santa, WARMING (H).
São Paulo: Campiñas, HEINER nr. 551 (Rg).

245. *Dryopteris lugubris* (Kze.) C. Chr. Ind. 276. 1905. — Fig. 38.

Syn. *Polypodium lugubre* Kze. msc.

Aspidium lugubre Mett. Aspid. nr. 230. 1858.

Aspidium catacolobum Kze.; Ettingsh. Farnkr. 182 tab. 126 f. 9, 10 1865 (t. spec. in hb. PRESL).

Type from Rio, leg. POHL (Herb. PRESL!)

In general habit very like *D. nephrodioides* and *D. Eggersii* but well-marked by its creeping rhizome and the presence of small brown scales on the rachis and costæ beneath. Stipe 20–30 cm. long and like the rachis quadrangular and densely pulverulent by very short stellate hairs. Lamina up to 1 m long, dull-green, firmly herbaceous or membranous, the upperside throughout shortly pubescent, the costæ strigose by antrorse hairs, the underside less hairy, most so on costulæ and veins, the hairs generally simple; costæ beneath pulverulent by short stellate hairs and furnished with some few stellato-ciliate small scales. Pinnæ sessile 15–18 cm long, $2\frac{1}{2}$ –3 cm broad, acute or shortly acuminate, alternate, remote, subhorizontal, the lower ones not reflexed, their base shortly attenuate in the lower, truncate in the upper ones, incised about $\frac{2}{3}$ of the

way down into entire, acute or subobtusate segments; upper basal segment of lower pinnæ generally a little prolonged and close to the rachis, the lower basal one shorter and remote from rachis. Veins simple, 14—16-jugate, often prominent on the upper side, the basal pair connivent to sinus. Sori medial; indusium small, deciduous, setose. Sporangia glabrous.

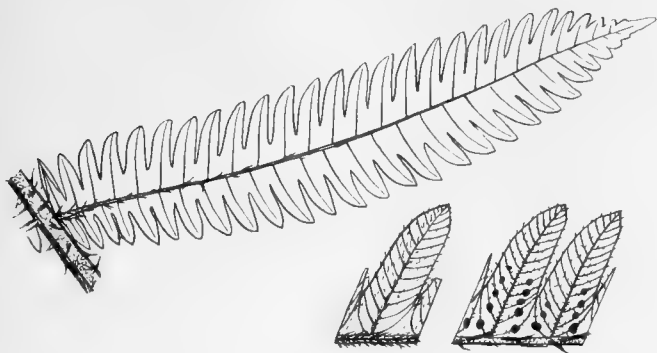


Fig. 38. *D. lugubris* (Kze.) C. Chr., leg. POHL. Pinna $\times \frac{4}{5}$ and segments seen from both surfaces, $\times 1\frac{1}{2}$.

The typical form of *D. lugubris* here described is rather common in Southern Brazil. It was referred to *Nephr. tetragonum* by BAKER in Flor. bras. which also includes our *D. scabra*, from which species it differs in pubescence and size. I have seen the following specimens:

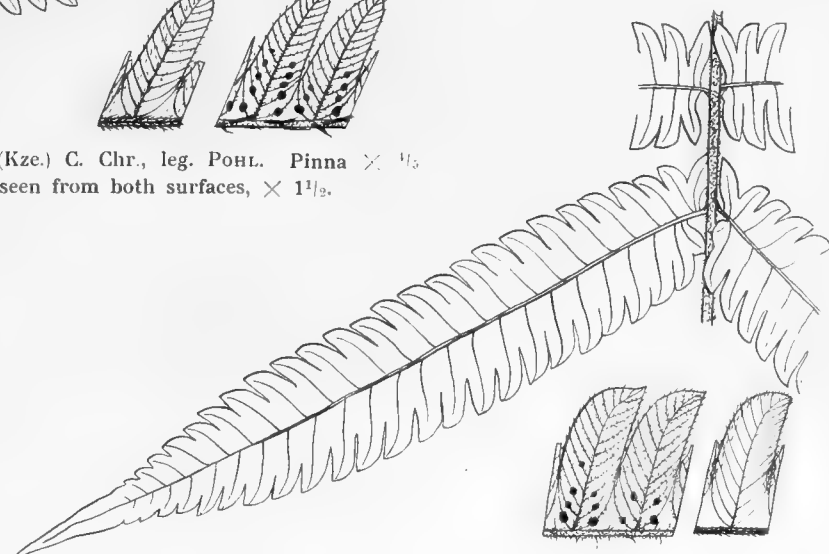


Fig. 39. *D. lugubris* var. *devolvens* (Bak.). Basal pinnæ $\times \frac{4}{5}$ and segments seen from both surfaces, $\times 1\frac{1}{2}$ (GLAZIOU 15766).

Minas Geraes: Caldas, MOSÉN nr. 2157, 2159 (Rg), 2160 (H, Rg); G. A. LINDBERG nr. 558 (B); REGNELL nr. III 1450 c (Rg) — Lagoa Santa, WARMING (H). — São Paulo: Serra de Caracal, MOSÉN nr. 2158 (Rg, S) — Matto Virgem, Rio Claro, LÖFGREN nr. 652 (H).

In some of these typical specimens the hairs of the surfaces sometimes are stellate; therefore I refer the following form as a variety to *D. lugubris*.

var. *quadrangularis* (Fée).

Syn. *Aspidium quadrangulare* Fée, Cr. vasc. Brés. 1: 145 tab. 50 fig. 2. 1869. Rio, GLAZIOU nr. 962 (Herb. COSSON, Paris, H).

Underside of lamina throughout like rachis and costæ above densely greyish puberulous by stellate hairs. The lower 3—4 veins run to sinus. Scales fewer. Otherwise typical.

var. *joinvillensis* Ros. Hedwigia 43: 225. 1904.

Syn. *Dryopteris joinvillensis* Ros. Hedwigia 46: 120. 1907.

Sta. Catharina: Joinville, SCHMALZ nr. 100 (R).

A very large form: pinnæ 25×4 cm; veins 18—20 to a side. Rachis gemmiferous. Pubescence about as in the former variety, but stellate hairs fewer and scales more numerous.

var *devolvens* (Bak.) — Fig. 39.

Syn. *Nephrodium devolvens* Bak. Journ. of Bot. 1885: 217.

Dryopteris devolvens C. Chr. Ind. 261. 1905.

Differs from the type by 1) size: lamina 3—4 dcm long; pinnæ 12—15 cm long, $2\frac{1}{2}$ cm broad, 2) the basal pinnæ being distinctly reflexed, 3) lamina upwards suddenly narrowed into a broad, pinnatifid apex, 4) segments close, even contiguous, acute, subfalcate; veins 10—12 to a side. In pubescence it does not differ materially; still the scales of rachis and costæ beneath are very few, and the rachis and costæ beneath are clothed with fine, simple hairs besides the stellate hairs. The upperside is hairy towards the margins, the underside finely downy by simple hairs. Veins not prominent.

A distinctly looking variety, well-marked by its close, acute segments and the upwards suddenly narrowed lamina, but I find it impossible to distinguish it specifically from *D. lugubris*. Several of the specimens referred to the type show now one now another of the characters, which mark the variety.

The following specimens are rather uniform:

Rio: GLAZIOU nr. 15766 (B, H, type-number) — Minas Geraes: Caldas, Mosén nr. 2156 (Rg, Hb. Brux.) — São Paulo: Piritabo (?), F. W. BAUER nr. 49 (R) — Capivary, GERDER nr. 88 (R) — Campiñas, HEINER nr. 568 (Rg) — Toledo, ULBRICHT nr. 51 (Rg; proliferous) — Matto Grosso: Santa Anna da Chapada, MALME nr. 2131 (Rg).

Synonyms of *D. lugubris* are probably:

Aspidium coadunatum Klf. Enum. 239. 1824 and

Nephrodium inaequale Schrad. Gött. gel. Anz. 1824: 869, both from the vicinity of Rio. A description of the former by METTENIUS is to be found in MS in (B) and from this description it seems to be identical with var. *devolvens*. The name of KAULFUSS has priority, if his species is a form of our *D. lugubris*, and I have no doubt that this is the case. We have thus here a common Brazilian species, which is described as new at least 6 times, viz 1824 by KAULFUSS (*coadunatum*) and SCHRADER (*inaequale*), 1858 by METTENIUS (*lugubre*), 1869 by FÉE (*quadrangulare*), 1885 by BAKER (*devolvens*) and 1907 by ROSENSTOCK (*joinvillensis*). This shows how necessary it is to compare specimens with descriptions and authentic specimens of described species before describing them as new.

Aspidium nitidulum Kze.; Ettingsh. Farnkr. 188 tab. 123 f. 3. 1865 is perhaps the same.

246. *Dryopteris glochidiata* (Mett.) C. Chr. n. sp. — Fig. 40 a.

Syn. *Aspidium glochidiatum* Mett. msc.

Type from Southern Brazil, leg. SELLOW (B!).

Species *D. nephrodioidi* (Kl.) habitu, magnitudine proxime affinis, differt: lamina subtus pilis minutis erectis glochidiformibus ubique praedita, supra pilis maxime simplicibus brevissimis pubescente; soris subcostularibus, sporangiis setis glochidiformibus instructis.

It is possible that this proposed new species is a form of the common Brazilian *D. lugubris*, which it resembles as well as *D. nephrodioides* in habit, size and venation. Still it lacks the scales of rachis and costae beneath, which are found in *D. lugubris*, the sori are distinctly inframedial, and the sporangia are setose by anchor-shaped hairs similar to those, occurring on the under surface. Rhizome short-creeping; stipe 4—4½ decm long, quadrangular. Rachis and costae beneath densely stellato-pulverulent. Pinnæ 10—11 cm long, 1½—1¾ cm broad. Veins 10 each side, the two lower ones connivent to sinus.

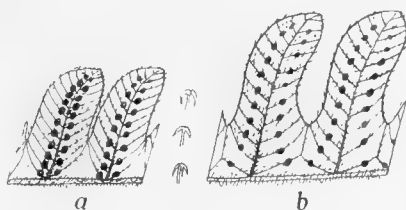


Fig. 40. Segments of a. *D. glochidiata* n. sp., with anchor-hairs from the surface, and b. *D. ancyrlothrix* Ros. $\times 1\frac{1}{2}$.

247. *Dryopteris ancyrlothrix* Ros. in Fedde, Repert. 7: 305. 1909. — Fig. 40 b.

Type from Ecuador: in monte Guayapurima, SPRUCE nr. 4748 (RB!).

A peculiar, thin-leaved species, resembling *D. glochidiata* by the anchor-shaped, erect hairs of the underside and rachis but otherwise very different by the longer pinnæ, the glabrous upperside and sporangia, exindusiate medial sori, the basal veins united etc. The rachis is not densely pulverulent as in *D. nephrodioides* and related species, still the species has its proper place here.

249. *Dryopteris nephrodioides* (Kl.) Hieron. Hedwigia 46: 327. 1907 (excl. var.).

Syn. *Aspidium nephrodioides* Klotzsch, Linnaea 20: 370. 1847.

Type from Venezuela, Caripe, in sylvis humidis, MORITZ nr. 206 (B!).

A variable species but easily distinguished from almost all other species by the dense pubescence of stellate hairs throughout the whole plant; the hairs of the rachis bear on a short stalk 5—6 short, normally simple branches, those of the surfaces are sessile and bear 3—5 long horizontal branches, which are adpressed to the leaf tissue; generally these hairs can only be seen by aid of the microscope; in certain forms rachis upwards, costae and veins above and the margins are furnished with some few, deciduous, long, simple setae.

The oblique rhizome bears at the top several leaves on long stalks; the leaves have a pinnatifid apex and a large number of alternate short-stalked linear pinnæ, which are long acuminate, a little reduced towards the base and incised

about $\frac{2}{3}$ of the way down to the costa; texture herbaceous, colour dark-green or grey-green. Segments numerous, oblique, obtuse or subacute. Veins 10—11-jugate, the basal ones connivent to sinus. Sori inframedial or submedial, furnished with a persistent, stellato-pilose indusium. Sporangia glabrous.

This species is closely allied to *D. lugubris*, from which it differs mainly by the absence of scales on rachis and costæ beneath and by fewer veins. KLOTZSCH's original specimen is apparently a small leaf of a species, which is known by pteridologists under other names. Best agreeing with the type are some plants from

Trinidad, heights of Aripo, Bot. Gard. Herb. nr. 325, 326, 327 (W).

The common West-Indian form of the species is probably quite typical; it differs from the original specimen by its large size; lamina 1 m or more long with 25 or more pairs of pinnae, which are 20—25 cm long, 2 cm broad; costæ above densely stellato-pubescent but without long, simple hairs; sori medial; basal pair of veins often united, which especially is to be found in the Jamaican and Cuban specimens, but in the same pinna one may find the basal veins free, connivent and anastomosing. This form is

f. *guadalupensis* (Fée).

Syn. *Nephrodium guadalupense* Fée 11 mém. 89 tab. 24 fig. 3. 1866.

Dryopteris guadalupensis O. Ktze.; C. Chr. 269.

Trinidad: Bot. Gard. Herb. nr. 4118 (C).

St. Vincent: H. H. & G. W. SMITH nr. 1348 (C).

Guadeloupe: L'HERMINIER nr. 34 et 132 (B); Père DUSS nr. 4040 (B, C, W), 4032 (RB), 4453 (B, W).

Martinique: Père DUSS nr. 4750 (W).

Porto Rico: BALBIS (B).

Jamaica: Brighton near Hope Bay, ALEX. MOORE (W); JENMAN (W); HART nr. 303 (W).

Cuba: Monte Verde nr. 1009 (B, S), 1010 pt. (B).

var. *Biolleyi* (Christ).

Syn. *Aspidium Biolleyi* Christ, Prim. Fl. Costaric. 3: 31. 1901.

Aspidium guadalupense Christ, Bull. Soc. bot. Belg. 35: 210. 1896.

Dryopteris guadalupensis Christ, Bull. Boiss. II. 7: 264. 1907 (with full description).

Dryopteris asterothrix Rosenstock, Fedde Repert. 7: 305. 1909.

This is the most developed form of the species and it can be easily confounded with *D. megalodus*, from which it scarcely can be distinguished by other characters than its pinnatifid apex and its generally narrower but larger and more deeply cut pinnae. From f. *guadalupensis*, which it resembles in most characters it differs 1) by the hairs of the underside; they are anchor-shaped, i. e. they bear on a short stalk 2—3 very short recurved branches; the costæ beneath are besides the stellate hairs furnished with long, simple ones; the hairs of the upper surface are like those of the type; 2) basal pair of veins almost constantly united, and 3) indusium small, deciduous, often not seen.

- Guatemala: Cubilquitz, v. TUERCKHEIM ed. J. D. S. nr. 8812 (C); nr. II. 1173 (B)
- Costa Rica: Tuis près Turrialba, PITTIER nr. 11243 (C, type, W) — Forêts de Tsaki, Talamanca, PITTIER nr. 9439 (C, W) — eodem loco, TONDUZ nr. 9464 (B, C, W) — Cartago; POLAKOWSKY nr. 423 (B) — forests of Virris, near la Banilla, BOLLLEY (C) — Hacienda Guacimo, TONDUZ nr. 14585 (W).
- Panama: HAYES nr. 65 (B, W); MAXON nr. 5772 (W), 5748 (W).
- Colombia: Sta. Marta, H. H. SMITH nr. 996 (C) — Salto del Diablo, STÜBEL nr. 677 (B) — Muzo, STÜBEL nr. 526 (B) — La Vega, LINDIG nr. 332 (B) — Tocarema, LINDIG nr. 237 (B).
- Ecuador: Andes quitenses, Canelos; SPRUCE nr. 5258 (RB) — Balao, EGGERS nr. 14729 (B, W) — FRASER (B).
- Peru: secus rivulum Cachi-yacu, SPRUCE nr. 4659 (RB = *D. asterothrix* Ros.; a form with the underside more decidedly stellato-pubescent, else typical).
- Amazonas: Rio Juruá, Miry, ULE nr. 5760 (B, C).
- Bahia: LUSCHNATH nr. 116 et 117 (B).

2. Eugoniopteris

C. Chr. Biolog. Arbejder tilegnede Eug. Warming 84. 1911.

249. *Dryopteris tristis* (Kze.) O. Ktze. Rev. 2: 814. 1891; C. Chr. Ind. 298. (pt.). — Fig. 41 c.

Syn. *Polypodium triste* Kunze, Linnaea 9: 47. 1834.

Aspidium triste Mett. Aspid. nr. 229. 1858.

Nephrodium triste Hk. sp. 4: 104. 1862; Hk. Bak. Syn. 266 (pt.).

Type from Peru; in sylvis primævis Huallagae ad Mission Tocache, leg. POEPPIG 1959 (non vidi).

In Herb. Mett. (B) there is found a sketch of the original specimen and from this and the original diagnosis I have got a fair idea of this species, which has been said to have a wide distribution throughout the whole tropical America. — Rhizome obliquely erect or creeping with a few scales, which bear stellate hairs. Lamina with terminal pinna, dark-green, membranous (not coriaceous); rachis and stem pulverulent by stellate hairs; costæ and veins of underside with several patent, long, whitish hairs, which, however, are deciduous, and several specimens appear to be glabrous. Lower pinnæ shortly stalked and with a characteristic long cuneate entire base, the upper ones mostly truncate at the base. Pinnæ 15—20 cm long, $2\frac{1}{2}$ —3 cm broad, much acuminate, incised $\frac{1}{2}$ to $\frac{1}{3}$ of the way down into subfalcate, acute, close segments. Veins simple, 12—14 to a side, the lower 4 running out to the sinus. Sori small, medial; indusium very small, ciliated by long, simple hairs, soon falling and in several specimens not found. Sporangia glabrous.

D. tristis is distributed from Guiana to Peru, while it does not occur in Southern Brazil; the plants therefrom referred to *D. tristis* belong to *D. monosora* and *D. cuneata*. I have not seen the true form from Central-America, although it

is possible that some of the doubtful forms referred as varieties to *D. tetragona* really belong to *D. tristis*. For the present I prefer to refer to *D. tristis* the specimens only which have medial sori and lower pinnæ with a long cuneate base. They are the following:

Peru: Tarapoto, SPRUCE nr. 4091 (RB) — in valle fluv. Rio Húallaga, STÜBEL nr. 1090 (v. *glabrata* Hieron. *Hedwigia* 46: 324), 1090 a (B).

Ecuador: FRASER (B).

Colombia: Castañal, SONNTAG (B, C) — Magdalena, LINDIG nr. 377 b, 383 (B) — Ocaña, ENGEL nr. 256 (B); SCHLIM nr. 129 et 657 (B) — inter Honda et Bogotá, STÜBEL nr. 391 (B); LINDEN nr. 1010 (B) — Sta. Marta, H. H. SMITH nr. 2562 (C).

Panama: HILLEBRAND nr. 195 (B, C) — Bocas del Toro, HART nr. 21 a (W); MAXON nr. 4651, 4682 4705, 5756 (W).

Costa Rica: Rio Hondo, plains of Sta. Clara, 100 m, COOK and DOYLE nr. 561 (W).

Surinam: Paramaribo, KAPPLER nr. 1776 (B).

Brasilia: Pará, Rio Maracá (near Guiana), M. GUEDES nr. 567 (C — bud on rachis) — in sylvis fl. Marañon, SPRUCE s. n. (RB).

250. *Dryopteris nicaraguensis* (Fourn.) C. Chr. Ind. 279. 1905. — Fig. 41 d.

Syn. *Phegopteris nicaraguensis* Fourn. Bull. Soc. Fr. 19: 252. 1872.

Type from Nicaragua: Chontales, leg. P. Lévy nr. 460 bis (Herb. Coston, Paris!).

The type-specimen, with which agree a few other specimens, belongs to a species, which seems to be a very distinct one. It is marked by its large size: stipe up to 8 dcm long, lamina 5 dcm, pinnæ 20—24 cm long by $3\frac{1}{2}$ —4 cm broad at the middle, by the grey-green or yellowish green colour and firm, papyraceous texture of the lamina and by the prominent, stramineous costæ and costulæ. Veins 12—18-jugate, those of the three lower pairs connivent to sinus, below which they are separated by a hyaline membrane, folded in the dried specimens. Rachis very finely downy, especially upwards, like the costæ beneath, or nearly glabrous; surfaces otherwise glabrous, opaque. Pinnæ 8—10 to a side, at distances of 4—5 cm, the inferior ones shortly stalked, (terminal pinna similar in shape), incised about half-way to the costa into falcate, subacute, entire segments, 8 mm broad, the lower ones of the lower pinnæ gradually reduced, those of the basal or two lower pairs forming a sinuate wing to the costa. Sori small, inframedial, apparently exindusiate. Sporangia glabrous.

The main differences between the typical form and *D. tristis* are seen in the colour and texture of the lamina, in the base of the lower pinnæ and in the position of the sori. Unfortunately all these characters seem to be combined in different ways, and I have examined a number of specimens, which as well may be referred to *D. nicaraguensis* as to *D. tristis*. Other specimens, partly mentioned under *D. tetragona*, connect this species with *D. nicaraguensis*, and I have failed to find good distinctive characters, by which these intermediate forms can be referred to their proper species. To *D. tetragona* var. 3 and 5 I have referred the forms having

setose sporangia and thinner texture; the doubtful forms enumerated below agree with *D. nicaraguensis* in having glabrous sporangia and firmer texture.

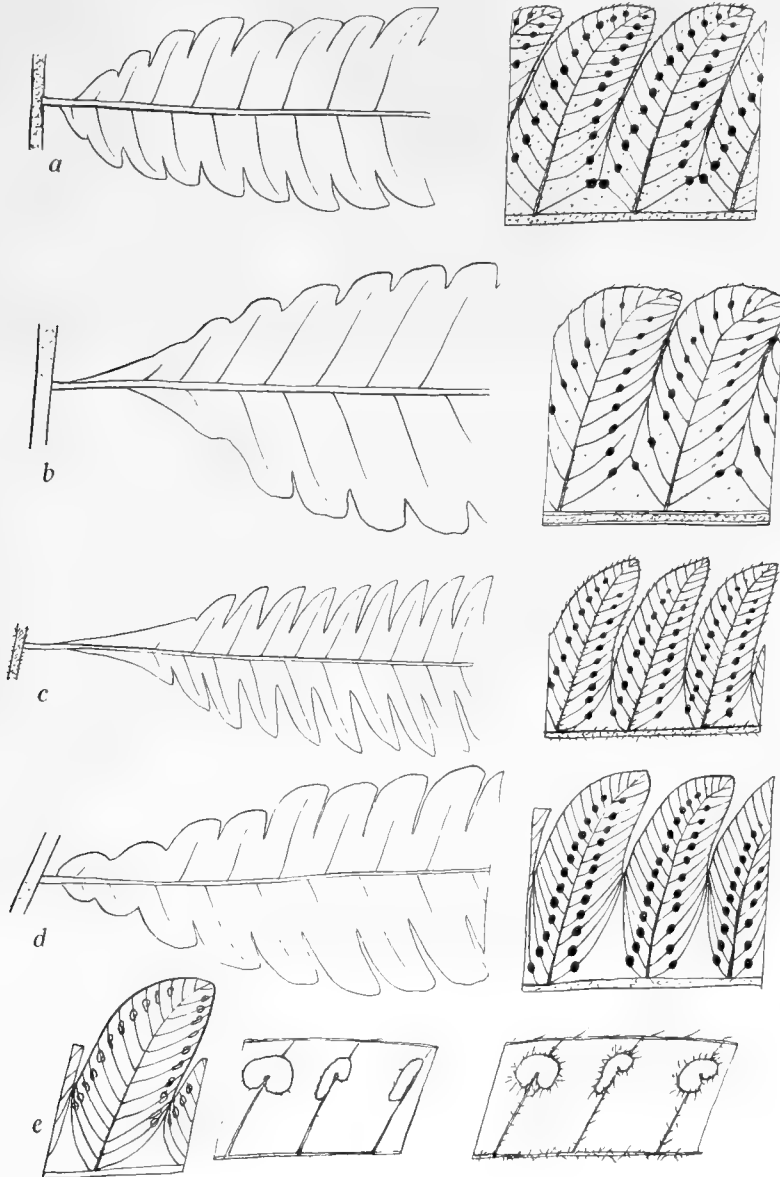


Fig. 41. Base of normal pinna ($\times \frac{4}{5}$) and segments ($\times 1\frac{1}{2}$) from the middle of a pinna of
 a. *D. megalodus* (Schk. Urb. — b. *D. leucophlebia* (Christ) C. Chr. — c. *D. tristis* (Kze.)
 O. Ktze. — d. *D. nicaraguensis* (Fourn.) C. Chr. — e. Segment of *D. Schaffneri* (Fée) ($\times 1\frac{1}{2}$)
 from the type with a fragment showing three different indusia and a fragment of the
 hairy form ($\times 4$).

As typical I consider the following specimens:

Nicaragua: San Juan del Norte, CHARLES L. SMITH nr. 2098 (exactly typical).

Costa Rica: Puerto Limon, Hot de la Uvita, PITTIER nr. 12703 (C, W) — Suerre, Llanuras de Santa Clara, 300 m.; J. DONNELL SMITH nr. 6929 (W) — Rio Turrialba, DONNELL SMITH nr. 5093 (W).

Formæ:

Guatemala: HEYDE nr. 731 (W; segments acute).

Panama: Chagres, FENDLER nr. 403 (= HAYES nr. 36 (W, texture thinner, lobes narrower, base of lower pinnæ about as in *D. tristis*).

var. **minor**. Pinnæ ca. 15 cm long by 2 cm broad, dark-green. Veins about 10 to each side, the lower 4 connivent to sinus. — Resembles very much *D. tetragona* var. 2, from which it differs by its erect rhizome, which is up to 10 cm high.

Guatemala: Alta Verapaz, Vicinity of Secanquim, about 450 m., MAXON and HAY nr. 3196, 3217 (W).

251. *Dryopteris paucipinnata* (Donn. Smith) Maxon, Contr. U. S. Nat. Herb. 13: 19. 1909.

Syn. *Nephrodium Fendleri* var. *paucipinnatum*, Donnell Smith, Bot. Gaz. 12: 134. 1887.

Type from Guatemala: Alta Verapaz, Petet, leg. H. von TÜRCKHEIM, ed. DONN. SMITH nr. 667 and 767 (W, B). Other specimens from the same region were collected by DONNELL SMITH nr. 1556 (W) and v. TÜRCKHEIM nr. II. 1690 (W).

Described in detail by MAXON (loc. cit.) and by him rightly separated from *D. Fendleri* as a distinct species. It is not at all allied to *D. Fendleri*, which belongs to a different group, but intermediate between *D. Schaffneri* or *D. tristis* and *D. oblitterata*. It resembles *D. Schaffneri* in its submarginal sori and distinct, persistent indusia, *D. tristis* in the cuneate base of the pinnæ and in venation, *D. oblitterata* in the general habit. It differs from all these species by its practically quite glabrous leaf, rigid to coriaceous texture and warted under-surface, from the two first-named species by its less incised pinnæ, which is pinnatifidly cut about $\frac{1}{3}$. From *D. Fendleri* it differs *inter alia* by its remote, alternate pinnæ with a cuneate base, venation and lack of aërophore and febrils beneath. Generally the lowermost 4—6 veins are connivent to sinus, but occasionally the lower pair are united. Sporangia glabrous.

252. *Dryopteris Fraseri* (Mett.) O. Ktze. Rev. 2: 812. 1891; C. Chr. Ind. 266.

Syn. *Aspidium Fraseri* Mett.; Kuhn, Linnaea 36: 109. 1869.

Nephrodium Fraseri Bak. Syn. 495. 1874.

Nephrodium brachyodon Sodiro, Cr. vasc. quit. 268. 1893!

Type from Ecuador, leg. FRASER 1859 (B!); identical specimens from the same country were collected by STÜBEL nr. 762 (B) and SODIRO (C).

A large species, very characteristic in habit but otherwise closely allied to the other large species of this group. Pinnæ at distances of up to 10 cm., opposite,

dark-green, membranous, 15–20 cm long, 4–5 cm broad, sessile or very shortly stalked, lower ones much reflexed, incised scarcely halfway to the costa into a little falcate, obtuse lobes. Terminal pinna rather distinct with a bud at base. Rachis slender, sparsely soft-hairy, lamina otherwise quite glabrous, but the surfaces often minutely warted. Veins 15–16-jugate, the lower 3–4 pairs connivent to a cartilaginous membrane below the sinus; the basal posterior vein raises always from the costa. Sori a little infra-medial, indusium small, deciduous, ciliate. Sporangia glabrous.

253. *Dryopteris cuneata* n. sp. — Fig. 42.

Type from Brazil: São Paulo, Santos, H. Mosén nr. 3545 (Rg fol. fert.; S fol. ster.).

Rhizomate repente, dense radicante, apice squamis rufis, latis pilis furcatis sparse ciliatis, dense onusto. Stipite $6\frac{1}{2}$ dcm longo, angulato, stramineo, glabro. Lamina ovata, ca. 4 dcm longa, pinnata cum impari, pure viridi (infra pallidiore), herbacea vel submembranacea, glaberrima. Pinnis lateralibus 7-jugis, inferioribus oppositis, 4 cm inter se remotis, breviter petiolulatis, superioribus alternis, supremis sessilibus, mediis 20 cm longis, medio 3 cm latis, acuminatis, basi cuneata subintegra, ad medium pinnatifidis. Laciniis approximatis, subfalcatis, 5 mm latis, obtusis vel subacutis. Venis simplicibus, 9–11-jugis, inferioribus 4–5 ad sinum conniventibus. Soris inframedialibus, parvis, indusiis persistentibus rufis sparse ciliatis tectis. Sporangiiis glabris.

This new species seems different from all other Brazilian species. It is nearest *D. monosora*, which it resembles in shape of pinnae and venation, but it is entirely glabrous and without scales on rachis, has a distinct terminal pinna, the sori are inframedial and the stipe much longer. In general habit it resembles *D. paucipinnata*, but it is much more thin-leaved and the sori are inframedial.

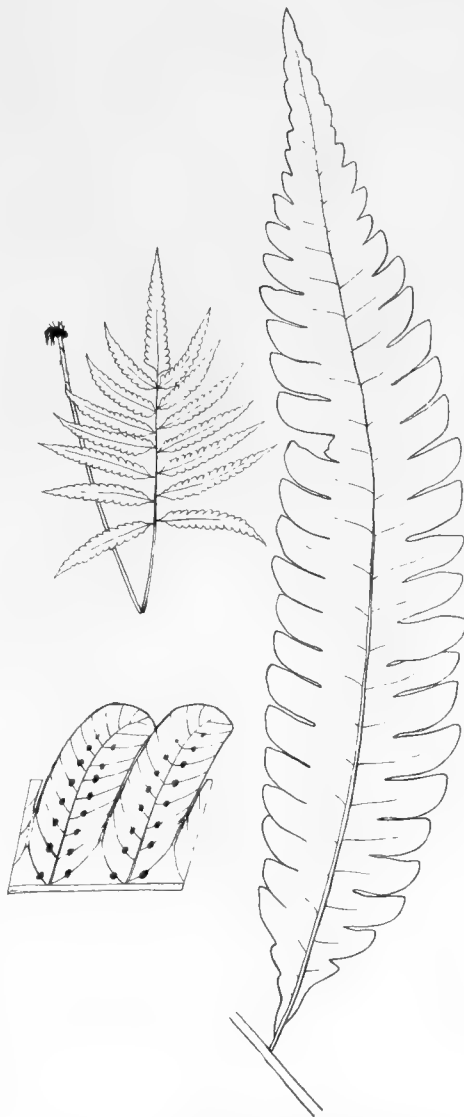


Fig. 42. *D. cuneata* n. sp. Pinna $\times \frac{1}{2}$, segments $\times 1\frac{1}{2}$ and an entire leaf, showing its habit, much reduced.

254. **Dryopteris Schaffneri** (Fée) C. Chr. Ind. 291. 1905. — Fig 41 e.

Syn. *Nephrodium Schaffneri* Fée, 8 mém. 108, 1857; 10 mém. 44. tab. 43.

Aspidium varians Mett.; Kuhn, *Linnaea* 36: 114. 1869!

Aspidium imbricatum Fourn. Mex. pl. 1: 96. 1872!

Dryopteris imbricata C. Chr. Ind. 271. 1905.

Type from Mexico: Mirador, leg. SCHAFFNER nr. 244 (non vidi).

A distinct species, in habit not unlike *D. tristis*, but well-marked by its peculiar indusia; in other respects it is very variable. Rhizome creeping. Lamina with 3–6 pairs of lateral pinnæ and a similar terminal one. Pinnæ 15–25 cm long, 3–4 cm broad, stalked, incised about halfway to the midrib into falcate, acute or obtuse segments. Pubescence variable; some specimens are practically quite glabrous (= var. *glabrescens* Fourn. l. c.); others have long, whitish, patent hairs on costæ and veins beneath; rachis shortly pulverulent; under-surface verrucose. Venation variable; veins about 15 to a side, simple or, not rarely furcate, the lowest 3–6 connivent to sinus, those of the basal pair often truly anastomosing. Sori nearest the margin, furnished with a persistent, flat, ciliate, variable indusium; in same pinnæ one can find as well reniform as athyroid or even asplenoid indusia.

I have no doubt that *A. varians* Mett. = *A. imbricatum* Fourn. (not *Polypodium imbricatum* Liebm., which is *D. tetragona*) is identical with *N. Schaffneri* Fée; this is the glabrescent form, while *A. varians* is hairy as described above. -- The species is apparently a rare one and confined to the humid forests of south-eastern Mexico. I have seen the following specimens:

Mexico: San Luis Potosi, VIRLET nr. 82 (B, Herb. Mus. Paris) — Misantra, L. HAHN nr. 623 (B, Herb. Mus. Paris) — Córdoba, KERBER nr. 86 a (B) — Mirador, SCHAFFNER, MÜLLER, SARTORIUS (B) — Vicinity of Gómez, Fárias, Tamaulipas, EDW. PALMER nr. 298 (W).

255. **Dryopteris vivipara** (Raddi) C. Chr. Index 300. 1905 — Fig. 43 c.

Syn. *Polypodium viviparum* Raddi, Pl. bras. 1: 22 tab. 32. 1825.

Polypodium diversifolium Sw. Vet. Akad. Handl. 1817: 60 (non alior.).

Polypodium proliferum Klf. Enum. 107. 1824.

(For other synonyms see Index Filicum).

Type from Southern Brazil. I have seen the original specimens of *P. diversifolium* Sw. from Minas Geraes, leg. FREYREHS (S), which no doubt is the same as *P. viviparum* Raddi.

One of the most distinct species of the whole group, easily distinguished from related species by its glabrous frond, proportionally small (8–10 cm × 1–2 cm) pinnæ, which are nearly entire or very shallowly serrulate or crenate, and by the cartilaginous, often thickened margins. The short-creeping or decumbent rhizome is clothed with stellato-pilose scales; rachis often gemmiferous. Pinnæ alternate, the lower ones short-stalked, rounded or short-cuneate at base, not warted beneath. Veins 3–6-jugate, the two lower pairs generally united, the lowermost pair, which

always spring out from the costule 1 mm above the costa (fig. 43 c), occasionally meniscioid. Sori about medial, the lowermost ones still often near the point, where the basal veins meet and generally 2—3 mm remote from the costa, thus leaving a disc along the middle of the pinna free of sori. Indusium not seen; sporangia glabrous.

D. vivipara varies mainly in texture and shape of pinnæ. Some forms are thinly membranaceous, others coriaceous. From the type I cannot distinguish certainly

Aspidium macropus Mett. Fil. Lechl. 2: 20. 1859, based on CLAUSSEN nr. 112 (B). METTENIUS found it different from *D. vivipara* by its having a small indusium, which I have not found. It is a large form with pinnæ up to 15 cm long; the rhizome seems to be erect and is densely clothed with red-brown scales. To me it is only a large form of typical *D. vivipara*. A more distinct variety is

var. platypes (Fée).

Syn. *Goniopteris platypes* Fée, Cr. vasc. Br. 1: 106 tab. 33 "fig. 3", left-hand figure. 1869.

Differs from the type by its glossy, dark-green papyraceous leaf and large size. Pinnæ up to 20 cm long, 3—3½ cm broad, cuneate at base, margins serrulate. Veins about 6, much upcurved, the anterior basal one often springing out from the costa. — The type specimen of this variety (GLAZIOU nr. 2402), which BAKER most remarkably referred to *D. tetragona* looks a distinct species, but it is connected with the type by numerous intermediate forms.

D. vivipara is a common species in Southern Brazil and it is one of the very few species, which occur in almost identical forms in the Andes. I name some collector-numbers:

Brazil: Rio, GLAZIOU nr. 409 (H), 7321 (B); SELLOW nr. 198 (B); BURCHELL nr. 936, 968 (B); CLAUSSEN nr. 2112 (B); MOSÉN nr. 85 pt. (B, H, Rg), 2682 Rg; REGNELL nr. 260 (Rg) — Sta. Catharina, Blumenau, H. SCHENK nr. 171 (C) — Paraná, Villa Nova, ANNIES, Rosenst. Fil. Austr. Bras. nr. 104 (Rg, W).

Columbia: Cundinamarca, WERCKLÉ 1906 (C); KARSTEN nr. 60 (B); LINDIG nr. 193 (B).

Costa Rica: Rio Surubres, 400 m., A. et C. BRADE nr. 420 (R).

var. platypes (Fée).

Brazil: Rio, GLAZIOU nr. 2402 (Herb. COSSON, Paris, H); MOSÉN nr. 85 part. (H) — JELINEK nr. 140 part. (B).

256. *Dryopteris straminea* (Bak.) C. Chr. Index 294. 1905. — Fig. 43 b.

Syn. *Polypodium salicifolium* Hook. sp. 4: 242. 1862.

Polypodium stramineum Bak. Syn. 316. 1867.

Type from Venezuela: Tovar, leg. FENDLER nr. 474 (Kew!).

Nearly exactly *D. vivipara* in size, texture, entirely glabrous frond, the cartilaginous margins and gemmiferous rachis, but veins all free, about 4 to a side, the two basal ones terminating in the leaf-tissue and the posterior one springing out from the costa. Pinnæ crenate or slightly lobed. It is not unlikely a variety of *D. vivipara*.

257. *Dryopteris Goeldii* n. sp. — Fig. 43 a.

Type from Brazil: Linkes Ufer d. Parahyba, leg. GOELDI 1887 nr. 3 (C).

Rhizomate oblique-erecto, squamis paucis stellato-ciliatis instructo. Stipitibus ad 30 cm longis, glabris. Lamina ovata, 30—40 cm longa, 20 cm lata, pure viridi, firmo-herbacea, rachi minute stellato-puberula excepta glaberrima, ad insertionem pinnae lateralis supremæ gemmulifera, pinnata cum pinna terminali subhastata.

Pinnis 6—7-jugis, inter se 3 cm remotis, breviter petiolulatis, lanceolatis, 10 cm longis, $1\frac{1}{2}$ cm latis, acuminatis, ad basin parum attenuatis, saepe auriculatis, ad tertiam partem incis. Lobis obliquis, acutis vel obtusiusculis. Venis 8—9-jugis, basalibus duabus semper anastomosantibus, sequentibus duabus ad sinum conniventibus. Soris fere medialibus vel potius inframedialibus. Indusiis minimis, pilis bi-trifurcatis ciliatis, mox evanidis. Sporangii glabris.

A critical species, in certain characters resembling several other species. It differs from *D. tetragona* by its less cut pinnae and glabrous costae, from *D. juruensis* by more pinnae and not warted surfaces, from *D. pyramidata* by distant pinnae, terminal pinna, glabrous costae and venation, from *D. anoptera*, which it resembles in venation, by glabrous sporangia and absence of scales, from *D. vivipara* by thinner texture and distinctly lobed pinnae, from *D. scabra* by venation, glabrous frond etc.

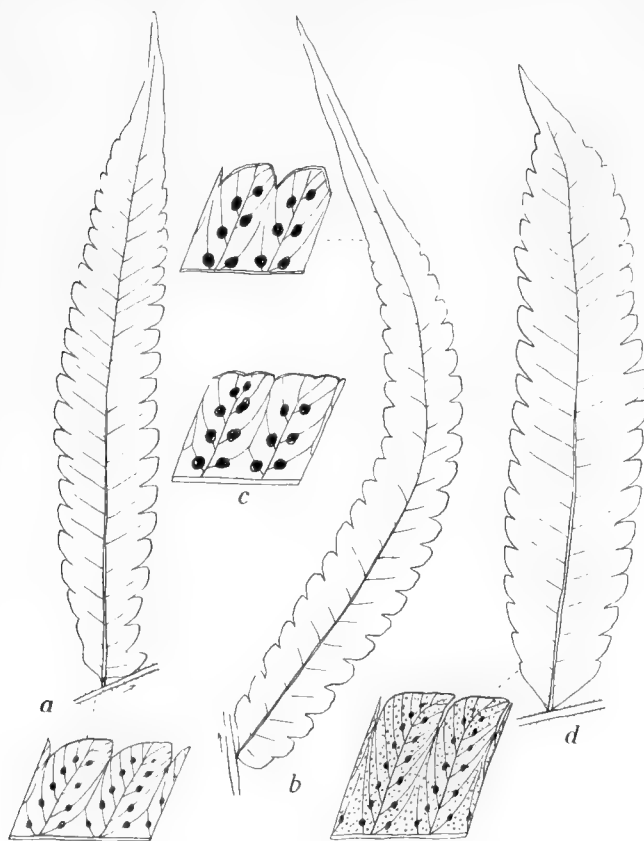


Fig. 43. Pinnæ ($\times \frac{4}{5}$) and segments ($\times 1\frac{1}{2}$) of a. *D. Goeldii* n. sp. — b. *D. straminea* (Bak.) C. Chr. and d. *D. juruensis* n. sp. c. Segment of *D. vivipara* (Raddi) C. Chr. $\times 1\frac{1}{2}$.

Still its nearest ally is *D. straminea* and it may be the same species and like it a variety of *D. vivipara*; it differs by its united basal veins and more numerous veins, more deeply cut pinnae of thinner texture and margins not so thickened.

258. *Dryopteris juruensis* n. sp. — Fig. 43 d.

Type from Amazonas: Auf Erdboden, im Walde bei Bom Fim, Rio Juruá, Oct. 1900, leg. E. ULE, Herb. Brazil. Amazonas expedition nr. 5325 (B!; C) — SPRUCE nr. 3905, ad fluv. Marañon (B) is exactly the same.

Rhizomate oblique-erecto, squamis stellato-pilosis sparse onusto. Stipitibus subfasciculatis, foliorum sterilium 10—20 cm, fertileum 30—40 cm longis, angularibus, sparse et decidue squamosis, glabris. Lamina ovata, 15—18 cm longa, 12—15 cm lata, griseo-viridi, membranacea, rachi minute stellato-puberula costis pinnarum subtus sparse et microscopice hirtis exceptis glaberrima, sed paginis densissime verrucosis, ad insertionem pinnæ lateralis supremæ gemma magna squamosa bulbillifera, pinnata cum impari. Pinnis lateralibus 2—4-jugis, alternis, infimis breviter petiolulatis, supremis sessilibus, lanceolato-ellipticis, 12 cm longis, 2—2½ cm latis, ad basin breviter contractis, versus apicem acuminatis, grosse serrulatis vel lobulatis. Lobis approximatis, subfalcatis, obtusis vel antice acutis, 5—6 mm longis, 4—5 mm latis. Venis simplicibus, valde ascendentibus, 8—9-jugis, inferioribus 3—4 ad sinum conniventibus, omnibus liberis et saepe interruptis, infimis interdum anastomosantibus meniscioides. Soris parum inframedialibus, indusiis minimis, ciliatis, raro repertis. Sporangii glabris.

This new species agrees with *D. obliterata* in shape and cutting of the pinnæ and partly also in venation, but it differs considerably from that species, its nearest relative, by the few pinnæ, gemmiferous rachis, thinner texture, both surfaces being densely warted by small raised points, and by its variable venation. Most often the veins are all free, but one can in the same pinna find all intermediate states of venation, from all veins being free to 2—3 pairs anastomosing and then the lowermost pair is often meniscoid. The anterior basal vein springs constantly out from the costa, which is not the case in *D. obliterata*. In all specimens seen the fertile leaves have a longer stipe than the sterile ones.

259. *Dryopteris obliterata* (Sw.) C. Chr. Ind. 280. 1905.

Syn. *Polypodium obliteratum* Sw. Prodr. 132. 1788; Fl. Ind. occ. 1660; Bak. in Hk. Icon. plant. tab. 1669. Jenm. Bull. Dept. Jam. n. s. 4: 132. 1897.

Phegopteris obliterata Mett. Pheg. nr. 46. 1858.

Polypodium faucium Liebm. Mex. Bregn. 57 (Vid. Selsk. Skr. V. 1: 209). 1849.

Type from Jamaica, leg. SWARTZ (S!).

Rhizome shortly creeping with a few densely stellato-pilose scales. Stipes subdistant, 4—5 dm long, quadrangular, greyish-stramineous, glabrous. Lamina with 6—10 pairs of shortly stalked, alternate, lateral pinnæ and a similar but often larger terminal pinna, firmly membranous or papyraceous, sometimes nearly coriaceous, generally greyish green, microscopically puberulous on rachis and costæ beneath, otherwise glabrous; hairs of rachis stellate. Pinnæ 12—20 cm long by 2—3 cm broad, acuminate, shortly attenuate towards the base, serrate or shallowly lobed. Teeth oblique, acute, not much longer than broad. Veins 6—8-jugate, the lower two pairs generally thinly anastomosing and sending a branch to the narrow membrane, to which the next 2—3 pairs are connivent; in Central-American specimens the lowest or sometimes the two lower pair of veins are meniscoid. Sori in two convergent rows, the lower ones being about medial, the upper ones gradually approaching the

costule. Sporangia with a few simple, deciduous setæ or in some specimens glabrous. "In the early stage of growth trace of a rudimentary involucre is observable, which however soon disappears" (JENMAN loc. cit.). I have not found indusia.

This species, of which the plate of Hk. Icon. t. 1669 gives a fair illustration, can be considered the type of a small group including the species nr. 257—261 of this monograph, which connect *D. tetragona* with *D. Poiteana*. From *D. tetragona* they differ by venation and less cut pinnæ, from *D. Poiteana* by few or no meniscioid veins. The Central-American specimens of *D. obliterata* approach *D. Poiteana* more than the West-Indian ones, and they also resemble *D. paucipinnata*, from which they differ by position of sori, venation, texture and generally exindusiate sori.

D. obliterata is confined to Jamaica, Cuba, Mexico and Northern Central-America. I have seen the following specimens:

- Jamaica: SWARTZ (H, S); MAXON nr. 831, 1799 (= UNDERWOOD nr. 2772), 1846, 1924, 2876 (W), 2375 (C, W); UNDERWOOD nr. 117 (W); CLUTE nr. 246 (W); HART nr. 336 (W); O. HANSEN (CC, H).
 Cuba: WRIGHT nr. 1010 (C, CC, S, W) — Santiago, POLLARD and E. & W. PALMER nr. 96 (W) — Pinar del Rio, PALMER and RILEY nr. 72 (C, H, W), 251 (W) — Santa Catalina, CALDWELL and BAKER nr. 7028 (W) — Isla de Pinos: A. H. CURTISS nr. 345 (W); A. A. TAYLOR nr. 9 (W); PALMER and RILEY nr. 1040 (W).
 Mexico: Barranca de Jovo, LIEBMANN (H B, *P. faucium* Liebm.) — SCHAFFNER (B) — Coatzacoalcos, isthmus of Tehuantepec, CHAS. L. SMITH nr. 2052 (W).
 Guatemala: Puerto Barrios, MAXON and HEY nr. 3059 (C, W) — Rio Dulce, DONNELL SMITH nr. 1559 (W) — Livingston, KELLERMANN nr. 4856 (W) — Puerto Barrios, B. L. ROBINSON nr. 472 (W) — near Cacao, H. S. BARBER nr. 186 (W).
 Honduras: Bonacca, GODMAN and SALVIN (B) — San Pedro Sula, C. THIEME, ed. DONN. SMITH nr. 5685 (W); 5693 part. (B, not W).

260. *Dryopteris nigrescentia* (Jenman) C. Chr. Ind. 279. 1905 — Fig. 44.

Syn. *Polypodium nigrescentium* Jenman, Gard. Chron. III. 17: 100. 1895;
 Bull. Dept. Jam. n. s. 4: 132. 1897.

Type from Jamaica, leg. JENMAN (W!).

A doubtful species, of which I have seen 5 leaves only. It may be an abnormal, local form of *D. obliterata*, which it resembles in the essential characters. It differs by the stouter rhizome, smaller leaves (10 cm long and broad at base), dark-green colour, wrinkled, serrulate, opposite and sessile pinnæ, and shortened often rudimentary terminal pinna. Veins 4—5 to each side, the lowest 2—3 pairs anastomosing. Sporangia glabrous. Rachis more conspicuously stellato-puberulous.

261. *Dryopteris Rolandii* n. sp. — Fig. 45.

Ecuador: Rio de Ventanas prope Guayaquil, leg. SPRUCE nr. 5718 (type in RB).

Rhizomate? Stipitibus validis, brunneo-stramineis, supra trisulcatis, 6½ dcm longis, ubique dense stellatim puberulis ad insertionem sparse squamosis; pilis brevissimis 3—6 furcatis, squamis angustis pilis stellatis dense instructis. Lamina 4 dcm longa, pinnata cum impari, herbacea, pure viridi; rachi tenui stellatim pilosa; pinnis 7-jugis, suboppositis vel superioribus alternis, sessilibus, inter se 5—6 cm



Fig. 44. Entire leaf of *D. nigrescentia* (Jenm.)
C Chr., $\times \frac{4}{10}$ and fragment $\times 1\frac{1}{2}$.

remotis, lanceolatis, 15 cm longis medio 3 cm latis, inferioribus e medio utrinque attenuatis, superioribus basi subtruncatis, breve acuminatis, supra ad costam pilis antrorsis simplicibus vel furcatis setosis, ad venas setis paucis instructis, inter venas glabris, subtus ubique — præsertim ad costam — pilis brevibus mollibus raro furcatis sparse pubescentibus, marginibus ciliatis, grosse lobatis. Lobis obliquis, approximatis, obtusis, leviter crenatis, mediis $\frac{1}{2}$ cm longis et latis, basali superiore rachin tangente, inferiore a rachi 4—5 mm remoto. Venis simplicibus, 9—10 jugis, inferioribus 2—3 jugis anastomosantibus, basilibus meniscioides. Soris luteis, medialibus, exindusiatis, sporangiis pilis 5—6 simplicibus acutis setosis.

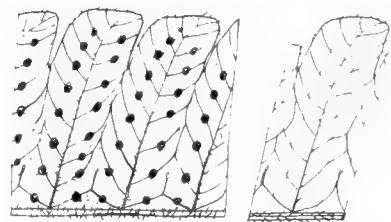
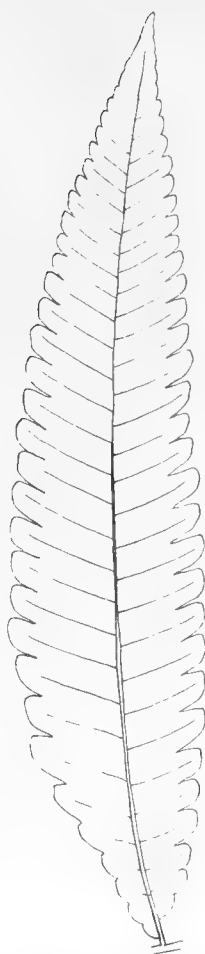


Fig. 45. *D. Rolandii* n. sp. Pinna $\times 1$; and fragments seen from both surfaces $\times 1\frac{1}{2}$.

Closely allied to *D. tetragona*, *D. megalodus* and *D. Poiteana*; it resembles *D. megalodus* in venation and cutting but differs by its densely setose capsules and lack of the adpressed stellate hairs on the leaf-tissue of the under-surfaces; from *D. Poiteana* it differs by the dense stellate pubescence of stipe and rachis, the deeper incised pinnæ and fewer anastomosing veins; as a rule only the basal pair of veins are united into a free excurrent vein. From *D. juruensis* it differs by its not-granulose surfaces and the not-bulbiferous rachis.

262. *Dryopteris tetragona* (Sw.) Urban, Symb. Ant. 4: 20. 1903; C. Chr. Ind. 297.

Syn. *Polypodium tetragonum* Sw. Prod. 132. 1788; Schkuhr, Kr. Gew. 1: tab. 18.

? *Polypodium androgynum* Poir. Enc. 5: 535. 1804.

Polypodium subtetragonum Link, Hort. Berol. 2: 105. 1833!

Polypodium imbricatum Liebm. Mex. Bregn. 58 (Vid. Selsk. Skr. V. 1: 210). 1849!

(For other synonyms see Index Fil.).

Type from Jamaica leg. SWARTZ (S!). As SWARTZ has named different forms *D. tetragonum* I consider that form typical, which is figured by SCHKUHR and which agrees perfectly with the original diagnosis.

D. tetragona has been considered a very variable species of a wide distribution, and this is to a certain degree true, but I have found that about half the number of the hundreds of specimens named *tetragona* and examined by me belong to more or less allied species, which it is not difficult to distinguish from *D. tetragona*. The common West-Indian typical form of this shows the following specific characters:

Rhizome shortly creeping or obliquely erect with brown scales, which are covered with stellate hairs. Stipes fasciculated like the rachis stramineous, quadrangular, slightly and very shortly hairy by stellate hairs. Leaves somewhat dimorphous, the pinnæ of the fertile leaves being narrower (2 cm br.) than those of the sterile ones (2½ cm br.). Lamina ovate with 6–12 pairs (generally 8) of lateral pinnæ and a similar terminal one. Pinnæ opposite or nearly so, short-stalked, oblong, acuminate, the lower ones narrowed towards the base, about 10 cm long by 2–2½ cm broad, incised ½ or ⅔ of the way down into close, acute, entire segments, herbaceous, dark-green, the under-surface generally nitid, glabrous on both surfaces, excepting the costæ (which are flat with two low furrows) and costulæ beneath, which are slightly hairy by spreading, simple, acute hairs, ciliate at the margins. Veins simple, 8–10-jugate, the basal pair united and sending a branch to the sinus, the two next free and reaching the margin immediately above the sinus. Sori inframedial, exindusiate; sporangia setose.

The form here described is very common in the West-Indies and there fairly constant; nearly identical forms are found on the continent from Mexico to Ecuador; the specimens herefrom have, however, generally a more dull under-surface. In the following I enumerate the islands and countries, from which I have seen specimens and add some of the more important collector-numbers.

West-Indian Islands: Trinidad, CRÜGER (B), BROADWAY nr. 3294 (B) — Grenada, EGGERS nr. 6331 (C) — St. Vincent, EGGERS nr. 6540 (W) — Barbados, EGGERS nr. 7113 (C), 7194 (C, RB) — Martinique, SIEBER, Fl. Mart. nr. 240 (B), Père DUSS nr. 1567 (RB, W), 1580 (W) 37 (C "*arboricole*") — Dominica, F. E. LLOYD nr. 538, 686 (W) — Guadeloupe, Père DUSS nr. 4032, 4113 (W) — Montserrat (H) — St. Kitts (S) — Antigua (B) — Saba (S) — St. Croix (H, W) — St. Thomas, EGGERS nr. 85 (B, C, H, W) — San Jan, EGGERS nr. 3056 (H) — Porto Rico, SINTENIS nr. 883 (S, W), 2498 (W), 2164 (C), GOLL nr. 935 (W), UNDERWOOD and GRIGGS nr. 274 (W), Mr. and Mrs. HELLER nr. 613 (W) — Haïti, NASH and TAYLOR nr. 1230 (W), Dr. WEINLAND nr. 9 (B) — Jamaica, MAXON nr. 826, 2184, 2565 (W), UNDERWOOD nr. 2911 (W), HART nr. 330 (W) — Cuba: Prov. Oriente, WRIGHT nr. 817 (S), MAXON nr. 4228 (W), LINDEN nr. 2191 (B); Prov. Sta. Clara, PRINGLE nr. 129 (W); Prov. Pinar del Rio, PALMER and RILEY nr. 291 (H, W), van HERMANN nr. 2069 (W); Prov. Habana, van HERMANN nr. 5053 (W) — Isle of Pines, PALMER and RILEY nr. 865 (W).

Florida: Marion Co., Miss REYNOLDS (W).

Mexico: PALMER nr. 1129 (W) — Laguna del Negro, ROVIROSA nr. 566 (W) — S. Luis Potosi: Tamasopo, PRINGLE nr. 3959 (B, C, S, W) — Vera Cruz: Papantla, LIEBMANN (H = *Pol. imbricatum* Liebm.; this was by FOURNIER identified with *Asp. varians* Mett., which, however, is the same as *D. Schaffneri*; LIEBMANN's type specimen consists of a single sterile leaf and belongs to typical *tetragona*. *P. tetragonum* Liebm. is *D. tetragona* var. 2 (see below).

Guatemala: Escuintla, DONNELL SMITH nr. 2439, 2440 (W) — Los Amates, C. et E. SELER nr. 3360, 3361 (B) — Mazatenango, MAXON and HAY nr. 3629 (W).

Honduras: San Pedro Sula, C. THIEME ed. J. D. S. nr. 5693 part. (W).

Nicaragua: Ceria Granada, P. LÉVY nr. 55 (Brux.).

Costa Rica: Port Limon, COOK and DOYLE nr. 435 (W).

Panama: Chiriqui, HART (W).

Colombia: Magdalena, LINDIG nr. 377 (B) — Sta. Marta, H. H. SMITH nr. 995 (C) — Galipan, MORITZ nr. 292 (B).

Venezuela: MORITZ nr. 205 (RB), 210 (B) — Pto. Cabello, KARSTEN nr. 71 pt. (B) — La Guayra, E. OTTO nr. 424 pt. (B) — San Julian, LYON (W).

Surinam: HOSTMANN & KAPPLER nr. 1775 (B).

Ecuador: El Recreo, EGGERS nr. 14900 (B).

var. *guadalupensis* (Fée).

Syn. *Goniopteris guadalupensis* Fée, 11 mém. 64 tab. 17 fig. 2. 1866.

Like typical *tetragona* in colour and pubescence but larger: pinnæ 3—3½ cm broad, incised only ½ of the way down, and 2—3 pairs of veins anastomosing. Not unlike *D. megalodus* but without stellate hairs beneath.

Guadeloupe: L'HERMINIER nr. 127 (B, C), Père DUSS nr. 233 (C).

Grenada: RAWSON W. RAWSON (B).

Trinidad: BROADWAY nr. 3293 (B).

Jamaica: HART (W).

In Central-America a long series of forms is to be found, which I dare not refer to any of the accepted species. As most of these forms are known only from a single or a few specimens, which rarely are fully identical, I prefer to place them under *D. tetragona* as *formæ dubiæ* instead of describing them as new species. In these forms the most important characters of the species *tetragona*, *obliterata*, *nicaraguensis* and *tristis* are combined. A special mark of the majority of these forms is that the basal veins do not anastomose but are connivent to sinus, although the leaves in other respects very much resemble *D. tetragona*, and it is possible that

this species varies in venation, still it is remarkable that the West-Indian common form always has the basal pair of veins anastomosing. These free-veined Central-American forms were mentioned before, f. inst. by FOURNIER (Bull. Soc. Fr. 19: 253), who only with doubt referred them to *D. tetragona*. I am inclined to consider var. 2, 5 and 6 distinct species, and var. 1 a variety of *D. tetragona*.

var. 1. Resembles in size and colour the common *tetragona*, but the basal veins connivent to sinus; 12—14 veins; sori subcostular; sporangia glabrous.

Guatemala: San Felipe, Depart. Retalhuleu, 2050', DONNELL SMITH nr. 2733 (W).

var. 2. Size of *D. tetragona*, colour intense, green, opaque not shining; lower veins occasionally anastomosing but more often the 2—3 lower ones are connivent to sinus. Sori small, dark, almost medial; sporangia glabrous.

This is *Polyp. tetragonum* Liebm. Mex. Bregn. 211, and it is probable that LIEBMANN was right in considering it specifically distinct from *D. tetragona* = *P. imbricatum* Liebm.

Mexico: Vera Cruz, Buenaventura, H. ROSS nr. 1073 (CC) — La Junta, H. ROSS nr. 1113 (CC), ROVIROSA (W), LIEBMANN (H).

Similar forms are

Guatemala: San Felipe, Dep. Retalhuleu, coffee-plantation, MAXON and HAY nr. 3513 (W); Rhizome long horizontally creeping; sori costular.

Honduras: San Pedro Sula, C. THIEME ed. J. D. S. nr. 5694 (C); Pinnæ narrower and very remote; sporangia setose.

var. 3. Very like var. 2, but segments falcate and veins about 15; lower pinnæ with a cuneate entire base and all long acuminate. Sporangia setose; rhizome erect. Perhaps a variety of *D. nicaraguensis*.

Guatemala: Cubilquitz, 350 m, v. TUERCKHEIM ed. J. D. S. nr. 8646 (C, W) — Volcan Tecuamburro, HEYDE et LUX ed J. D. S. nr. 4685 (B, C, W) — San Felipe, DONNELL SMITH nr. 2733 (W) — near the Finca Sepacuite, COOK and GRIGGS nr. 372 (W).

var. 4. Intermediate between *D. tetragona* and *D. obliterata*. Pinnæ incised $\frac{1}{2}$, basal pair of veins anastomosing with a short meniscioid branch; above them 2—4 pairs of veins connivent to sinus; sori medial; sporangia densely setose.

Guatemala: Gualan, Depart. Zacapa, W. A. KELLERMANN nr. 4869 (W).

var. 5. Here I place a number of rather different forms, which are intermediate between *D. tetragona* and *D. nicaraguensis*. They resemble the former in size, the setose sporangia, and the truncate or slightly narrowed base of the pinnæ, and the latter in venation, the light colour and the semiterete costa of the pinnæ beneath. As a rule the pinnæ are incised $\frac{1}{3}$ — $\frac{1}{2}$ with 6—8 veins connivent to sinus. Sori inframedial, small, yellowish. Texture firmer than *D. tetragona*, thinner than *D. nicaraguensis*.

Costa Rica: Puerto Viejo, PITTIER nr. 7489 (B, W), 6937 (C, W) — Guacimo, Llanos de Sta. Clara, TONDUZ nr. 14578 (W) — forêts de Bornea, PITTIER nr. 4824 (W) — Suerre, Llanuras de Sta. Clara, DONNELL SMITH nr. 6897 (W) — forêts de Tuis, TONDUZ nr. 11312 (C) — Jiménez, ALFARO nr. 16479 (C).

Panama: Bocas del Toro, J. HART nr. 55 (W).

var. 6. Not unlike *D. nicaraguensis* but texture thinner and segments less falcate, acute; lower pinnæ with a long cuneate, entire base as in *D. tristis*. Rachis and costæ beneath very shortly pulverulent by stellate hairs. Sori subcostular, setose by many multibranched hairs. Probably a new species, but the specimens are too fragmentary for a description.

Honduras: San Pedro Sula, 1500', C. THIEME ed. J. D. S. nr. 5693 part. (C, W).

In Brazil several forms occur, which most authors have referred to *D. tetragona*, but which are positively specifically different; see *D. scabra*, *D. incompleta* and others.

Nephrodium aureo-viridum Jenman, W. Ind. and Guiana Ferns 238. 1908 is, as far as I can judge from a photograph and a fragment of JENMAN's type-specimen (from British Guiana), received from Miss SLOSSON, not safely distinguishable from *D. tetragona*. Its sori are said to be indusiate when young.

263. *Dryopteris megalodus* (Schkuhr) Urban, Symb. Ant. 4: 21. 1903;
C. Chr. Ind. 277. — Fig. 41 a.

Syn. *Polypodium megalodus* Schkuhr, Kr. Gew. 1: 24 tab. 19 b. 1806.

Goniopteris quadrangularis Fée, 11 mém. 63 tab. 16 fig. 3. 1866.

(For other synonyms see Ind. Fil.).

SCHKUHR characterized this species by pointing out the presence of stellate hairs on the underside; as the species here named *D. megalodus* differs from other species of *Eugoniopteris*, *D. leucophlebia* excepted, by that character, and as SCHKUHR's plate very well illustrates our species, I have no doubt that I understand the species of SCHKUHR rightly. As suggested in Index Fil. *Polypodium pennatum* Poir. Enc. 5: 535. 1904 is probably the same species and, if so, POIRET's name has priority. I have seen the original specimen of it in Herb. LAMARCK (Mus. Paris) but unfortunately my notes do not permit me to identify it with *D. megalodus* with absolute certainty; it may be also *D. nephrodioides*. I prefer, therefore, to name the species by that name, under which it has been known for a century.

D. megalodus is not closely allied to *D. tetragona*, with which BAKER united it. It resembles that species mostly by its lower pair of veins anastomosing under a broad angle. From *D. glandulosa*, with which it has very often been confounded, it differs by its stellate hairs, venation and absence of aërophores.

The short-creeping rhizome is sparsely clothed with castaneous, small scales, which are stellato-pubescent throughout. Stipe and rachis often quadrangular and slightly puberulous by very small and soon deciduous stellate hairs. Pinnæ few, seldom 10 to a side, distinctly stalked, 15–25 cm long, 3–4 broad, herbaceous, dark-green, incised about $\frac{1}{3}$ to the costa into falcate, obtuse, faintly crenate, close lobes, glabrous above, costæ and veins beneath minutely puberulous by stellate hairs, leaf-tissue of the underside with microscopic stellate hairs or glabrous. Venation somewhat variable; veins 12–16 to a side, simple, the basal pair always anastomosing and sending a branch to a cartilagineous membrane, which extends from the sinus

one third of the way to the costa; along the edges of this membrane run the following 2—3 veins from the same costule to sinus; as a rule they run closely side by side, but sometimes they are found to be united. This being the case the venation is very similar to that of *D. glandulosa*. Sori medial, small, exindusiate. Sporangia glabrous; receptacle with stellate hairs.

The typical form of *D. megalodus* is probably common in most of the Lesser Antilles. In Central-America a larger form occurs, which can not always easily be distinguished from *D. nephrodioides* var. *Biolleyi*, especially if the apex of the frond is wanting. True *D. megalodus* has a distinct terminal pinna similar to the lateral ones and is exindusiate. METTENIUS says that he has seen indusium in *D. megalodus* (Fil. Lechl. II. 21.), but the specimens examined by him (now in B) all belong to *D. nephrodioides*.

Specimens seen:

Trinidad: HART nr. 3771 (W), 4120 (C); FENDLER nr. 21 (W); PREUSS nr. 1463 (B); BROADWAY nr. 3648 (RB).
 St. Vincent: H. H. and G. W. SMITH nr. 858 (C); EGGERS nr. 6641 (W).
 Martinique: SIEBER Syn. Fil. exs. nr. 160 (B).
 Porto Rico: BALBIS (B); EGGERS nr. 974 b (B, RB).
 San Domingo: MAYERHOFF nr. 103 (B); PRENLELOUP nr. 727 (C); WRIGHT, PARRY and BRUMMEL sine num. (W).
 Cuba: Valley of Rio Bayamita, MAXON nr. 3973 (W) — Farallones of la Perla, MAXON nr. 4387 (W) — above Jaguey, Yateras, MAXON nr. 4415 (W) — Monte Verde, WRIGHT nr. 1010 pt. (S).
 Venezuela: Caracas, MORITZ nr. 50 (B) — Lower Orinoco, RUSBY and SQUIRES nr. 388 (B, W); STEVENS (W) — Puerto Cabello, APPUN nr. 29 (RB).
 Colombia: Sta. Marta, H. H. SMITH nr. 2690 (C).
 Ecuador: RIMBACH nr. 104 (R).
 Costa Rica: WERCKLÉ (C).
 Guatemala: Cubilquitz, Alta Verapaz, v. TUERCKHEIM, ed. J. DONN. SMITH nr. 8812 (W).

264. *Dryopteris leucophlebia* (Christ) C. Chr. Ind. 274. 1905. — Fig. 41 b.

Syn. *Aspidium leucophlebium* Christ, Bull. L'Herb. Boiss. II. 4: 961. 1904.

Type from Costa Rica, leg. WERCKLÉ 1904 (C!).

A most distinct species, in cutting not unlike *D. hastata*, but much larger, in pubescence like *D. megalodus*, but less cut. In the type specimen, the leaf differs in shape from the other species of this section, with which it otherwise best corresponds, by its upper pinnæ being sessile with a broad base, adnate to rachis, widest on the lower side, and upwards at least confluent with the hastate lobed apex, thus not having a distinct terminal pinna. Such a one, however, present in another specimen from Costa Rica, which otherwise does not at all differ. — Pinnæ (the lower ones shortly stalked and cuneate at base) about 20 cm long by 4 cm broad, shallowly lobed, pale-green, firmly herbaceous, apparently glabrous, but by using a strong lense both surfaces are seen to be furnished with scattered, microscopical 4—6 branched hairs, which are best seen on the costæ and costulæ beneath. Rachis very finely puberulous by sessile, branched, deciduous hairs. Lobes obtusely rounded, subfalcate, crenate. Veins 10—12, the two basal opposite ones anastomosing under

a subacute angle in an excurrent branch, to which the next 2—3 alternately joint, and which run to the base of a hyaline membrane, to which the following 3—4 veins run out. Sori medial, small; sporangia setose by 2—4 short, simple setæ. I have seen a trace of a small, ciliate indusium.

Costa Rica, without locality, WERCKLÉ 1904 and 1905 (C) — Jiménez, Llanos de Santa Clara, Comarca de Limón, 200 m, DONNELL SMITH nr. 5094 (W) — Suerre, Llanuras de Santa Clara, 300 m, DONNELL SMITH nr. 6928 (W).

265. *Dryopteris Poiteana* (Bory) Urban, Symb. Ant. 4: 20. 1903; C. Chr. Ind. 285.

Syn. *Lastrea Poiteana* Bory, Dict. class. 9: 233. 1826.

Polypodium crenatum Sw. Prod. 132. 1788; Fl. Ind. occ. 1661. Hk. Bak.

Syn. 315; Jenm. Bull. Dept. Jam. n. s. 4: 133. 1897 (non Forskål 1775).

Phegopteris crenata Mett. Fil. Lips. 84. 1856.

[PLUMIER, Fil. tab. 111].

The *type* of *Lastrea Poiteana* Bory I have not seen, but there is no reason to doubt that it is the same as *Polypodium crenatum* Sw., which was collected by SWARTZ in Jamaica (S!) and which I take for the type of the species.

A well-known species, well described by JENMAN (loc. cit.) and others. The creeping rhizome is naked or clothed with some few scales, which bear some furcate hairs on the edges. The lamina, consisting of 3—6 pairs of lateral pinnæ about 4 cm broad and similar terminal one is more or less soft-hairy beneath, especially on costæ and veins, glabrous or with a few setæ on the veins above; hairs simple. Costules prominent, stramineous. Margins subentire, crenate or broadly and shallowly serrate, rarely lobed. Veins 6—8-jugate, distant, the lower 2—4 pairs up-curved and anastomosing under an acute angle and meniscioid, the next 2—3 pairs alternately united into a common branch or often interrupted before meeting the opposite vein. Sori a little below the middle of the vein; sporangia when young furnished by 4—6 long, simple hairs. In the same sorus one finds as well quite young sporangia as ripe ones and intermediate states.

D. Poiteana varies mainly in pubescence; some specimens are almost glabrous and then resemble *D. meniscioides*, others, especially the andine specimens, much hairy and difficult to distinguish from *D. Ghiesbreghtii*; still I think it possible to determine specimens of these three species by the venation; in *D. Poiteana* rarely more than 4 pairs of veins are meniscioid and meet under acute angles; in the two other species 8—10 or more pairs of veins are meniscioid and meet under broad angles, and their sporangia seem to be glabrous even as young.

Goniopteris Rivoirei Fée, Gen. 2535 1850—52; 11 mém. tab. 18 fig. 2 from Gueloupe seems according to the figure and a specimen in (B) so named to be a small form of *D. Poiteana* with large sori. JENMAN believed it to be *D. obliterated*, while BAKER (Ann. of Bot. 5: 460. 1891) restored it as a species. It must, however, be remarked, that the two reduced figures of the whole plant of *Gon. Rivoirei* and

G. hastata on Fée's plates 18 f. 1—2 are confounded. — The veins of *G. Rivoirei* are not so upcurved as in typical *D. Poiteana*.

D. Poiteana has a wide distribution; it is found in most West-Indian islands and from Guatemala to Peru and Northern Brazil. I enumerate here some of the more important collector-numbers seen by me.

West-Indies. Trinidad: HART nr. 560 (C); Bot. Gard. Herb. nr. 36 (W); FENDLER nr. 20 (W); EGGERS (H) — Tobago: EGGERS nr. 5808 (W) — Grenada: EGGERS nr. 6114 b (C); SHERRING nr. 110 (W) — St. Vincent: EGGERS nr. 6650 (W) — Martinique: Père DUSS nr. 1565 (C, W), 4118 (W) — Dominica: EGGERS s. n. (W) — Guadeloupe: L'HERMINIER nr. 124 (B); Père DUSS nr. 16 (C; proliferous), 4069 b (W) — Montserrat: RYAN (H) — St. Croix: ISERT (H); RYAN (H); BØRGESSEN (CC) — St. Thomas: (H) — Porto Rico: SENTENIS nr. 8751 (W), 5965 (C, W); EGGERS nr. 1342 (B, C); GOLL nr. 131, 315, 340 (W); A. A. HELLER nr. 6178 (W); Mr. and Mrs. HELLER nr. 612 (W); UNDERWOOD and GRIGGS nr. 887 (W) — Haiti: MEYERHOFF (B); WRIGHT, PARRY and BRUMMEL (W) — Jamaica: SWARTZ (S); HART nr. 237 (W); CLUTE nr. 245 (W) — Cuba: Pinar del Rio, PALMER and RILEY nr. 499 (W); Oriente, WRIGHT nr. 3963 (S).

Guatemala: Los Amates, Dept. Izabal, KELLERMANN nr. 4855 (W).

Honduras: San Pedro Sula, C. THIEME, ed. DONN. SMITH nr. 5686 (B, C, W).

Costa Rica: Matina, PITTIER nr. 9749 (W). — A variety with the sori close to the costule, multi-setose sporangia and the veins often interrupted, not confluent. — WERCKLÉ (CC).

Panama: Bocas del Toro, HART nr. 56 (W).

Colombia: Sta. Marta, H. H. SMITH nr. 1043 (C, Rg).

Ecuador: Andes quitenses, SODIRO (C) — El Recreo, EGGERS nr. 15123 (B, W) — Balao, EGGERS nr. 14207 (W).

Peru: Tabalosos, STÜBEL nr. 1088 (B).

Venezuela: Caripe, MORITZ nr. 57, 192, 197 (B) — San Julian, M. W. LYON (W) — Puerto Cabello, LANSBERG nr. 67 (B); FUNCK nr. 7 (B).

Guiana: Surinam, Paramaribo, KAPPLER nr. 1755 (B) — Cayenne, LEPRIEUR (B).

Brazil: Pará, Serra de Baturité, HUBER nr. G. 129 (C) — Bahia, SALZMANN (C); LUSCHNATH nr. 122 (B).

266. *Dryopteris meniscioides* (Liebm.) C. Chr. Ind. 277. 1905.

Syn. *Polypodium meniscioides* Liebm. Vid. Selsk. Skr. V. 1: 211 (seors. 59). 1849. Hk. Bak. Syn. ed. II. 314; Rovirosa, Pteridografia del Sur de Mexico 241 pl. 41 (bona!).

Type from Mexico: Teotalcingo, Dept. Oajaca, leg. LIEBMANN nr. 2407 (H!). Besides the specimens of LIEBMANN's collection I have only seen one more from Chiapas, G. MUNCH nr. 149 (C).

Perhaps a variety of *D. Poiteana*, which does not seem to occur in Mexico. It differs 1) by its perfectly glabrous frond, which is rigidly membranous or subcoriaceous, 2) its crenate or very shallowly serrulate pinnæ, which are gradually narrowed from the middle to the base (in *D. Poiteana* the pinnæ have a short cuneate or rounded base), and 3) by venation. Veins 8—10-jugate, of which about the 8 pairs are meniscioid, but united under more acute angles than in the following species. Sori a little inframedial, small and not rarely oblong; sometimes 2 or even 3 sori are found on the same vein, and these being confluent the species resembles

a *Meniscium*. It is really an intermediate link between *Goniopteris* and *Meniscium* and therefore I agree with METTENIUS in reducing *Meniscium* to a subgenus of *Dryopteris*. The difference is found mainly in the position of sori; if the sori are born on the vein near the point, where two opposite veins unite, the two opposite sori become with age confluent and thus we have a meniscioid sorus. — *D. meniscioides* has 8—10 lateral pinnæ, about 20 cm long and $3\frac{1}{2}$ cm broad. Their edges are cartilaginous, a little thickened and sparsely ciliate. — *Goniopteris rostrata* Fée, referred hereto in my Index, belongs to *D. glandulosa*.

267. *Dryopteris Ghiesbreghtii* (Linden) C. Chr. Ind. 267. 1905.

Syn. *Polypodium Ghiesbreghtii* Linden; Bak. Syn. 315. 1867.

Goniopteris mollis Fée, Gen. 252. 1850—52; 11 mém. tab. 24 fig. 1. 1866.

Polypodium crenatum Hk. Fil. exot. tab. 84. 1859.

Type from Mexico: Teapa, Tabasco leg. LINDEN nr. 1499 (B!).

A larger plant than *D. Poiteana* with only 2—5 pairs of lateral sessile pinnæ, which are 20—30 cm long by 5—8 cm broad, from subentire to irregularly serrulate or lobed. Stipe 3—4 dm long, raised from a creeping, nearly scaleless rhizome, glabrous. Pinnæ slightly strigose on the veins above, densely soft-hairy beneath. Costæ and costules as a rule not stramineous. Veins in 10—12 pairs, all, the 2—3 uppermost pairs excepted, anastomosing under broad angles, generally all meniscioid. Sori about medial on the vein, in some specimens distinctly inframedial, in others nearer the anastomosing point. Sporangia glabrous even as young but intermixed with long hairs from the receptacle.

Apparently a very distinct species when only the typical form is considered, but it is not always easy to distinguish from the andine forms of *D. Poiteana*. The best characters is the venation and the glabrous sporangia. The species is confined to Central-America, from Southern Mexico to Costa Rica, a region where *D. Poiteana* does not occur (Mexico) or is rare. I have seen the following specimens besides numerous cultivated ones.

Mexico: Tabasco, LINDEN nr. 1499 (B); ROVIROSA nr. 575 (W).

Guatemala: Alta Verapaz, Cubilquitz, v. TUERCKHEIM, ed. DONNELL SMITH nr. 8648 (C, W), II 876 (W); Sacolal, v. TUERCKHEIM ed. DONN. SMITH nr. 1407 (W); Chamiquin, v. TUERCKHEIM nr. 550 (W); near Finca Sepacuite, COOK and GRIGGS nr. 281 and 664 (W); Secanquim, MAXON and HAY nr. 3132 (W); Puerto Barrios, CHAS. C. DEAM nr. 450 (W); near Cacao, BARBER nr. 194 (W).

Nicaragua: Canada Yasira, Dept. Metagalpa, E. ROTHSCHUH nr. 117 (B).

Costa Rica: Talamanca, TONDUZ nr. 8659 (W); Hacienda de Guacimo, TONDUZ nr. 14572 (W); Puerto Viejo, PITTIER nr. 7503 (W); Jiménez, ALFARO nr. 157 (W), 16517 (C); Port Limon, COOK and DOYLE nr. 419 (W); Río Hondo, COOK and DOYLE nr. 496 (W).

Unknown species of *Goniopteris*.

1. *Goniopteris macrocladia* Fée, Cr. vasc. Brés. 1: 106 tab. 33 right-hand figure ("fig. 1"). 1869 — Brazil: Sta Catharina, ALBURQUERQUE. It seems to be distinct from all Brazilian species known to me.
2. *Nephrodium nemorale* Sodiro, Cr. vasc. quit. 267. 1893; *Dryopteris nemoralis* C. Chr. Ind. 279 — Ecuador.
3. *Polypodium Urbani* Sod. l. c. 301; *Dryopteris Urbani* C. Chr. Ind. 299 — Ecuador.
4. *Aspidium hemiotis* Christ, Hedwigia 45: 191. 1906 — Amazonas.

Subgenus 10. *Meniscium* (Schreber).

The old genus *Meniscium* is a well-known and apparently one of the most distinct groups of ferns. The species from the Old and the New World referred to it are, however, certainly not very close relatives, and I now firmly believe that they are even not congeneric, and, moreover, that the American species can not be generically separated from *Goniopteris*. In this view I agree with METTENIUS (see Fil. Lechl. II. 19). Within the second group of *Goniopteris*, *Eugoniopteris*, we find an unbroken row of species, from free-veined species to such species as *D. Ghiesbreghtii* and *D. meniscioides*, the venation of which is perfectly meniscioid. The other important character of *Meniscium*: the confluent sori, also is insufficient as distinguishing mark between *Goniopteris* and *Meniscium*. Two species as *D. meniscioides* and *D. ensiformis* described below are really so closely related that it should be very unnatural to place them in two different genera. Still I have failed to find stellate hairs in all species of *Meniscium* and this together with the whole uniform character of the species justify the segregation of the species from the subgenus *Goniopteris* and the referring of them to a proper subgenus, *Meniscium*. If one should prefer to separate both these subgenera from the others, they should certainly be united into a single genus.

The Old World's species of *Meniscium*, as commonly understood, are considerably different from the American ones, and I have no doubt that they must be referred to *Cyclosorus* being a specialized group of that subgenus. Thus the two subgenera (or genera) *Cyclosorus* and *Goniopteris* includes each a series of species, from free-veined forms to such having meniscioid venation. The American species of *Cyclosorus* do not show a venation intermediate between the simple, goniopteroid venation of *D. mollis* and meniscioid veins, but such forms we find in Asia. I shall here only refer to such forms as *Polypodium urophyllum* Wall. and *Meniscium cuspidatum* Bl.

My material of the American species of *Meniscium* is not so comprehensive as that of the other subspecies of *Dryopteris*. I can not, therefore, give here a review of the species. The group is represented in America by perhaps a dozen species,

not a few of the forms in Ind. Fil. referred to *D. reticulata* being valid species. METTENIUS (Fil. Leschl. II. 21—25) has given a review of the species known till 1859; and I can, as far as I know the species, fully agree with his treatment. Since then several species have been described, and below I describe two others as new, the first of these being an interesting intermediate between the true species of *Meniscium* and *D. meniscioides* belonging to *Goniopteris*.

268. *Dryopteris ensiformis* n. sp. — Fig. 46.

Type from Costa Rica: Lisières des pâturages à La Palma, 1459 m, leg. Ad. TONDUZ nr. 12533 (C).

Rhizomate? Stipitibus ad basin 2 cm crassis, ad 1 m longis, glabris. Lamina visa incompleta, pinnata, omnino glabra, coriacea, brunneo-viridi. Pinnis inter se 6 cm remotis, ensiformibus, 30—35 cm longis, ad 5 cm latis, apice caudata, basi abrupte rotundata, subcordata subsessilibus, infra aërophoro instructa (?); marginibus irregulariter repandulis, cartilagineis. Venis secundariis 4 mm circiter inter se remotis, curvatis, tertiariis simplicibus, 10—12-jugis, omnibus more *Meniscii* anastomosantibus; radiis late clavatis, liberis vel saepe omnibus confluentibus venam venis secundariis parallellam formantibus. Sori parvis, atrorufis, in venis medialibus, rotundis, raro confluentibus, exindusiatis. Sporangia receptaculisque glabris.

This new species is a peculiar fern, in general habit a *Meniscium*, but in venation and sori not unlike *D. meniscioides*, from which it differs by the much closer secondary and tertiary veins and by the subcordate base of the pinnæ, which have nearly the same breadth from base to short of the apex. It also resembles *D. (Meniscium) Andreana* (Sod.) C. Chr., which in sori is a true *Meniscium*. The most remarkable features of our new species are 1) the presence of an aërophore, 2) the edges of the pinnæ and 3) the venation. Ad 1). At the insertion of the costæ is to be found a large, blackbrown spot, which evidently is the scar of a fallen aërophore. Ad 2). The pinnæ are bordered by a broad, cartilagineous line. Ad 3). The excurrent veinlet formed by two anastomosing veins is always much longer than found in other species of *Meniscium*; often it reaches nearly to the next cross-vein and then ends in a broad clavate apex, in which a brown, oblong pellucid spot is seen on the upperside; but often the veinlet is continued to the next cross-vein and the veinlets between two secondary veins then together form a continued vein parallel to the secondary veins. In this case the veinlets are thickened above the middle and enclose a brown pellucid spot as described, above which they again narrow.

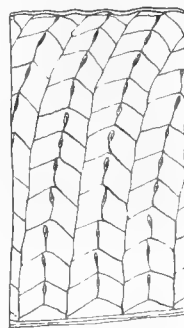
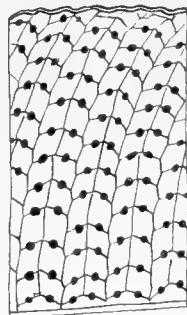


Fig. 46. Fragments of *D. ensiformis* n. sp. seen from both surfaces $\times 1\frac{1}{2}$.

269. *Dryopteris Andreana* (Sod.) C. Chr. Ind. 252. 1905.

Syn. *Meniscium Andreanum* Sodiro, Rec. 71. 1883; Cr. vasc. quit. 392. 1893.

Type from Ecuador, Andes quitenses, leg. SODIRO (C!).

Closely related to the preceding species, but its sori all meniscioid, the base of the sessile pinnæ not subcordate but bluntly rounded with the lower side of the upper pinnæ adnate to rachis, without aërophore (?), rachis, costæ on both sides and secondary veins beneath sparsely pubescent by very short, simple hairs and the venation somewhat different. Secondary veins at distances of 4–6 mm with about 17–18 transverse, close and rather convex tertiary veins; the free, included veinlet short, rarely extending beyond the middle of the areole, sometimes nearly obsolete, its apex seen from above often broadly clavate as in *D. ensiformis*. Receptacle glabrous. — Pinnæ 20–40 cm long, 5–7 cm broad (18–24 by 3–5 cm according to SODIRO). — The species is not very near *D. reticulata*.

270. *Dryopteris pachysora* Hieron. Hedwigia 46: 351 tab. 7 fig. 19. 1907.

Type from Ecuador, Mt. Abitagua, STÜBEL nr. 897 (B!).

Differs from *D. Andreana* but agrees with the following two species by its long-stalked pinnæ with a subequally cuneate base. Transverse veins 12–14, the free veinlets long, extending beyond the middle of the areole, its apex scarcely clavate, or occasionally reaching the next cross-vein. The confluent sori very short and thick. Costæ above very sparsely setose, the leaf otherwise glabrous.

271. *Dryopteris falcata* (Liebm.) C. Chr. comb. nov.

Syn. *Meniscium falcatum* Liebm., Vid. Selsk. Skr. V. 1: 183. 1849.

Phegopteris falcata Mett. Fil. Lechl. II. 23. 1859.

Meniscium Jungersenii Fée, Gen. 223. 1850–52; Fourn. Mex. Pl. 1: 73 (as *Jurgensenii*).

Type from Mexico: Depart. Oajaca, Lacobe, LIEBMANN (H!). Also Costa Rica, Buenos Ayres, PITTIER nr. 4839 (C).

A very large species, widely different from typical *D. reticulata*. Rachis angular, glabrous, brownish-stramineous. Pinnæ linear-elongate, 40 cm long, $3\frac{1}{2}$ cm broad, dark-green, thinly membranous, apex long acuminate, margins undulato-crenulate, base subequally cuneate, long-stalked (petiole 2–2 $\frac{1}{2}$ cm long), upper surface, the very sparsely setose costa excepted, glabrous, costa beneath short-hairy. Secondary veins at distances of about 3 mm with 12–13 transverse, a little convex veins; the included veinlets as a rule free but terminating in a subclavate apex above the middle of the areole, or not rarely prolonged and united with the next cross-vein. Sori small, nearly round, placed at the base of the free veinlet. Receptacle glabrous.

Differs from *D. pachysora* by its much longer pinnæ of thinner texture, the costæ hairy beneath and its small sori, from *D. reticulata* by its long and long-stalked pinnæ with a cuneate, subequal base, pubescent costæ and short sori.

272. *Dryopteris lingulata* n. sp.

Syn. *Phegopteris Andreana* Christ, Prim. Fl. Costar. 3: 35. 1901.

Type from Costa Rica: Forêts de la vallée du Rio Hondo près Madre de Dios, 200 m, leg. PITTIER nr. 10349 (C).

Rhizomate? Stipite incompleto 60 cm longo, griseo, glabro, anguloso. Lamina 60—70 cm longa, chartacea, laete-viridi, costis supra minute puberulis exceptis omnino glabra, impari-pinnata. Pinnis lateralibus, 5-jugis, 12 cm inter se remotis, lingulatis seu elliptico-lanceolatis, 30—35 cm longis, medio 7 cm latis, caudato-acuminatis, marginibus undulatis, subintegris inferioribus, petiolo 2—4 cm longo stipitatis, basi cuneata, supremis sessilibus basi inferiore ad rachin adnata, superiore rotundato-cuneata. Venis secundariis 6—8 mm inter se distantibus, tertiariis 16—17 inter se 3 mm remotis convexis; venulis liberis apice late clavato ultra medium areolae in parenchymate desinentibus, seu ad venam transversam superiorem productis. Soris lunatis, angustis, saepe 4—5 mm longis. Receptaculis glabris.

Differs from the two preceding species, which it resembles by the long-stalked inferior pinnæ with a cuneate base, by the shape of the pinnæ and its long, narrow, convex sori. A single pinna resembles not a little an entire, small leaf of *D. gigantea*, from which it differs by its glabrous rachis and costæ beneath.

273. *Dryopteris gigantea* (Mett.) C. Chr. Index 267. 1905.

Syn. *Meniscium giganteum* Mett. Fil. Lechl. 1: 19. 1856.

Type from Peru: St. Gavan, LECHLER nr. 2292 (B!, S). Other specimens from Ecuador, SODIRO (C), Columbia, Costa Rica, PITTIER nr. 1163 and 9448 (C).

Well-marked by its entire frond, hairy midrib and principal veins, and by the cross-veins being covered with sporangia from end to end. It appears that the species also can be pinnate with 1—3 pairs of lateral pinnæ.

The following forms, in Index Fil. referred to *D. reticulata*, are certainly good species (for synonymy see Ind. Fil.).

274. *Dryopteris reticulata* (L.) Urb.

The typical form of this is West-Indian, but very similar forms are found in Central-America. In Brazil the species scarcely occurs.

275. *Dryopteris sorbifolia* (Jacq.) Hieron. Hedwigia 46: 350. 1907.

Area: Mexico along the Andes to Peru.

HIERONYMUS refers to this species *Meniscium arborescens* Willd. and *M. Kappelerianum* Fée and he regards *Phegopteris mollis* Mett. as a variety.

276. *Meniscium macrophyllum* Kze. — Bahia and Guiana.

277. *Phegopteris membranacea* Mett. — Peru, LECHLER nr. 1785.

278. *Dryopteris longifolia* (Fée) Hieron. *Hedwigia* 46: 351. 1907.

Meniscium longifolium Fée, Cr. vasc. Brés. 1: 84 tab. 25 fig. 2 (an Desv. Prodr. 223. 1827). — Brazil, GLAZIOU nr. 1747 (C) and several other specimens.

This is not very near *D. reticulata*; it resembles in general habit *D. falcata*, but the petioles of the pinnæ are shorter and their base rounded or very shortly cuneate, the main difference is, however, the pubescence. The underside is finely pubescent throughout and the receptacle is rather densely setose. By this last character it differs from all species mentioned above but agrees with a specimen from Plateau de Goyaz, GLAZIOU nr. 22631 (C), which probably belongs to an undescribed species. It is scarcely different from *D. longifolia* in pubescence, in shape of pinnæ it resembles closely typical *D. reticulata*. A third form, also from Goyaz, GLAZIOU nr. 22633 (C) has similarly pilose receptacles but long, narrow pinnæ not unlike those of *D. angustifolia*.

The following two species have always been considered good species.

279. *Dryopteris angustifolia* (Willd.) Urban.

West-Indies, Honduras, Ecuador.

280. *Dryopteris serrata* (Cav.) C. Chr. Ind. 291.

Brazil, Guiana, Costa Rica—Ecuador.

Unknown to me are

Meniscium Salzmanni Fée, Gen. 223 — Bahia.

M. chrysodioides Fée, Gen. 225 — America austr., coll. PAMPLIN 55.

M. elongatum Fée, Cr. vasc. Brés. 1: 83 tab. 25 fig. 1. 1869 — Brazil, GLAZIOU 1169.

M. sessilifolium Pohl; Fée, l. c. 84 — Brazil, POHL.

Unknown species of uncertain position.

The descriptions of the following species are too short or do not mention such characters, which might indicate the right position of the species described in my classification.

1. *Goniopteris lucida* Fée Gen. 253 — Guadeloupe.

2. *Phegopteris ciliata* Fée, Gen. 248 — Cuba, LINDEN nr. 270.

3. *Aspidium tenuiculum* Fée, Gen. 292 — Cuba?, LINDEN nr. 2.

4. *Phegopteris amplificata* Fée, 8 mém. 89 — Mexico, SCHAFFNER nr. 219.

5. *Phegopteris melanorachis* Fée, 8 mém. 91 — Mexico, SCHAFFNER nr. 238.

Known as sterile only; probably an *Alsophila*.

6. *Aspidium cheiloptotium* Fée, 8 mém. 103 — Mexico, SCHAFFNER (nr. 499 t. FOURN.).

7. *Aspidium Van Heurckii* Fourn. Pl. Mex. 1: 97 — Mexico, BOTT. nr. 1461 “Differt a præcedente (*A. Kunzei* Fée = *Dryopteris cheilanthoides* var. *resinoso-foetida*) indusio villosa”, the whole “description”.

Additional notes to § 1 Eudryopteris.

- (1) Recently I have received from Dr. H. Ross, Munich, a copy of his paper: Contributions à la flore du Mexique (Société scientifique “Antonio Alzate”, Mémoires 32. 1912), wherein my new species *Dryopteris Rossii*, mentioned above pag. 72, is described on pag. 179 and figured on plate XII.
- (2) To § *Eudryopteris* belong our common European *D. spinulosa* with its subspecies *dilatata*. A specimen of the subspecies was collected by Mrs. VALENTINER in 1909–1911 on the Falkland Islands. The specimen, which I have seen (Kew), is so exactly like our commonest North-European form, that I am nearly convinced, that the species was introduced into the far Islands.

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Names of accepted species in italics. An asterisk before the name indicates that the species is illustrated in the present work or in my other papers on American *Dryopteris*. In the following list both the number of the species and the page are quoted. The names without reference to page are synonyms, which are to be found in my earlier papers. *Aspidium lasiesthes* Kze, f. inst., is a name not found in the present work; the figure 84 below indicates that the name is a synonym of species nr. 84 *D. oligocarpa*. Under this species we find (p. 136) a reference to "Revision nr. 5", where the synonym is quoted.

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<i>tenuifolia</i> Pr.	34	96	<i>conspersum</i> Schrad.	190	176	<i>longipilosum</i> Sod.		156
Leptogramma (subge-			<i>corazonense</i> Bak.	150		<i>lustratum</i> Hieron.	80	
nus 8)		196	<i>costale</i> Bak.	185	169	<i>macradenium</i> Sod.	172	
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10)		268	<i>crenulaeum</i> Jenm.		156	<i>macrourum</i> Schott.	190	176
Meniscium			<i>crinitum</i> Sod.	34	97	<i>mexicanum</i> Pr.	4	68
<i>Andreanum</i> Sod.	269	270	<i>Cumingianum</i> J. Sm.	206	208	<i>microsorium</i> Hook.	50	109
<i>arborescens</i> Willd.	275		<i>decrescens</i> Bak.	169		<i>negligens</i> Jenm.		131
<i>chrosodioides</i> Fée.		272	<i>deflexum</i> Pr.	79		<i>nemorale</i> Sod.		268
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<i>falcatum</i> Liebm.	271	270	<i>dejectum</i> Jenm.	188	171	<i>nimbatum</i> Jenm.		116
<i>giganteum</i> Mett.	273	271	<i>deversum</i> Ros.	190	179	<i>nitidulum</i> Bak.	226	229
<i>Jungersenii</i> Fée.	271	270	<i>devolvens</i> Bak.	245	246	<i>Nockianum</i> Jenm.	89	
<i>Kapplerianum</i> Fée.	275	271	<i>diplazioides</i> Hook.	122		<i>oppositum</i> Fée.		195
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<i>macrophyllum</i> Kze.	276	271	<i>dissimile</i> Schrad.	190	176	<i>paludosum</i> Liebm.	201	193
			<i>Eggersii</i> Hieron.	238	240	<i>panamensis</i> Pr.	67	
			<i>elegantulum</i> Sod.		156	<i>patens</i> Jenm.	192	180

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RECHERCHES

SUR LES

NOMBRES DE BERNOULLI

PAR

NIELS NIELSEN

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER. 7. RÆKKE, NATURV. OG MATHEMATISK AFD. X. 3



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Introduction.

La méthode classique d'EULER¹⁾, suivie dans l'étude des nombres de BERNOULLI, des coefficients des tangentes et des nombres d'EULER ou coefficients des sécantes, est fondée sur les séries de puissances obtenues pour les fonctions méromorphes élémentaires

$$(1) \quad \operatorname{tg} x, \quad \frac{1}{x} - \cot x, \quad \frac{x}{\sin x}, \quad \frac{1}{\cos x}, \quad \frac{x}{1 - e^{-x}}, \quad \frac{2}{1 + e^{-x}}$$

combinées avec les propriétés fondamentales de ces fonctions et avec les séries de puissances toujours convergentes obtenues pour les fonctions entières

$$(2) \quad \sin x, \quad \cos x, \quad e^{-x}.$$

Or, désignons par a un nombre complexe quelconque, puis combinons les séries (1) avec celles obtenues pour les puissances

$$(3) \quad \left(\frac{\sin x}{x}\right)^a, \quad (\cos x)^a, \quad \left(\frac{1 - e^{-x}}{x}\right)^a, \quad \left(\frac{1 + e^{-x}}{2}\right)^a,$$

nous verrons que les formules classiques, concernant les nombres susdits, ne sont que des représentants isolés d'une infinité de formules de ce genre.

L'illustre GÖPEL²⁾, déjà en 1843, a remarqué de telles généralisations des formules classiques, et il ajoute:

Da man sich dem Obigen zufolge Recursionsformeln in beliebiger Menge verschaffen kann, so möchte es nicht von grosser Erheblichkeit sein, deren neue aufzusuchen; es gelänge denn eine solche aufzufinden, die einen tieferen Blick in den Bau dieser Zahlen verstatte.

¹⁾ Opuscula analytica, t. II, p. 257—274; Saint-Petersbourg 1785. Voir, pour des recherches ultérieures d'autres géomètres, la belle Monographie de M. LOUIS SAALSCHÜTZ: Vorlesungen über die Bernoullischen Zahlen. Berlin 1893.

²⁾ Archiv de Grunert, t. 3, p. 65; 1843.

Cette réserve de GÖPEL, dans ses remarques relatives à une Note de SCHLÖMILCH ¹⁾, est très intéressante; car HERMITE ²⁾ et STERN ³⁾ ont appliqué, précisément de la manière indiquée par GÖPEL, les formules récurrentes de MOIVRE et de JACOBI pour les nombres de BERNOULLI.

De plus, il faut ajouter que des formules récurrentes, d'une forme très bizarre nous le verrons, sont très utiles et dans la théorie des nombres en question et dans la théorie des nombres en général. C'est-à-dire qu'il est impossible de condamner dès à présent comme étant sans valeur une telle formule récurrente.

Quant à la méthode classique que nous venons de mentionner, nous remarquons expressément, qu'il nous semble peu naturel et peu systématique de fonder l'étude des nombres rationnels en question sur des éléments purement transcendants. De plus, une théorie élémentaire et directe est beaucoup plus simple et générale que la méthode classique.

On pourrait dire que les résultats classiques concernant les nombres B_n , T_n et E_n sont trouvés par hasard, sans méthodes générales et systématiques.

En effet, on sait que des formules très analogues sont trouvées séparément et par des considérations très différentes, tandis que de telles formules sont en réalité des cas particuliers d'une même formule générale, comme nous le verrons plusieurs fois dans les pages suivantes.

Comme un exemple caractéristique de ce genre, nous prenons les formules récurrentes incomplètes, déduites dans le chapitre IV à l'aide d'un seul polynôme entier. De cette manière nous obtenons d'un seul coup toutes les formules connues de ce genre et un grand nombre d'autres.

Pour citer un autre exemple, nous remarquons que les formules récurrentes générales contenues dans les paragraphes 11, 12 et 13, savoir les formules d'EULER et de BERNOULLI concernant des sommes de puissances numériques, nous donnent comme des cas particuliers un nombre de formules classiques déduites séparément et par des considérations très différentes.

Remarquons en passant que les formules de BERNOULLI et d'EULER ne sont que des cas particuliers des deux équations aux différences finies qui figurent dans nos définitions de $\varphi_n(x)$ et $\chi_n(x)$.

On sait que la méthode développée dans le paragraphe 7 est très connue — pour des valeurs entières de la variable x .

La méthode du paragraphe 9, nouvelle peut-être, est essentielle dans la théorie des nombres B_n , T_n et E_n .

En effet, nous déduisons à l'aide d'une formule récurrente, d'une certaine forme très générale, pour les B_n ou les T_n une identité algébrique contenant une variable complexe, identité qui est une conséquence immédiate de la formule numérique

¹⁾ Archiv de Grunert, t. 3, p. 9 18; 1843.

²⁾ Journal de Crelle, t. 81, p. 93—95; 1876.

³⁾ Ibid. t. 84, p. 267 269; 1878.

susdite. De cette manière nous pouvons traiter, en même temps, les trois groupes de nombres B_n , T_n et E_n .

Remarquons que les formules très générales et très remarquables de M. HAUSSNER¹⁾ sont de la forme susdite²⁾ et que la plupart des formules récursives connues ont la même propriété.

Dans un Mémoire récent³⁾ j'ai étudié les suites harmoniques comme des généralisations des suites formées des $\varphi_n(x)$ et des $\chi_n(x)$, mais sans appliquer aux nombres B_n , T_n et E_n les résultats généraux ainsi obtenus.

Une telle application est le but principal du présent Mémoire, tandis que je me réserve de revenir, à une autre occasion, sur les applications des résultats ainsi trouvés à la formule sommatoire d'EULER et MACLAURIN et à la théorie des nombres.

On voit que ma méthode est entièrement élémentaire, parce qu'elle n'exige que la formule binomiale d'un exposant positif entier et, ce qui est la même chose, la formule de TAYLOR pour un polynome entier.

Quant aux résultats ainsi obtenus, ils me semblent exiger une révision profonde de toute notre connaissance relative aux nombres de BERNOULLI. On peut comparer encore notre généralisation du théorème de LIPSCHITZ.

J'ai ajouté la Table des simples formules récursives pour la clarté du texte. La Table ne contient que des formules de la forme la plus simple, et il faut remarquer que la plupart de ces formules sont obtenues en donnant, dans des formules beaucoup plus générales, des valeurs particulières à un paramètre.

Or, les formules de la Table étant aussi simples que leurs représentants épars connus et classiques, il est évident que ces formules classiques sont trouvées par hasard, sans méthodes systématiques et générales.

J'indique par un astérisque ajouté au numéro, que je me rapelle avoir vu autrefois la formule récursive en question; cependant je ne peux pas prétendre que ces indications soient complètes.

¹⁾ Göttinger Nachrichten 1893, p. 777—809.

²⁾ J'ai réussi à généraliser, par la méthode susdite, les formules de M. HAUSSNER qui se présentent comme des représentants isolés d'une grande classe de formules de ce genre. Le Mémoire en question paraîtra dans les *Berichte der kgl. sächsischen Gesellschaft der Wissenschaften zu Leipzig*.

³⁾ Le Mémoire paraîtra dans les *Annales de l'École Normale*.

Copenhague, le 29 septembre 1912.

Niels Nielsen.

CHAPITRE I.

Les fonctions de Bernoulli et d'Euler.

§ 1. Remarques sur les suites harmoniques.

Nous désignons comme suite harmonique une suite illimitée de polynomes entiers

$$(1) \quad f_0(x), f_1(x), f_2(x), \dots, f_n(x), \dots$$

qui satisfont aux deux conditions suivantes:

1° $f_n(x)$ est toujours du degré n par rapport à x .

2° Supposons $n > 1$, nous aurons constamment

$$(2) \quad f_n'(x) = f_{n-1}(x).$$

Désignons par

$$(3) \quad a_0, a_1, a_2, \dots, a_n, \dots$$

une suite illimitée quelconque, telle que $a_0 \neq 0$, puis posons pour $n \geq 0$

$$(4) \quad f_n(x) = \sum_{s=0}^{s=n} \frac{a_s x^s}{(n-s)!},$$

il est évident que les polynomes $f_n(x)$ ainsi définis forment une suite harmonique.

Inversement, on voit sur-le-champ que les éléments d'une suite harmonique quelconque se présentent sous la forme (4), où a_s dépend seulement de son indice s , de sorte que les a_s forment une suite ordinaire.

Supposons que les éléments de la suite harmonique (1) se déterminent à l'aide des expressions (4), nous disons que la suite (3) est la base de la suite harmonique (1), propriété que nous désignons par le symbole

$$(5) \quad [f_n(x), a_n];$$

de plus, nous désignons par l'autre symbole

$$(6) \quad [a_n]$$

la base de la suite harmonique (5), savoir la suite ordinaire (3).

On sait que M. APPELL¹⁾ a étudié, le premier à un point de vue systématique, les suites harmoniques. Dans mon Mémoire susdit j'ai donné d'autres propriétés des suites harmoniques.

Nous nous bornerons à indiquer ici les deux théorèmes suivants, qui sont évidents du reste:

I. Pour un élément quelconque de la suite harmonique (1) nous aurons la série de TAYLOR

$$(7) \quad f_n(x+h) = \sum_{s=0}^{s=n} \frac{h^s}{s!} \cdot f_{n-s}(x).$$

¹⁾ Annales de l'École Normale (2) t. 9, p. 119—144; 1880.

II. Les deux suites harmoniques $[f_n(x), a_n]$ et $[g_n(x), b_n]$ sont identiques, pourvu que nous ayons pour tous les n

$$(8) \quad f_n(0) = g_n(0).$$

En effet, la formule (8) donnera $a_n = b_n$.

§ 2. Les fonctions de Bernoulli.

Nous définissons la suite des polynomes de BERNOULLI

$$(1) \quad \varphi_0(x), \varphi_1(x), \varphi_2(x), \dots, \varphi_n(x), \dots$$

comme la suite harmonique, dont les éléments satisfont, pour $n \geq 1$, à l'équation aux différences finies

$$(2) \quad \varphi_n(x) - \varphi_n(x-1) = \frac{x^{n-1}}{(n-1)!}.$$

Démontrons tout d'abord que les équations (2) déterminent parfaitement la base $[a_n]$ de la suite harmonique (1).

A cet effet, il est évident qu'un polynome entier quelconque qui satisfera à l'équation (2) est précisément du degré n par rapport à x . Ordonnons ensuite, à l'aide de la formule binomiale, selon des puissances ascendantes de x le polynome $\varphi_n(x-1)$, puis cherchons, au premier membre de (2), le coefficient de la puissance x^{n-p-1} , nous aurons :

$$(3) \quad a_0 = 1; \quad \sum_{s=0}^{s=p} (-1)^s a_{p-s} = 0;$$

c'est-à-dire que la base $[a_n]$ susdite est déterminée.

Nous aurons par exemple

$$(4) \quad a_0 = 1, \quad a_1 = \frac{1}{2}, \quad a_2 = \frac{1}{12}, \quad a_3 = 0.$$

Cela posé, nous aurons immédiatement le théorème suivant :

I. Désignons par K une constante arbitraire, le polynome le plus général qui satisfait à l'équation aux différences finies

$$(5) \quad f(x) - f(x-1) = \sum_{s=0}^{s=n-1} a_{n,s} x^{n-s-1}, \quad n > 1$$

se présente sous la forme

$$(6) \quad f(x) = K + \sum_{s=0}^{s=n-1} a_{n,s} (n-s-1)! \varphi_{n-s}(x).$$

Ce théorème établi, il est très facile de démontrer cet autre, essentiel dans nos recherches suivantes :

II. Soit $[f_n(x), a_n]$ une suite harmonique quelconque, il existe une autre suite harmonique parfaitement déterminée $[F_n(x), A_n]$, telle que nous aurons pour $n > 1$ constamment

$$(7) \quad F_n(x) - F_n(x-1) = f_{n-1}(x);$$

car nous aurons pour tous les n :

$$(8) \quad F_n(x) = \sum_{s=0}^{s=n} a_s \varphi_{n-s}(x).$$

En effet, la formule (6) donnera pour $n \geq 0$:

$$F_{n+1}(x) = K + \sum_{s=0}^{s=n} a_s \varphi_{n-s+1}(x),$$

de sorte que nous obtenons, en différentiant par rapport à x , précisément la formule (8).

Cela posé, il est très facile de discuter la base $[a_n]$ des fonctions de BERNOULLI, définie par les équations (3).

A cet effet, nous étudions la suite harmonique dont les éléments sont les polynomes

$$(9) \quad f_n(x) = (x+1)\varphi_{n-1}(x) - (n-1)\varphi_n(x), \quad f_0(x) = 1;$$

nous trouvons, après un simple calcul direct:

$$f_n(x) - f_n(x-1) = \varphi_{n-1}(x) - \sum_{s=0}^{s=n-1} a_s x^{n-s-1} (n-s-1)!,$$

de sorte que le théorème II donnera, en vertu de (4):

$$(10) \quad (x+1)\varphi_{n-1}(x) - n\varphi_n(x) = \sum_{s=1}^{s=n} a_s \varphi_{n-s}(x).$$

Cherchons ensuite, dans les deux membres de (10), le coefficient de la puissance x^{n-p} , nous aurons pour $p \geq 1$:

$$(n-p)! \left[\sum_{s=1}^{s=p} a_s a_{p-s} - (n-p-1)! : (n-p)! \right],$$

d'où, en vertu des valeurs particulières (4):

$$(11) \quad (p+1)a_p = \sum_{s=2}^{s=p-2} a_s a_{p-s}, \quad p > 4.$$

Cela posé, la valeur particulière $a_3 = 0$ donnera, par la conclusion ordinaire de m à $m+1$, l'expression générale:

$$(12) \quad a_{2p+1} = 0, \quad p \geq 1;$$

posons ensuite

$$(13) \quad a_{2p} = \frac{(-1)^{p-1} B_p}{(2p)!}, \quad p \geq 1,$$

nous aurons, en vertu de (11), la formule réursive:

$$(14) \quad (2p+1)B_p = \sum_{s=1}^{p-1} \binom{2p}{2s} B_s B_{p-s}, \quad p \geq 2,$$

ce qui montrera que les nombres rationnels B_n sont constamment positifs.

Introduisons dans (3) les expressions (4), (12) et (13), nous aurons:

$$(15) \quad \frac{p-1}{2} \cdot \sum_{s=1}^{\leq \frac{p}{2}} (-1)^{s-1} \binom{p-1}{2s} B_s, \quad p \geq 2;$$

c'est-à-dire que notre définition des nombres de BERNOULLI coïncide avec la définition classique de ces nombres.

La base $[a_n]$ des fonctions de BERNOULLI étant déterminée, nous aurons les expressions suivantes:

$$(16) \quad \left\{ \begin{array}{l} \varphi_0(x) = 1, \quad \varphi_1(x) = x + \frac{1}{2}, \\ \varphi_n(x) = \frac{x^n}{n!} + \frac{1}{2} \cdot \frac{x^{n-1}}{(n-1)!} + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s x^{n-2s}}{(2s)! (n-2s)!}, \end{array} \right.$$

et l'équation aux différences finies (2) donnera par conséquent:

$$(17) \quad \left\{ \begin{array}{l} \varphi_0(x-1) = 1, \quad \varphi_1(x-1) = x - \frac{1}{2}, \\ \varphi_n(x-1) = \frac{x^n}{n!} - \frac{1}{2} \cdot \frac{x^{n-1}}{(n-1)!} + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s x^{n-2s}}{(2s)! (n-2s)!}. \end{array} \right.$$

Remarquons encore que la formule (10) se présente sous la forme

$$(18) \quad \left(x + \frac{1}{2}\right) \varphi_{n-1}(x) = n \varphi_n(x) + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \varphi_{n-2s}(x), \quad n \geq 2,$$

formule qui nous sera très utile dans nos recherches suivantes.

§ 3. Les fonctions d'Euler.

La suite des polynômes d'EULER

$$(1) \quad \chi_0(x), \chi_1(x), \chi_2(x), \dots, \chi_n(x), \dots$$

est définie par les équations aux différences finies

$$(2) \quad \chi_n(x) + \chi_n(x-1) = \frac{x^n}{n!}, \quad n \geq 0.$$

En effet, on voit immédiatement que $\chi_n(x)$ est précisément du degré n par rapport à x ; posons

$$(3) \quad \chi_n(x) = \sum_{s=0}^{s=n} \frac{\beta_s x^{n-s}}{(n-s)!},$$

ce qui est toujours possible; nous aurons, en vertu de (2):

$$(4) \quad \beta_0 = \frac{1}{2}; \quad 2\beta_p + \sum_{s=1}^{s=p} \frac{(-1)^s \beta_{p-s}}{s!} = 0,$$

ce qui nous détermine β_p comme fonction de p seulement; nous aurons par exemple

$$(5) \quad \beta_0 = \frac{1}{2}, \quad \beta_1 = \frac{1}{4}, \quad \beta_2 = 0, \quad \beta_3 = -\frac{1}{48}, \quad \beta_4 = 0.$$

Cela posé, nous aurons le théorème suivant:

I. Les fonctions $\chi_n(x)$ d'Euler forment une suite harmonique.

L'analogie entre les fonctions de BERNOULLI et celles d'EULER est évidente.

De plus, nous aurons les deux théorèmes suivants:

II. Il existe un seul polynôme entier de x qui satisfait à l'équation aux différences finies

$$(6) \quad g(x) + g(x-1) = \sum_{s=0}^{s=n} c_{n,s} x^{n-s},$$

savoir le polynôme

$$(7) \quad g(x) = \sum_{s=0}^{s=n} c_{n,s} (n-s)! \chi_{n-s}(x).$$

III. Soit $[g_n(x), c_n]$ une suite harmonique, les équations aux différences finies

$$(8) \quad G_n(x) + G_n(x-1) = g_n(x), \quad n \geq 0$$

déterminent parfaitement une autre suite harmonique $[G_n(x), C_n]$; car nous aurons pour tous les n

$$(9) \quad G_n(x) = \sum_{s=0}^{s=n} c_s \chi_{n-s}(x).$$

Cela posé, il est très facile de discuter la base $[\beta_n]$ des fonctions d'EULER.

En effet, nous verrons, en vertu de (2), que les polynômes

$$(10) \quad g_n(x) = (x+1)\chi_n(x) - (n+1)\chi_{n+1}(x)$$

qui forment une suite harmonique satisfont aux équations aux différences finies

$$(11) \quad g_n(x) + g_n(x-1) = \chi_n(x) = \sum_{s=0}^{s=n} \frac{\beta_s x^{n-s}}{(n-s)!},$$

ce qui donnera, en vertu de (9):

$$(12) \quad (x+1)\chi_n(x) = (n+1)\chi_{n+1}(x) + \sum_{s=0}^{s=n} \beta_s \chi_{n-s}(x),$$

d'où, pour $p \geq 3$, la formule récursive

$$(13) \quad p\beta_p = - \sum_{s=1}^{s=p-2} \beta_s \beta_{p-s-1}.$$

Appliquons maintenant la valeur particulière $\beta_2 = 0$, la conclusion ordinaire de m à $m+1$ donnera, en vertu de (13), l'expression générale

$$(14) \quad \beta_{2p} = 0, \quad p > 1;$$

posons ensuite pour $p \geq 1$

$$(15) \quad \beta_{2p-1} = \frac{(-1)^{p-1} T_p}{(2p-1)! 2^{2p}}, \quad T_1 = 1,$$

nous aurons, en vertu de (13), la formule récursive

$$(16) \quad T_{p+1} = \sum_{s=0}^{s=p-1} \binom{2p}{2s+1} T_{s+1} T_{p-s}, \quad p \geq 1,$$

ce qui montrera que les T_n sont des positifs entiers, et que T_n est, pour $n > 1$, toujours un nombre pair.

Introduisons dans (4) les expressions (14) et (15), nous aurons en posant $p = 2n+1$, respectivement $p = 2n$:

$$(17) \quad \left\{ \begin{array}{l} 2T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s} 2^{2s} T_{n-s+1} = (-1)^n 2^{2n+1}, \\ \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2s} T_{n-s} = (-1)^{n-1} 2^{2n-1}; \end{array} \right.$$

c'est-à-dire que notre définition des coefficients des tangentes coïncide avec la définition classique de ces nombres.

La base $[\beta_n]$ des fonctions d'EULER ainsi déterminée, nous aurons les expressions suivantes:

$$(18) \quad \left\{ \begin{array}{l} \chi_0(x) = \frac{1}{2}, \quad \chi_1(x) = \frac{x}{2} + \frac{1}{4}, \\ \chi_n(x) = \frac{1}{2} \cdot \frac{x^n}{n!} - \sum_{s=1}^{\leq \frac{n+1}{2}} \frac{(-1)^{s-1} T_s x^{n-2s+1}}{(2s-1)! (n-2s+1)! 2^{2s}}, \end{array} \right.$$

et l'équation aux différences finies (2) donnera par conséquent:

$$(19) \quad \left\{ \begin{array}{l} \chi_0(x-1) = \frac{1}{2}, \quad \chi_1(x-1) = \frac{x}{2} - \frac{1}{4}, \\ \chi_n(x-1) = \frac{1}{2} \cdot \frac{x^n}{n!} - \sum_{s=1}^{\leq \frac{n+1}{2}} \frac{(-1)^{s-1} T_s x^{n-2s+1}}{(2s-1)! (n-2s+1)! 2^{2s}}. \end{array} \right.$$

Remarquons encore que la formule (12) se présente sous la forme

$$(20) \quad \left(x + \frac{1}{2}\right) \chi_n(x) = (n+1) \chi_{n+1}(x) + \sum_{s=1}^{\leq n+1} \frac{2}{(2s-1)!} (-1)^{s-1} T_s \chi_{n-2s+1}(x), \quad n > 1,$$

formule qui nous sera très utile dans nos recherches suivantes.

§ 4. Formules de Bernoulli et d'Euler.

Posons dans les équations aux différences finies

$$(1) \quad \varphi_n(x) - \varphi_n(x-1) = \frac{x^{n-1}}{(n-1)!}, \quad \chi_n(x) + \chi_n(x-1) = \frac{x^n}{n!}$$

successivement

$$x+1, x+2, x+3 \dots, x+p$$

au lieu de x , puis posons pour abréger

$$(2) \quad s_n(x, p) = \sum_{s=1}^{s=p} (x+s)^n, \quad \sigma_n(x, p) = \sum_{s=1}^{s=p} (-1)^{p-s} (x+s)^n,$$

nous aurons respectivement:

$$(3) \quad \begin{cases} \varphi_n(x+p) - \varphi_n(x) = \frac{1}{(n-1)!} s_{n-1}(x, p), \\ \chi_n(x+p) - (-1)^p \chi_n(x) = \frac{1}{n!} \sigma_n(x, p), \end{cases}$$

formules qui nous permettent d'exprimer sous forme explicite les deux sommes de puissances numériques

$$(4) \quad s_n(p) = \sum_{s=1}^{s=p} s^n, \quad \sigma_n(p) = \sum_{s=1}^{s=p} (-1)^{p-s} s^n.$$

Nous aurons en effet:

$$(5) \quad s_n(0, p) = s_n(p), \quad s_n(-1, p) = s_n(p-1),$$

$$(6) \quad \sigma_n(0, p) = \sigma_n(p), \quad \sigma_n(-1, p) = \sigma_n(p-1),$$

ce qui donnera, en vertu de (3), les expressions cherchées

$$(7) \quad s_n(p) = n! (\varphi_{n+1}(p) - \varphi_{n+1}(0)),$$

$$(8) \quad \sigma_n(p) = n! (\chi_n(p) - (-1)^p \chi_n(0)).$$

JACQUES BERNOULLI¹⁾ a donné explicitement les dix premières formules de la forme (7), savoir les formules qui correspondent à $1 < n < 10$, et c'est précisément dans ces formules que les nombres de BERNOULLI, savoir les

$$B_1 \ B_2 \ B_3 \ B_4 \ B_5$$

se sont présentés pour la première fois dans l'Analyse.

¹⁾ Ars conjectandi, p. 97; Bâle 1713.

BERNOULLI indique aussi la formule générale (5), mais sans démonstration, sans éclaircissements sur la nature des coefficients du polynome $\varphi_n(x)$.

La formule (8) est due à EULER¹⁾. On voit du reste que cette dernière formule est plus compliquée que celle de BERNOULLI, parce que sa forme dépend de la parité du nombre p . Dans nos recherches suivantes sur les fonctions $\chi_n(x)$ ou sur les nombres $\sigma_n(p)$ nous retrouvons la même inconvénient.

JACOBI²⁾ semble avoir introduit le premier une variable continue au lieu du positif entier p qui figure au second membre de (7). En effet, il a considéré la fonction

$$(9) \quad \varphi_{2n}(x) + \frac{(-1)^n B_n}{(2n)!}.$$

OSTROGRADSKY³⁾, MALMSTÉN⁴⁾, DIENGER⁵⁾ ont de même considéré une variable continue dans notre formule numérique susdite; mais c'est RAABE⁶⁾ qui a donné la première monographie des fonctions de BERNOULLI.

On doit à JACOBI et à MALMSTÉN des remarques importantes relatives à la variation de $\varphi_n(x)$. OSTROGRADSKY a représenté les $\varphi_n(x)$ comme des polynômes entiers de la variable $x+1$, représentations qui sont des conséquences immédiates des formules (17) du paragraphe 2 et (19) du paragraphe 3 pour $\varphi_n(x-1)$, respectivement $\chi_n(x-1)$.

RAABE⁷⁾ a introduit dans le second membre de (8) une variable continue; plus tard HERMITE⁸⁾ a résolu, à un autre point de vue, le même problème, tandis que M. F. ROGEL⁹⁾ a donné une monographie des fonctions d'EULER.

Il est digne d'intérêt que les deux suites harmoniques, formées par les polynômes

$$s_n(x, p) \quad , \quad \frac{\sigma_n(x, p)}{n!}$$

nous permettent de généraliser les formules de BERNOULLI et d'EULER.

A cet effet, remarquons tout d'abord que $s_n(x, p)$ est toujours du degré n par rapport à x , tandis que $\sigma_n(x, p)$ est du degré n respectivement $n-1$, selon que p est impair ou pair.

Nous aurons par exemple:

$$(10) \quad \sigma_1(x, 2p) = p, \quad \sigma_1(x, 2p+1) = x+p+1.$$

¹⁾ Institutiones calculi differentialis, p. 499; Saint-Petersbourg 1755.

²⁾ Journal de Crelle, t. 12, p. 265—267; 1834.

³⁾ Mémoires de l'Académie de Saint-Petersbourg, (6) t. 2, p. 322—323; 1841.

⁴⁾ Journal de Crelle, t. 35, p. 60—67; 1847.

⁵⁾ Ibid. t. 34, p. 75—100; 1847.

⁶⁾ Die Jacob Bernoullische Function; Zurich 1848. Journal de Crelle, t. 42, p. 348—367; 1851. Mathematische Mittheilungen I—II; Zurich 1857—58.

⁷⁾ Mathematische Mittheilungen II, p. 129—138; 1858.

⁸⁾ Journal de Crelle, t. 116, p. 144; 1896.

⁹⁾ Prager Berichte 1892; 52 pages.

Les définitions (2) donnent immédiatement les équations aux différences finies

$$\begin{aligned}s_n(x, p) - s_n(x-1, p) &= (x+p)^n - x^n, \\ \sigma_n(x, p) + \sigma_n(x-1, p) &= (x+p)^n - (-1)^p x^n,\end{aligned}$$

de sorte que nous aurons de même:

$$\begin{aligned}s_n(x, p) + s_n(x-1, p) &= 2s_n(x, p) - (x+p)^n + x^n, \\ \sigma_n(x, p) - \sigma_n(x-1, p) &= 2\sigma_n(x, p) - (x+p)^n + (-1)^p x^n.\end{aligned}$$

Appliquons ensuite la formule binomiale et la série de TAYLOR (7) du paragraphe 1; les théorèmes II du paragraphe 2 et III du paragraphe 3 donnent pour $s_n(x, p)$ ces deux développements:

$$(11) \quad \frac{s_n(x, p)}{n!} = \sum_{r=0}^{r=n} \frac{p^{r+1}}{(r+1)!} \varphi_{n-r}(x),$$

$$(12) \quad \frac{s_n(x, p)}{n!} = 2p\chi_n(x) + \sum_{r=1}^{r=n} \frac{2s_r(p) - p^r}{r!} \chi_{n-r}(x).$$

Quant à $\sigma_n(x, p)$, nous aurons pour p impair:

$$(13) \quad \frac{\sigma_n(x, p)}{n!} = 2\chi_n(x) + \sum_{r=1}^{r=n} \frac{p^r}{r!} \chi_{n-r}(x),$$

$$(14) \quad \frac{\sigma_n(x, p)}{n!} = \sum_{r=0}^{r=n} \frac{2\sigma_{r+1}(p) - p^{r+1}}{(r+1)!} \varphi_{n-r}(x),$$

tandis que l'hypothèse p pair donnera:

$$(15) \quad \frac{\sigma_n(x, p)}{n!} = \sum_{r=0}^{r=n-1} \frac{p^{r+1}}{(r+1)!} \chi_{n-r-1}(x),$$

$$(16) \quad \frac{\sigma_n(x, p)}{n!} = \sum_{r=0}^{r=n-1} \frac{2\sigma_{r+2}(p) - p^{r+2}}{(r+2)!} \varphi_{n-r-1}(x).$$

Posons $x = 0$ dans les formules (11), (13) et (15): nous retrouvons les formules de BERNOULLI et d'EULER, tandis que les formules (12), (14) et (16) nous donnent l'inversion de (7) et (8), comme nous le verrons dans le paragraphe 13.

Dans ce qui suit nous avons à étudier aussi ces deux autres sommes de puissances numériques:

$$(17) \quad t_n(p) = 2^n s_n(-\tfrac{1}{2}, p) = \sum_{s=1}^{s=p} (2s-1)^n,$$

$$(18) \quad \tau_n(p) = 2^n \sigma_n(-\tfrac{1}{2}, p) = \sum_{s=1}^{s=p} (-1)^{s-1} (2p-2s+1)^n.$$

Conformément aux formules (3) nous posons:

$$(19) \quad s_n(x, 0) = \sigma_n(x, 0) = t_n(0) = \tau_n(0) = 0.$$

§ 5. Théorème de Jacobi.

Revenons maintenant aux équations aux différences finies qui figurent dans la définition de $\varphi_n(x)$, respectivement $\chi_n(x)$, puis changeons le signe de x , nous aurons

$$(-1)^n \varphi_n(-x-1) = (-1)^n \varphi_n(-x) + \frac{x^{n-1}}{(n-1)!},$$

$$(-1)^n \chi_n(-x-1) = (-1)^{n-1} \chi_n(-x) + \frac{x^n}{n!},$$

tandis que les définitions de $\varphi_n(x)$ et $\chi_n(x)$ donnent respectivement:

$$(-1)^n \varphi_n(-x) = \varphi_n(x) - \frac{x^{n-1}}{(n-1)!},$$

$$(-1)^{n-1} \chi_n(-x) = \chi_n(x) - \frac{x^n}{n!};$$

cest-à-dire que nous aurons le théorème suivant:

I. Les fonctions $\varphi_n(x)$ et $\chi_n(x)$ satisfont, pour tous les n , aux équations fonctionnelles

$$(1) \quad (-1)^n \varphi_n(-x-1) = \varphi_n(x), \quad (-1)^n \chi_n(-x-1) = \chi_n(x).$$

Soit maintenant $f(x)$ un polynôme entier quelconque du degré n par rapport à x , l'identité évidente

$$x = -\frac{1}{2} + \left(x + \frac{1}{2}\right)$$

donnera la série de TAYLOR

$$(2) \quad f(x) = \sum_{s=0}^{s=n} \frac{\left(x + \frac{1}{2}\right)^s}{s!} f^{(s)}\left(-\frac{1}{2}\right);$$

appliquons ensuite les identités

$$\begin{aligned} \left(x + \frac{1}{2}\right)^{2n} &= \left((x^2 + x) + \frac{1}{4}\right)^n, \\ \left(x + \frac{1}{2}\right)^{2n+1} &= \left(x + \frac{1}{2}\right) \cdot \left((x^2 + x) + \frac{1}{4}\right)^n, \end{aligned}$$

nous aurons, en vertu de (2), une représentation de la forme

$$(3) \quad f(x) = f_1(x^2 + x) + \left(x + \frac{1}{2}\right) f_2(x^2 + x),$$

valable pour un polynôme entier quelconque de x .

Cela posé, développons à l'aide de (3) les polynômes $\varphi_n(x)$ et $\chi_n(x)$; les formules (1) montrent qu'un des polynômes correspondants $f_1(x)$ ou $f_2(x)$ se réduira à zéro.

JACOBI¹⁾ a démontré, pour une valeur paire de n , la première des formules (1); de plus, il indique un développement de la forme²⁾

¹⁾ Journal de Crelle, t. 12, p. 267; 1834.

²⁾ loc. cit. p. 271. Voir aussi Nouvelles Annales, t. 7, p. 448; 1848, t. 10, p. 198-199; 1851.

$$(4) \quad s_{2n-1}(p) = \sum_{q=0}^{q=n-1} a_{n,q} (p^2 + p)^{n-q};$$

plus tard PROUHET¹⁾ a démontré la formule analogue

$$(5) \quad s_{2n}(p) = \left(p + \frac{1}{2}\right) \cdot \sum_{q=0}^{q=n} b_{n,q} (p^2 + p)^{n-q}.$$

Cependant, il n'est pas possible de donner sous une simple forme les coefficients $a_{n,q}$ et $b_{n,q}$ qui figurent aux seconds membres des formules (4) et (5), de sorte qu'il sera inutile de chercher pour $\varphi_n(x)$, et pour $\chi_n(x)$ aussi du reste, les développements obtenus à l'aide de la formule (3).

Remarquons en passant que MALMSTÉN²⁾, DIENGER³⁾, RAABE⁴⁾ ont démontré la première des formules (1), tandis que RAABE⁵⁾ a démontré aussi la seconde.

Quant aux équations fonctionnelles (1), nous aurons immédiatement:

$$(6) \quad \varphi_{2n+1}\left(-\frac{1}{2}\right) = 0, \quad \chi_{2n+1}\left(-\frac{1}{2}\right) = 0;$$

le dernier de ces résultats nous permet d'introduire les nombres d'EULER. Nous prenons comme définition

$$(7) \quad E_n = (-1)^n (2n)! 2^{2n} \chi_{2n}\left(-\frac{1}{2}\right), \quad n > 1,$$

ce qui donnera, en vertu de la formule (20) du paragraphe 3:

$$(8) \quad E_1 = 1; \quad E_n = T_n + \sum_{s=1}^{s=n-1} \binom{2n-1}{2s-1} T_s E_{n-s},$$

de sorte que les nombres d'EULER sont des positifs entiers.

Remarquons que les T_n , à l'exception de $T_1 = 1$, sont des nombres pairs, nous verrons que les E_n sont tous des nombres impairs.

Dans la formule (18) du paragraphe 3, posons $x = -\frac{1}{2}$, nous aurons, en vertu de (6) et (7), pourvu que n soit remplacé par $2n$:

$$(9) \quad E_n = (-1)^n \cdot \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} T_{n-s};$$

c'est-à-dire que notre définition des nombres d'EULER coïncide avec la définition classique de ces nombres.

¹⁾ Nouvelles Annales, t. 10, p. 199—200; 1851.

²⁾ Journal de Crelle, t. 35, p. 64; 1847.

³⁾ Ibid. t. 34, p. 99; 1847.

⁴⁾ Die Jacob Bernoullische Function, p. 18; Zurich 1848. Journal de Crelle, t. 42, p. 354; 1851. *Mathematische Mittheilungen* t. I, p. 48; Zurich 1857.

⁵⁾ *Mathematische Mittheilungen* t. II, p. 136; Zurich 1858.

Appliquons maintenant à $\chi_n(x)$ la formule (2), nous aurons:

$$(10) \quad \chi_n(x) = \frac{\left(x + \frac{1}{2}\right)^n}{n! 2^n} + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^s E_s}{(2s)! 2^{2s+1}} \cdot \frac{\left(x + \frac{1}{2}\right)^{n-2s}}{(n-2s)!};$$

cette formule, due à RAABE¹⁾, est essentielle dans nos recherches sur les nombres d'EULER.

Démontrons maintenant que les équations fonctionnelles (1) nous conduiront à des résultats intéressants concernant les deux fonctions $\varphi_n(px)$ et $\chi_n(px)$, dans lesquelles p désigne un entier quelconque, plus grand que l'unité.

En premier lieu, les équations aux différences finies qui figurent dans la définition des $\varphi_n(x)$ et des $\chi_n(x)$ donnent:

$$\varphi_n(-px-p) = \varphi_n(-px-1) - \frac{(-1)^{n-1}}{(n-1)!} \cdot \sum_{s=1}^{s=p-1} (px+s)^{n-1},$$

$$\chi_n(-px-p) = (-1)^{p-1} \chi_n(-px-1) - \frac{(-1)^n}{n!} \cdot \sum_{s=1}^{s=p-1} (-1)^{s-1} (px+p-s)^n;$$

multiplions ensuite par $(-1)^n$, puis divisons par p^{n-1} respectivement par p^n , nous aurons, en vertu des équations fonctionnelles (1):

$$(11) \quad \frac{(-1)^n}{p^{n-1}} \varphi_n(-px-p) = \frac{\varphi_n(px)}{p^{n-1}} - \frac{1}{(n-1)!} \cdot \sum_{s=1}^{s=p-1} \left(x + \frac{s}{p}\right)^{n-1},$$

$$(12) \quad \frac{(-1)^n}{p^n} \chi_n(-px-p) = \frac{(-1)^{p-1} \chi_n(px)}{p^n} + \frac{1}{n!} \cdot \sum_{s=1}^{s=p-1} (-1)^{s-1} \left(x + \frac{p-s}{p}\right)^n.$$

En second lieu, soient α et β deux variables complexes quelconques, les équations aux différences finies susdites donnent:

$$(13) \quad \varphi_n(p\alpha-\beta-1) - \varphi_n(p\alpha-\beta-p-1) = \frac{(-1)^{n-1}}{(n-1)!} \cdot \sum_{s=0}^{s=p-1} (\beta+s+1-p\alpha)^{n-1},$$

$$(14) \quad \chi_n(p\alpha-\beta-1) - (-1)^p \chi_n(p\alpha-\beta-p-1) = \frac{(-1)^n}{n!} \cdot \sum_{s=0}^{s=p-1} (-1)^s (\beta+s+1-p\alpha)^n.$$

Étudions d'abord la formule (13), ordonnons selon de puissances descendantes de α le second membre; le théorème II du paragraphe 2 donnera le développement suivant:

$$(15) \quad \varphi_n(p\alpha-\beta-1) = p^n \varphi_n(\alpha) + \sum_{r=0}^{r=n} \frac{(-1)^r p^{n-r-1}}{r!} s_r(\beta, p) \varphi_{n-r}(\alpha);$$

¹⁾ *Mathematische Mittheilungen* t. II, p. 133; Zurich 1858.

car le second membre de (13) représente, divisé par p^{n-1} , une suite harmonique par rapport à la variable α .

La formule (14) donnera par le même procédé, pour p pair :

$$(16) \quad \chi_n(p\alpha - \beta - 1) = \sum_{r=0}^{r=n} \frac{(-1)^r p^{n-r-1}}{(r+1)!} \sigma_{r+1}(\beta, p) \varphi_{n-r}(x),$$

tandis que nous aurons pour p impair :

$$(17) \quad \chi_n(p\alpha - \beta - 1) = p^n \chi_n(\alpha) + \sum_{r=1}^{r=n} \frac{(-1)^r p^{n-r}}{r!} \sigma_r(\beta, p) \chi_{n-r}(\alpha).$$

Cela posé, nous avons à déduire des trois développements que nous venons d'établir quelques résultats plus particuliers, mais très importants.

Posons tout d'abord $\alpha = 0$ et $\beta = x$, puis appliquons les équations fonctionnelles (1), il en résulte les formules

$$(18) \quad \varphi_n(x) = (p)^n \varphi_n(0) + \sum_{r=1}^{r=n} \frac{(-1)^{n-r} p^{n-r-1}}{r!} \varphi_{n-r}(0) s_r(x, p),$$

$$(19) \quad \chi_n(x) = \sum_{r=0}^{r=n} \frac{(-1)^{n-r} p^{n-r-1}}{(r+1)!} \varphi_{n-r}(0) \sigma_r(x, p),$$

$$(20) \quad \chi_n(x) = (p)^n \chi_n(0) + \sum_{r=1}^{r=n} \frac{(-1)^{n-r} p^{n-r}}{r!} \chi_{n-r}(0) \sigma_r(x, p),$$

qui représentent les inversions des formules (11), (13) et (15) du paragraphe 4; dans (19) et (20) il faut supposer, par conséquent, p pair respectivement impair.

Posons ensuite $\alpha = x$, $\beta = -1$, nous aurons ces trois développements :

$$(21) \quad \varphi_n(px) = p^n \varphi_n(x) + \sum_{r=1}^{r=n} \frac{(-1)^r p^{n-r-1}}{r!} s_r(p-1) \varphi_{n-r}(x),$$

$$(22) \quad \chi_n(px) = \sum_{r=0}^{r=n} \frac{(-1)^r p^{n-r-1}}{(r+1)!} \sigma_{r+1}(p-1) \varphi_{n-r}(x),$$

$$(23) \quad \chi_n(px) = p^n \chi_n(x) + \sum_{r=1}^{r=n} \frac{(-1)^r p^{n-r}}{r!} \sigma_r(p-1) \chi_{n-r}(x).$$

Posons au contraire $\beta = 0$, nous aurons des développements analogues pour $\varphi_n(px-1)$ et $\chi_n(px-1)$, formules qui peuvent être déduites des trois précédentes si nous remplaçons $s_r(p-1)$ et $\sigma_r(p-1)$ par les $s_r(p)$ et $\sigma_r(p)$ correspondantes.

§ 6. Formules de Raabe.

Comme une autre conséquence immédiate de nos développements précédents nous avons à démontrer les trois formules suivantes:

$$(1) \quad p^{n-1} \cdot \sum_{s=0}^{s=p-1} \varphi_n \left(\frac{x-s}{p} \right) = \varphi_n(x),$$

$$(2) \quad (2p+1)^n \cdot \sum_{s=0}^{s=2p} (-1)^s \chi_n \left(\frac{x-s}{2p+1} \right) = \chi_n(x),$$

$$(3) \quad (2p)^{n-1} \cdot \sum_{s=0}^{s=2p-1} (-1)^s \varphi_n \left(\frac{x-s}{2p} \right) = \chi_{n-1}(x),$$

où p désigne un positif entier quelconque.

A cet effet, nous désignons pour abréger par

$$F_n(x), \quad G_n(x), \quad H_n(x)$$

les expressions qui figurent aux premiers membres des trois formules en question. Remarquons que ces polynomes forment des suites harmoniques et que nous aurons de plus:

$$F_n(x) - F_n(x-1) = \frac{x^{n-1}}{(n-1)!},$$

$$G_n(x) + G_n(x-1) = \frac{x^n}{n!},$$

$$H_n(x) + H_n(x-1) = \frac{x^{n-1}}{(n-1)!};$$

les formules en question sont des conséquences immédiates de nos définitions des polynomes $\varphi_n(x)$ et $\chi_n(x)$.

Les formules (1) et (2) appartiennent à RAABE¹⁾, tandis que la troisième est peut-être nouvelle.

Posons dans (1) $p=2$, dans (3) $p=1$, nous aurons la formule importante

$$(4) \quad \chi_{n-1}(x) = 2^n \varphi_n \left(\frac{x}{2} \right) - \varphi_n(x),$$

d'où, en remplaçant n par $2n$ puis posant $x=0$, la relation suivante, due à EULER²⁾, entre les B_n et les T_n :

$$(5) \quad T_n = \frac{2^{2n}(2^{2n}-1)B_n}{2n}.$$

Remarquons que les définitions des $\varphi_n(x)$ et $\chi_n(x)$ donnent immédiatement ces valeurs numériques:

¹⁾ Die Jacob Bernoullische Function, pp. 23, 28; Zurich 1848. Journal de Crelle, t. 42, p. 356-357; 1851. Mathematische Mittheilungen, t. II, p. 134; Zurich 1858.

²⁾ Opuscula analytica, t. II, p. 273; Saint-Petersbourg 1785.

$$(6) \quad \left\{ \begin{array}{l} \varphi_{2n}(0) = \varphi_{2n}(-1) = \frac{(-1)^{n-1} B_n}{(2n)!} \\ \varphi_{2n+1}(0) = \varphi_{2n+1}(-1) = 0 \end{array} \right\} \quad n \geq 1,$$

$$(7) \quad \left\{ \begin{array}{l} \chi_{2n}(0) = \chi_{2n}(-1) = 0 \\ \chi_{2n+1}(0) = -\chi_{2n+1}(-1) = \frac{(-1)^n T_{n+1}}{(2n+1)! 2^{2n+2}} \end{array} \right\} \quad n \geq 1.$$

De plus, nous aurons :

$$(8) \quad \varphi_{2n} \left(-\frac{1}{2} \right) = \frac{(-1)^n (2^{2n}-2) B_n}{(2n)! 2^{2n}}, \quad \varphi_{2n+1} \left(-\frac{1}{2} \right) = 0,$$

$$(9) \quad \chi_{2n} \left(-\frac{1}{2} \right) = \frac{(-1)^n E_n}{(2n)! 2^{2n+1}}, \quad \chi_{2n+1} \left(-\frac{1}{2} \right) = 0;$$

la première des formules (8) peut être obtenue si nous posons dans (1) $p=2$ et $x=0$; les trois autres formules sont des conséquences immédiates de (6) et (7) du paragraphe 5.

Posons dans (4) $x=0$, puis remplaçons n par $2n$ respectivement par $2n+1$, nous aurons :

$$(10) \quad \varphi_{2n} \left(-\frac{1}{4} \right) = \varphi_{2n} \left(-\frac{3}{4} \right) = \frac{(-1)^n (2^{2n}-2) B_n}{(2n)! 2^{4n}},$$

$$(11) \quad \varphi_{2n+1} \left(-\frac{1}{4} \right) = -\varphi_{2n+1} \left(-\frac{3}{4} \right) = \frac{(-1)^n E_n}{(2n)! 2^{4n+2}}.$$

Posons dans (1) $p=3$, $x=0$ et dans (2) $p=1$, $x=0$, nous aurons :

$$(12) \quad \varphi_{2n} \left(-\frac{1}{3} \right) = \varphi_{2n} \left(-\frac{2}{3} \right) = \frac{(-1)^n (3^{2n}-3) B_n}{(2n)! 3^{2n} \cdot 2},$$

$$(13) \quad \chi_{2n-1} \left(-\frac{1}{3} \right) = -\chi_{2n-1} \left(-\frac{2}{3} \right) = \frac{(-1)^{n-1} (3^{2n}-3) T_n}{(2n-1)! 6^{2n} \cdot 2}.$$

Enfin, posons dans (1) $p=6$, $x=0$, puis appliquons (8) et (12), nous aurons :

$$(14) \quad \varphi_{2n} \left(-\frac{1}{6} \right) = \varphi_{2n} \left(-\frac{5}{6} \right) = \frac{(-1)^{n-1} (3^{2n}-3) (2^{2n}-2) B_n}{(2n)! 6^{2n} \cdot 2}.$$

Ces résultats numériques, essentiels dans les recherches très vastes et interminables de M. GLAISHER¹⁾, ne jouent qu'un rôle très modeste dans nos recherches suivantes. Nous préférons appliquer directement les formules fondamentales (1), (2) et (3) elles-mêmes.

¹⁾ Voir par exemple: Quarterly Journal of mathematics, t. 29, p. 1-168; 1897.

CHAPITRE II.

Les nombres de Bernoulli et d'Euler.

§ 7. Méthodes générales.

On voit fort bien comment on pourrait appliquer notre théorie précédente à l'évaluation des formules récurrentes pour les B_n , T_n et E_n .

A cet effet, nous prenons pour point de départ l'équation aux différences finies

$$(1) \quad f(x) - f(x-1) = \sum_{s=0}^{s=n-1} a_{n,s} x^{n-s-1},$$

de sorte que nous aurons, en vertu du théorème I du paragraphe 2:

$$(2) \quad f(x) = K + \sum_{s=1}^{s=n} a_{n,n-s} (n-s-1)! \varphi_s(x).$$

Appliquons ensuite la première des équations fonctionnelles (1) du paragraphe 5, nous aurons de même:

$$(3) \quad f(-x-1) = K + \sum_{s=n}^{s=1} (-1)^s a_{n,n-s} (n-s-1)! \varphi_s(x),$$

d'où, en additionnant puis soustrayant les formules (2) et (3):

$$(4) \quad \frac{f(x) + f(-x-1)}{2} = K + \sum_{s=1}^{\leq \frac{n}{2}} a_{n,n-2s} (2s-1)! \varphi_{2s}(x),$$

$$(5) \quad \frac{f(x) - f(-x-1)}{2} = \sum_{s=0}^{\leq \frac{n-1}{2}} a_{n,n-2s-1} (2s)! \varphi_{2s+1}(x).$$

Quant aux coefficients $a_{n,s}$ qui figurent au second membre de (1), nous aurons immédiatement:

$$(6) \quad \begin{cases} a_{n,n-1} = f(0) - f(-1), \\ (n-s-1)! a_{n,s} = f^{(n-s-1)}(0) - f^{(n-s-1)}(-1). \end{cases}$$

Posons dans (4) $x = 0$, dans (5) $x = -\frac{1}{4}$, nous aurons respectivement:

$$(7) \quad \frac{f(0) + f(-1)}{2} - K = \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} a_{n,n-2s}}{2s} \cdot B_s,$$

$$(8) \quad a_{n,n-1} - f\left(-\frac{1}{4}\right) + f\left(-\frac{3}{4}\right) = \sum_{s=1}^{\leq \frac{n-1}{2}} \frac{(-1)^{s-1} a_{n,n-2s-1}}{2^{4s+1}} E_s;$$

posons ensuite, dans (4)

$$x = -\frac{1}{2}, -\frac{1}{3}, \frac{1}{4}, -\frac{1}{6},$$

nous aurons d'autres formules récurrentes pour les B_n .

Appliquons maintenant à la formule (2) les formules générales (1) et (2) du paragraphe 6: nous aurons, en désignant par p un positif entier quelconque:

$$(9) \quad p^{n-1} \cdot \sum_{s=0}^{s=p-1} f\left(\frac{x-s}{p}\right) = p^n K + \sum_{s=1}^{s=n} a_{n,n-s} (s-1)! p^{n-s} \varphi_s(x),$$

$$(10) \quad (2p)^{n-1} \cdot \sum_{s=0}^{s=2p-1} (-1)^s f\left(\frac{x-s}{2p}\right) = \sum_{s=1}^{s=n} a_{n,n-s} (s-1)! (2p)^{n-s} \chi_{s-1}(x).$$

Posons par exemple $p=2$, respectivement $p=1$, l'hypothèse $x=0$ donnera:

$$(11) \quad \sum_{s=1}^{s=\frac{n}{2}} \frac{(-1)^{s-1} a_{n,n-2s}}{(2s)! 2^{2s}} B_s = \frac{f(0) - 2f\left(-\frac{1}{2}\right) + f(-1)}{4} = K,$$

$$(12) \quad \sum_{s=1}^{s=\frac{n}{2}} \frac{(-1)^{s-1} a_{n,n-2s}}{2^{4s-1}} T_s = \frac{f(0) - 2f\left(-\frac{1}{2}\right) + f(-1)}{2}.$$

On voit que les formules (8) et (12) ne contiennent pas la constante K .

Considérons maintenant l'autre équation aux différences finies

$$(13) \quad f(x) + f(x-1) = \sum_{s=0}^{s=n} b_{n,s} x^{n-s},$$

nous aurons

$$(14) \quad \begin{cases} b_{n,n} = f(0) + f(-1) \\ (n-s)! b_{n,s} = f^{(n-s)}(0) + f^{(n-s)}(-1) \end{cases}$$

et pour le polynôme $f(x)$ l'expression suivante:

$$(15) \quad f(x) = \sum_{s=0}^{s=n} b_{n,n-s} s! \chi_s(x).$$

Il est évident que nous aurons, dans ce cas, des formules analogues à (4) et (5).

Posons $x=0$, $x=-\frac{1}{2}$, nous aurons de même:

$$(16) \quad \sum_{s=1}^{s=\frac{n-1}{2}} \frac{(-1)^{s-1} b_{n,n-2s-1}}{2^{2s}} T_s = \frac{f(0) - f(-1)}{2},$$

$$(17) \quad \sum_{s=1}^{s=\frac{n}{2}} \frac{(-1)^{s-1} b_{n,n-2s}}{2^{2s+1}} E_s = \frac{f(0) - 2f\left(-\frac{1}{2}\right) + f(-1)}{2},$$

de sorte que les formules (12) et (17) donnent, pourvu que la fonction $f(x)$ qui figure dans les équations aux différences finies (1) et (13) soit la même, la relation intéressante

$$(18) \quad \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} b_{n, n-2s}}{2^{2s+1}} E_s = \sum_{s=1}^{\leq \frac{n-1}{2}} \frac{(-1)^{s-1} a_{n, n-s}}{2^{4s-1}} T_s.$$

Appliquons maintenant la formule (2) du paragraphe 6, nous aurons, en vertu de (15):

$$(19) \quad (2p+1)^n \cdot \sum_{s=0}^{s=2p} (-1)^s f\left(\frac{x-s}{2p+1}\right) = \sum_{s=0}^{s=n} b_{n, n+s} s! (2p+1)^{n-s} \chi_s(x).$$

Supposons que les polynômes qui figurent aux seconds membres des équations aux différences finies (1) et (13) ne forment pas des suites harmoniques, nous trouvons des formules nouvelles en différentiant plusieurs fois par rapport à x les deux formules (2) et (15).

Les difficultés qui se présentent dans l'application des méthodes susdites sont évidentes.

En effet, il faut connaître:

1° La valeur de $f(x)$ pour des valeurs particulières de l'argument x

2° Les coefficients $a_{n, s}$ respectivement $b_{n, s}$ qui figurent au second membre de la formule (1) respectivement de la formule (13)

3° La constante K^1 qui figure dans le développement (2).

Cette dernière difficulté est écartée dans le cas où $f(x)$ est l'élément général d'une suite harmonique.

En effet, dans ce cas il existe deux bases $[a_n]$ et $[b_n]$, telles que nous aurons:

$$(20) \quad a_{n, p} = \frac{a_p}{(n-p-1)!}, \quad b_{n, p} = \frac{b_p}{(n-p)!},$$

et le théorème II du paragraphe 2 donnera de même:

$$(21) \quad K = a_n.$$

Dans le cas où $f(x)$ est l'élément général d'une suite harmonique, nous ne trouvons pas de formules nouvelles en différentiant par rapport à x les formules (2) respectivement (15).

¹⁾ Posons $f(x) = A_{n, 0} x^{n-1} + A_{n, 1} x^{n-2} + \dots + A_{n, n-1} x + A_{n, n}$, nous aurons pour K l'expression suivante

$$K = \sum_{s=0}^s \frac{(-1)^s A_{n, s}}{n-s+1}.$$

§ 8. Sur une équation fonctionnelle.

Dans un cas, essentiel pour les recherches qui nous occupent ici, nous connaissons dès à présent les coefficients $a_{n,s}$ et $\beta_{n,s}$ qui figurent aux seconds membres des équations aux différences finies

$$(1) \quad f(x) - f(x-1) = \sum_{s=0}^{s=n-1} a_{n,s} x^{n-s-1},$$

$$(2) \quad f(x) + f(x-1) = \sum_{s=0}^{s=n} \beta_{n,s} x^{n-s},$$

pourvu que le polynôme du degré n par rapport à x

$$(3) \quad f(x) = \sum_{s=0}^{s=n} a_{n,s} x^{n-s}$$

soit donné.

C'est dans le cas où $f(x)$ satisfait à l'équation fonctionnelle

$$(4) \quad (-1)^n f(-x-1) = f(x).$$

En effet, changeons dans (4) le signe de x , nous aurons, en vertu de (3):

$$f(x-1) = (-1)^n f(-x) = \sum_{s=0}^{s=n} (-1)^s a_{n,s} x^{n-s},$$

ce qui donnera immédiatement:

$$(5) \quad f(x) - f(x-1) = 2 \cdot \sum_{s=0}^{s=\frac{n-1}{2}} a_{n,2s+1} x^{n-2s-1},$$

$$(6) \quad f(x) + f(x-1) = 2 \cdot \sum_{s=0}^{s=\frac{n}{2}} a_{n,2s} x^{n-2s},$$

de sorte que ces deux conditions sont certainement nécessaires pour que le polynôme $f(x)$, définie par la formule (3), satisfasse à l'équation fonctionnelle (4).

Inversement, prenons pour point de départ les deux équations aux différences finies (5) et (6), les théorèmes I du paragraphe 2 et III du paragraphe 3 donnent respectivement ces deux développements:

$$(7) \quad \frac{1}{2} f(x) = K_{n-1} + \sum_{s=0}^{s=\frac{n-1}{2}} a_{n,2s+1} (n-2s-1)! \varphi_{n-2s}(x),$$

$$(8) \quad \frac{1}{2} f(x) = \sum_{s=0}^{s=\frac{n}{2}} a_{n,2s} (n-2s)! \chi_{n-2s}(x).$$

Cela posé, appliquons le théorème I du paragraphe 5, nous aurons en vertu de (8):

I. L'équation aux différences finies (6) représente la condition suffisante et nécessaire pour que le polynome entier $f(x)$, défini par la formule (3), satisfasse à l'équation fonctionnelle (4).

Quant à la condition nécessaire (5), elle n'est suffisante que dans le cas où le degré n de $f(x)$ est un nombre pair. Soit, au contraire, n un nombre impair, il faut, en vertu de (7), ajouter la condition ultérieure

$$(9) \quad K_n = 0;$$

car l'équation (4) donnera, pour n impair:

$$f\left(-\frac{1}{2}\right) = 0;$$

et nous aurons de même:

$$\varphi_{2m+1}\left(-\frac{1}{2}\right) = 0.$$

Ordonnons maintenant selon des puissances descendantes de x le second membre de (7), nous aurons:

$$(10) \quad \left\{ \begin{array}{l} a_{n,0} = \frac{2}{n} a_{n,1}, \\ (-1)^{p-1} \left(\frac{a_{n,2p}}{2} - \frac{a_{n,2p+1}}{n-2p} \right) = \frac{1}{n-2p} \cdot \sum_{s=0}^{s=p-1} (-1)^s \binom{n-2s-1}{2p-2s} B_{p-s} a_{n,2s+1}, \end{array} \right.$$

tandis que la formule (8) donnera de même:

$$(11) \quad (-1)^p a_{n,2p+1} = \sum_{s=0}^{s=p} \frac{(-1)^s}{2^{2p-2s+1}} \binom{n-2s}{2p-2s+1} T_{p-s+1} a_{n,2s}.$$

On voit que les deux systèmes d'équations linéaires (10) et (11) entre les coefficients $a_{n,p}$ sont inverses l'un à l'autre.

Dans le cas où n est un nombre pair, on voit que le dernier coefficient $a_{n,n}$ de $f(x)$ ne figure dans aucune des formules (10) et (11); c'est-à-dire que nous pouvons donner à ce coefficient une valeur arbitraire.

L'application des formules (5) et (6), ou, ce qui est la même chose, des formules (7) et (8) à la théorie des nombres de BERNOULLI et d'EULER est évidente et se présente immédiatement en vertu des développements du paragraphe 7.

On voit du reste que les formules (7) et (8) sont, à ce point de vue, moins générales que les formules correspondantes (2) et (15) du paragraphe 7; car les deux premières formules ne nous donnent que des formules récursives qui correspondent à une valeur paire ou impaire de n , tandis que les deux dernières donneront de telles formules, quelle que soit la parité de n .

Soient maintenant

$$(12) \quad a_1, a_2, a_3, \dots, a_n$$

les racines de l'équation algébrique

$$f(x) = 0,$$

où $f(x)$ est le polynôme qui figure dans les formules (3) et (4), et soit a_s une des racines (12) qui n'est pas égale à $-\frac{1}{2}$; il existe une autre racine a_r , telle que

$$(13) \quad a_s = -1 - a_r.$$

Cela posé, désignons par m un positif entier quelconque, il est évident que les polynômes entiers

$$(14) \quad F_m(x) = \sum_{s=1}^{s=n} \frac{(x - a_s)^m}{m!}$$

qui forment une suite harmonique satisfont aux équations fonctionnelles

$$(-1)^m F_m(-x-1) = F_m(x).$$

Posons ensuite pour abréger

$$(15) \quad s_0 = n, \quad s_r = a_1^r + a_2^r + \dots + a_n^r,$$

$$(16) \quad t_r = \left(\frac{1}{2} + a_1\right)^r + \left(\frac{1}{2} + a_2\right)^r + \dots + \left(\frac{1}{2} + a_n\right)^r,$$

nous aurons évidemment:

$$(17) \quad t_{2r+1} = 0,$$

tandis que la formule (14) donnera:

$$F_m(x) = \sum_{q=0}^{q=m} \frac{(-1)^q s_q}{q!} \cdot \frac{x^{m-q}}{(m-q)!};$$

c'est-à-dire que nous aurons les deux développements

$$(18) \quad \frac{1}{2} F_m(x) = \sum_{q=0}^{\leq \frac{m}{2}} \frac{s_{2q+1}}{(2q+1)!} \varphi_{m-2q}(x),$$

$$(19) \quad \frac{1}{2} F_m(x) = \sum_{q=0}^{\leq \frac{m}{2}} \frac{s_{2q}}{(2q)!} \chi_{m-2q}(x).$$

Remplaçons maintenant, dans ces deux formules, m par $2m$ respectivement par $2m+1$, puis posons $x=0$, nous aurons respectivement:

$$(20) \quad (-1)^m \left(\left(m + \frac{1}{2}\right) s_{2m} + s_{2m+1} \right) = \sum_{q=0}^{q=m-1} (-1)^q \binom{2m+1}{2q+1} s_{2q+1} B_{m-q},$$

$$(21) \quad (-1)^{m-1} s_{2m+1} = \sum_{q=0}^{q=m} \frac{(-1)^q}{2^{2m-2q+1}} \binom{2m+1}{2q} s_{2q} T_{m-q+1},$$

formules qui sont analogues à (10) respectivement (11).

Remplaçons ensuite, dans (18) et (19), m par $2m$, puis posons $x = -\frac{1}{2}$, nous aurons de même:

$$(22) \quad (-1)^{m-1} \left(s_{2m+1} + \left(m + \frac{1}{2} \right) t_{2m} \right) = \sum_{q=0}^{q=m-1} (-1)^q \binom{2m+1}{2q+1} \frac{2^{2m-2q-2}}{2^{2m-2q}} s_{2q} B_{m-q},$$

$$(23) \quad (-1)^m (t_{2m} - s_{2m}) = \sum_{q=0}^{q=m-1} \frac{(-1)^q}{2^{2m-2q}} \binom{2m}{2q} s_{2q} E_{m-q}.$$

Il est très facile du reste de généraliser beaucoup les formules de ce genre.

§ 9. Les suites parfaites et les formules récursives.

Nous désignons comme parfaite une suite harmonique $[F_n(x), A_n]$, dont les éléments satisfont pour tous les n à l'équation fonctionnelle

$$(1) \quad (-1)^n F_n(-x-1) = F_n(x).$$

La suite harmonique définie à l'aide de la formule (14) du paragraphe 8 est par conséquent parfaite.

De plus nous aurons la proposition suivante:

I. Les suites harmoniques formées des fonctions de Bernoulli ou des fonctions d'Euler sont parfaites toutes deux.

Appliquons maintenant les résultats que nous venons de développer dans le paragraphe 8, nous aurons les deux équations aux différences finies

$$(2) \quad F_n(x) - F_n(x-1) = 2 \cdot \sum_{s=0}^{\leq \frac{n-1}{2}} \frac{A_{2s+1} x^{n-2s-1}}{(n-2s-1)!},$$

$$(3) \quad F_n(x) + F_n(x-1) = 2 \cdot \sum_{s=0}^{\leq \frac{n}{2}} \frac{A_{2s} x^{n-2s}}{(n-2s)!}$$

et les deux développements

$$(4) \quad \frac{1}{2} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} A_{2s+1} \varphi_{n-2s}(x),$$

$$(5) \quad \frac{1}{2} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} A_{2s} \chi_{n-2s}(x),$$

tandis que les éléments de la base $[A_n]$ satisfont aux conditions suivantes:

$$(6) \quad \begin{cases} A_1 = \frac{1}{2} A_0, \\ \sum_{s=0}^{n-1} \frac{(-1)^s A_{2s+1} B_{n-s}}{(2n-2s)!} = (-1)^{n-1} \left(\frac{1}{2} A_{2n} - A_{2n+1} \right), \end{cases}$$

ou, ce qui est la même chose :

$$(7) \quad \sum_{s=0}^{n-1} \frac{(-1)^s A_{2s} T_{n-s+1}}{(2n-2s+1)! 2^{2n-2s+1}} = (-1)^n A_{2n+1}.$$

On voit que la constante K_n qui figure dans la formule (7) du paragraphe 8 satisfait, dans le cas qui nous étudions ici, à la condition $K_{2n+1} = 0$; c'est-à-dire que nous aurons le théorème suivant :

II. La suite parfaite $[F_n(x), A_n]$ est déterminée, pourvu que nous connaissions une seule des deux suites infinies

$$(8) \quad A_0, A_2, A_4, \dots, A_{2n}, \dots$$

$$(9) \quad A_1, A_3, A_5, \dots, A_{2n+1}, \dots$$

Pour les fonctions $\varphi_n(x)$ de BERNOULLI la suite correspondante (9) ne contient que le seul élément

$$A_1 = \frac{1}{2},$$

tandis que la suite (8) qui correspond aux fonctions $\chi_n(x)$ d'EULER se réduira au seul élément

$$A_0 = \frac{1}{2}.$$

On voit qu'une suite parfaite quelconque conduira à des formules récursives de la forme (6) et (7).

Or, il est très facile de donner l'inversion des théorèmes indiqués par ces deux formules récursives.

En premier lieu, soient

$$a_0, a_1, a_2, \dots, a_n, \dots$$

$$b_0, b_1, b_2, \dots, b_n, \dots; \quad b_0 = 0$$

deux suites infinies, telles que nous aurons, pour $n \geq 1$, les formules récursives

$$(10) \quad \sum_{s=0}^{n-1} \frac{(-1)^s a_s B_{n-s}}{(2n-2s)!} = (-1)^{n-1} b_n,$$

puis posons pour $n \geq 0$

$$(11) \quad A_{2n} = 2(a_n + b_n), \quad A_{2n+1} = a_n,$$

nous aurons le théorème suivant :

III. La suite harmonique $[F_n(x), A_n]$, dont la base $[A_n]$ est déterminée par les formules (11), est parfaite, de sorte que nous aurons ces deux développements, valables pour tous les n :

$$(12) \quad \frac{1}{2} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} a_s \varphi_{n-2s}(x),$$

$$(13) \quad \frac{1}{4} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} (a_s + b_s) \chi_{n-2s}(x).$$

C'est-à-dire que nous pouvons remplacer la formule numérique (10) par les deux formules (12) et (13) qui contiennent la variable complexe x .

Remplaçons, par exemple, dans (13) n par $2n+1$, puis posons $x=0$, nous aurons, comme conséquence immédiate de (10), la formule récursive pour les T_n

$$(14) \quad \sum_{s=0}^{s=n} \frac{(-1)^s (a_s - b_s) T_{n-s+1}}{(2n-2s+1)! 2^{2n-2s}} = (-1)^n a_n.$$

En second lieu, soient

$$\begin{aligned} a_0, a_1, a_2, \dots, a_n, \dots \\ b_0, b_1, b_2, \dots, b_n, \dots \end{aligned}$$

deux suites infinies, telles que nous aurons, pour $n \geq 0$, les formules récursives

$$(15) \quad \sum_{s=0}^{s=n} \frac{(-1)^s a_s T_{n-s+1}}{(2n-2s+1)! 2^{2n-2s+1}} = (-1)^n b_n,$$

puis posons

$$(16) \quad A_{2n} = a_n, \quad A_{2n+1} = b_n,$$

nous aurons de même:

IV. La suite harmonique $[F_n(x), A_n]$, dont la base $[A_n]$ est déterminée par les formules (16), est parfaite, de sorte que nous aurons ces deux développements, valables pour tous les n :

$$(17) \quad \frac{1}{2} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} b_s \varphi_{n-2s}(x),$$

$$(18) \quad \frac{1}{2} F_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} a_s \chi_{n-2s}(x).$$

Remplaçons dans (17) n par $2n$, puis posons $x=0$, nous aurons, comme conséquence immédiate de (15) la formule récursive pour les B_n

$$(19) \quad \sum_{s=0}^{n-1} \frac{(-1)^s b_s B_{n-s}}{(2n-2s)!} = (-1)^{n-1} \left(\frac{1}{2} a_n - b_n \right).$$

Dans ce qui suit nous désignons comme régulières les formules récursives de la forme (10) respectivement (15) pour les B_n et les T_n , tandis que les autres formules récursives seront désignées comme étant irrégulières.

Remarquons que la plupart des formules récursives linéaires connues pour les B_n et les T_n sont régulières, il est évident que les suites parfaites jouent un rôle fondamental dans la théorie des nombres susdits.

Cependant, il faut remarquer que la plupart des suites parfaites que j'ai trouvées par ces considérations ne présentent qu'un intérêt médiocre pour l'Algèbre.

Le polynome étudié dans le paragraphe 14 semble être une exception à la règle générale.

Il est très facile d'indiquer d'autres suites parfaites. Soit par exemple $p \geq 2$ un nombre entier, je dis, que les polynomes

$$(20) \quad f_n(x) = \frac{1}{n!} \cdot \sum_{s=1}^{p-1} \left(x + \frac{s}{p} \right)^n,$$

$$(21) \quad g_n(x) = \frac{1}{n!} \cdot \sum_{s=1}^{p-1} (-1)^{s-1} \left(x + \frac{p-s}{p} \right)^n.$$

forment des suites parfaites.

En effet, nous aurons immédiatement :

$$(22) \quad (-1)^n f_n(-x-1) = f_n(x), \quad (-1)^n g_n(-x-1) = (-1)^p g_n(x),$$

et nous verrons que $f_n(x)$ est toujours du degré n , $g_n(x)$ du degré n respectivement $n-1$, selon que p est pair ou impair.

Appliquons les définitions (20) et (21), les formules (11) et (12) du paragraphe 5 se présentent sous la forme suivante :

$$\begin{aligned} \frac{(-1)^n}{p^{n-1}} \varphi_n(-px-p) &= \frac{1}{p^{n-1}} \varphi_n(px) + f_{n-1}(x), \\ \frac{(-1)^n}{p^n} \chi_n(-px-p) &= \frac{(-1)^{p-1}}{p^n} \chi_n(x) + g_n(x), \end{aligned}$$

ce qui donnera le théorème suivant :

VI. Soit $p \geq 1$ un nombre entier, les polynomes

$$(23) \quad F_n(x) = \frac{2}{p^{n-1}} \varphi_n(px) + f_{n-1}(x),$$

$$(24) \quad G_n(x) = \frac{2}{p^n} \chi_n(px) - (-1)^p g_n(x)$$

forment des suites parfaites.

On voit que $F_n(x)$ est toujours du degré n , tandis que $G_n(x)$ est du degré $n-1$ respectivement n , selon que p est pair ou impair.

CHAPITRE III.

Formules complètes linéaires.

§ 10. Formules régulières pour les B_n .

Il est très intéressant, ce me semble, que la plupart des formules récurrentes classiques connues pour les nombres de BERNOULLI sont une conséquence immédiate de l'identité

$$(1) \quad \varphi_m(x) = \frac{x^m}{m!} + \frac{x^{m-1}}{2 \cdot (m-1)!} + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^s B_s x^{m-2s}}{(2s)! (m-2s)!},$$

formule qui nous donnera un grand nombre d'autres formules récurrentes aussi élégantes que les précédentes.

En effet, désignons par p un positif entier quelconque, puis posons pour abréger

$$(2) \quad A_m(x, p) = \frac{m+1}{2} (2s_m(x, p) - (x+p)^m) - (x+p)^{m+1},$$

nous aurons, en posant dans (1) $x+p$ au lieu de x , la formule générale

$$(3) \quad \sum_{s=1}^{\leq \frac{m}{2}} (-1)^{s-1} \binom{m}{2s} (x+p)^{m-2s} B_s = m! \varphi_m(x) + A_{m-1}(x, p),$$

formule qui est essentielle dans la théorie des nombres de BERNOULLI.

Première application. Supposons $x=0$, puis posons pour abréger

$$(4) \quad A_m = A_m(0, p) = \frac{m+1}{2} (2s_m(p) - p^m) - p^{m+1},$$

je dis, que nous aurons les quatre formules récurrentes générales:

$$(5) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} p^{2s+1} B_{n-s} = (-1)^{n-1} A_{2n},$$

$$(6) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+2}{2s+2} p^{2s+2} B_{n-s} = (-1)^{n-1} A_{2n+1},$$

$$(7) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} p^{2s+2} B_{n-s} = (-1)^{n-1} (A_{2n+1} - p A_{2n}),$$

$$(8) \quad (2n+1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} p^{2s} B_{n-s} = (-1)^{n-1} \left(\frac{A_{2n}}{p} - A_{2n-1} \right).$$

En effet, posons dans (3) $x=0$, $m=2n+1$ respectivement $m=2n+2$, nous aurons les formules (5) et (6); soustrayons (5) de (6), il en résulte la formule (7).

Quant à la formule (8), nous posons dans (6) $n-1$ au lieu de n , puis soustrayons le résultat ainsi obtenu de la formule (5) divisée par p .

Ces formules générales nous donnent les cas particuliers suivants:

1° $p=1$; nous aurons:

$$A_m = \frac{m-1}{2}, \quad A_m - A_{m-1} = \frac{1}{2},$$

ce qui donnera les formules 1, 2, 3 et 4 de la Table. Les trois premières de ces formules sont dues respectivement à MOIVRE¹⁾, à JACOBI²⁾ et à STERN³⁾.

La formule de MOIVRE est la première formule récurrente connue pour les nombres de BERNOULLI. On voit que notre définition des B_n , savoir la formule (15) du paragraphe 2, donnera et la formule de MOIVRE et celle de JACOBI.

2° $p=2$; nous aurons dans ce cas:

$$A_m = (m-3)2^{m-1} + m + 1, \quad A_m - 2A_{m-1} = 2^{m-1} - (m-1),$$

ce qui nous conduira aux formules 5, 6, 7 et 8 de la Table; la formule 5 est due à G.-F. MEYER⁴⁾.

On voit, en vertu de la formule (11), du paragraphe 4, que les suites parfaites qui correspondent aux formules régulières (5) et (6) ont respectivement les éléments généraux

$$(9) \quad s_n(x, p) + \frac{(-1)^n s_n(-x-1, p)}{n!},$$

$$(10) \quad s_{n+1}(x, p) + \frac{(-1)^n s_{n+1}(-x-1, p)}{(n+1)!}.$$

Deuxième application. Supposons, dans notre formule (3), $x = -\frac{1}{2}$, puis posons pour abréger:

$$(11) \quad A_m = m(2t_{m-1}(p) - (2p-1)^{m-1}) - (2p-1)^m.$$

où $t_m(p)$ est la somme de puissances numériques, définie dans la formule (17) du paragraphe 4, nous aurons, par la méthode expliquée dans la première application, les formules générales

$$(12) \quad (2^{2n+1} - 2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 2^{2n-2s} (2p-1)^{2s} B_{n-s} = (-1)^{n-1} A_{2n},$$

$$(13) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s-1} 2^{2n-2s} (2p-1)^{2s} B_{n-s} = \frac{(-1)^{n-1} A_{2n+1}}{2p-1},$$

¹⁾ Miscellanea analytica, complementum, p. 6; Londres 1730.

²⁾ Journal de Crelle, t. 12, p. 265; 1834.

³⁾ Ibid. t. 84, p. 267; 1878.

⁴⁾ Die Bernoullischen Zahlen (Thèse de doctorat); Göttingue 1859.

$$(14) \quad ((2n-1)2^{2n}+2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2n-2s} (2p-1)^{2s} B_{n-s} - (-1)^{n-1} \left(\frac{A_{2n+1}}{2p-1} - A_{2n} \right),$$

$$(15) \quad (2^{2n+1}-2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 2^{2n-2s} (2p-1)^{2s} B_{n-s} = (-1)^{n-1} (A_{2n} - (2p-1)A_{2n-1}).$$

1° $p=1$; la formule (11) donnera

$$A_m = m-1, \quad A_m - A_{m-1} = 1,$$

d'où les formules 9, 10, 11 et 12 de la Table. La formule 9 est due à STERN¹⁾, tandis que la formule 10 appartient à EULER²⁾.

2° $p=2$, ce qui donnera:

$$A_m = (m-3)3^{m-1} + 2m, \quad A_m - 3A_{m+1} = 3^{m-1} - (4m-6),$$

et nous trouvons par conséquent les formules 13, 14, 15 et 16 de la Table.

Les suites parfaites qui correspondent aux deux formules régulières (12) et (13) ont respectivement les éléments généraux

$$(16) \quad \frac{1}{2^{n-2}} \varphi_n(2x) + \frac{s_{n-1} \left(x - \frac{1}{2}, p \right) + (-1)^n s_{n-1} \left(-x - \frac{1}{2}, p-1 \right)}{n!},$$

$$(17) \quad \frac{s_n \left(x - \frac{1}{2}, p \right) + (-1)^n s_n \left(-x - \frac{1}{2}, p-1 \right)}{n!}.$$

Troisième application. Posons, dans la formule générale (3),

$$x = -\frac{a}{4}; \quad a=1 \text{ ou } a=3,$$

nous aurons les formules générales

$$(18) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} (4p-a)^{2s+1} 4^{2n-2s} B_{n-s} = (-1)^{\frac{a+1}{2}} E_n - (-1)^n 4^{2n+1} A_{2n} \left(-\frac{a}{4}, p \right),$$

$$(19) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} (4p-a)^{2s} 4^{2n-2s} B_{n-s} = (2^{2n}-2)B_n - (-1)^n 4^{2n} A_{2n-1} \left(-\frac{a}{4}, p \right).$$

1° $p=1$; nous aurons les formules 17, 18, 19 et 20 de la Table; la formule 17 est due à WORPITZKY³⁾.

2° $p=2$; nous trouvons les formules 21, 22, 23 et 24 de la Table.

Les suites parfaites qui correspondent aux deux formules régulières (18) et (19) ont respectivement les éléments généraux

¹⁾ Journal de Crelle, t. 26, p. 90; 1843.

²⁾ Opuscula analytica, t. II, p. 264-265; Saint-Petersbourg 1785.

³⁾ Journal de Crelle, t. 94, p. 224; 1883.

$$(20) \quad \frac{(-1)^{\frac{a-1}{2}}}{2^n} \chi_n \left(2x - \frac{1}{2} \right) + \frac{s_n \left(x - \frac{a}{4}, p \right) + (-1)^n s_n \left(-x - \frac{a}{4}, p-1 \right)}{n!},$$

$$(21) \quad \frac{1}{4^{n-1}} \varphi_n(4x) - \frac{1}{2^{n-1}} \varphi_n(2x) + \frac{s_{n-1} \left(x - \frac{a}{4}, p \right) + (-1)^n s_{n-1} \left(-x - \frac{a}{4}, p-1 \right)}{(n-1)!}.$$

Quatrième application. Soit, dans la formule (3),

$$x = -\frac{a}{3}, \quad a = 1 \text{ ou } a = 2,$$

nous aurons la formule récursive générale

$$(22) \quad \frac{3}{2} (3^{2n} - 1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} (3p-a)^{2s} 3^{2n-2s} B_{n-s} = (-1)^{n-1} 3^{2n} A_{2n-1} \left(-\frac{a}{3}, p \right),$$

dont la suite parfaite correspondante à l'élément général

$$(23) \quad \varphi_n(3x) = \frac{s_{n-1} \left(x - \frac{a}{3}, p \right) + (-1)^n s_{n-1} \left(-x - \frac{a}{3}, p-1 \right)}{(n-1)!}.$$

1° $p = 1$; nous aurons les formules 25 et 26 de la Table.

2° $p = 2$, ce qui donnera les formules 27 et 28 de la Table.

Cinquième application. Les hypothèses

$$x = -\frac{a}{6}, \quad a = 1 \text{ ou } a = 5$$

nous conduiront, en vertu de (3), à la formule récursive générale

$$(24) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} (6p-a)^{2s} 6^{2n-2s} B_{n-s} = \frac{(3^{2n}-3)(2^{2n}-2)}{2} B_n + (-1)^{n-1} 6^{2n} A_{2n-1} \left(-\frac{a}{6}, p \right),$$

à laquelle correspond la suite parfaite formée des polynômes

$$(25) \quad \frac{\varphi_n(6x)}{6^{n-1}} = \frac{\varphi_n(3x)}{3^{n-1}} - \frac{\varphi_n(2x)}{2^{n-1}} + \frac{s_{n-1} \left(x - \frac{a}{6}, p \right) + (-1)^n s_{n-1} \left(-x - \frac{a}{6}, p-1 \right)}{(n-1)!}.$$

1° $p = 1$; nous aurons les formules 29 et 30 de la Table.

2° $p = 2$, ce qui nous conduira aux formules 31 et 32 de la Table.

On voit que les formules récursives que nous venons de développer sont contenues dans les trois formules générales de la forme

$$(26) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} 4^{2n-2s} p^{2s} B_{n-s} = a_{n,p} B_n + b_{n,p} E_n + c_{n,p},$$

$$(27) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 4^{2n-2s} p^{2s} B_{n-s} = a'_{n,p} B_n + b'_{n,p} E_n + c'_{n,p},$$

$$(28) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} 6^{2n-2s} p^{2s} B_{n-s} = a_{n,p} B_n + \beta_{n,p}.$$

Dans ces formules générales p désigne un positif entier quelconque.

§ 11. Formules régulières pour les T_n .

Dans nos recherches sur les coefficients des tangentes nous prenons pour point de départ la fonction d'EULER

$$(1) \quad \chi_m(x) = \frac{1}{2} \cdot \frac{x^m}{m!} + \sum_{s=1}^{\leq \frac{m+1}{2}} \frac{(-1)^{s-1} T_s x^{m-2s+1}}{(2s-1)! (m-2s+1)! 2^{2s}};$$

remplaçons x par $x+p$, où p désigne un positif entier quelconque, puis posons pour abrégé

$$(2) \quad A_m(x, p) = 2^{m+1} \sigma_m(x, p) - (2p+2x)^m,$$

nous aurons la formule générale:

$$(3) \quad \sum_{s=1}^{\leq \frac{m+1}{2}} (-1)^{s-1} \binom{m}{2s-1} (2p+2x)^{m-2s+1} T_s = A_m(x, p) + (-1)^p m! 2^{m+1} \chi_m(x).$$

Première application. Supposons, dans (3), $x = -\frac{1}{2}$, puis posons en vertu de (2):

$$(4) \quad A_m = 2\tau_m(p) - (2p-1)^m,$$

où $\tau_m(p)$ est la somme de puissances numériques définie dans la formule (18) du paragraphe 4; nous aurons, par le procédé ordinaire, ces quatre formules récursives:

$$(5) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} (2p-1)^{2s+1} T_{n-s} = (-1)^{p-1} E_n + (-1)^{n-1} A_{2n},$$

$$(6) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} (2p-1)^{2s} T_{n-s} = (-1)^{n-1} A_{2n-1},$$

$$(7) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} (2p-1)^{2s+1} T_{n-s} = (-1)^{p-1} E_n - (-1)^n (A_{2n} - (2p-1) A_{2n-1}),$$

$$(8) \quad T_{n+1} = \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} (2p-1)^{2s+2} T_{n-s} = (-1)^{p-1} (2p-1) E_n - (-1)^n ((2p-1) A_{2n} - A_{2n+1}).$$

1° $p=1$; nous aurons:

$$A_m = 1, \quad A_m - A_{m-1} = 0,$$

ce qui nous conduira aux formules 33, 34, 35, 36 de la Table.

2° $p = 2$, ce qui donnera :

$$A_m = 3^m - 2, \quad A_m - 3A_{m-1} = 4;$$

nous trouvons les formules 41, 42, 43 et 44 de la Table.

3° $p = 3$; les formules (7) et (8) nous conduiront aux formules 49, 50 de la Table.

Les suites parfaites qui correspondent aux formules régulières (5) et (6) ont respectivement les éléments généraux

$$(9) \quad (-1)^p 2\chi_{n+1} \left(x - \frac{1}{2} \right) + \frac{\sigma_{n+1} \left(x - \frac{1}{2}, p \right) + (-1)^n \sigma_{n+1} \left(-x - \frac{1}{2}, p-1 \right)}{(n+1)!},$$

$$(10) \quad \frac{\sigma_n \left(x - \frac{1}{2}, p \right) + (-1)^n \sigma_n \left(-x - \frac{1}{2}, p-1 \right)}{n!}.$$

Deuxième application. Supposons, dans la formule générale (3), $x = 0$, puis posons pour abréger

$$(11) \quad A_m = 2^{m+1} \sigma_m(p) - (2p)^m,$$

nous aurons, pourvu que p soit un nombre impair :

$$(12) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} (2p)^{2s+1} T_{n-s} = (-1)^{n-1} A_{2n},$$

$$(13) \quad 2T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s} (2p)^{2s} T_{n-s+1} = (-1)^n A_{2n+1},$$

$$(14) \quad 2n T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s+1} (2p)^{2s} T_{n-s+1} = (-1)^n \left(\frac{A_{2n+2}}{2p} - A_{2n+1} \right),$$

$$(15) \quad 2T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n}{2s} (2p)^{2s} T_{n-s+1} = (-1)^n (A_{2n+1} - 2p A_{2n}).$$

1° $p = 1$, nous trouvons :

$$A_m = 2^m, \quad A_m - 2A_{m-1} = 0,$$

ce qui donnera les formules 37, 38, 39 et 40 de la Table.

2° $p = 3$; les formules (14) et (15) donnent les formules 51 et 52 de la Table.

Supposons que p soit un nombre pair, nous aurons :

$$(16) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} (2p)^{2s+1} T_{n-s} = (-1)^{n-1} A_{2n},$$

$$(17) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} (2p)^{2s+2} T_{n-s} = (-1)^{n-1} A_{2n+1},$$

$$(18) \quad 2n T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} (2p)^{2s} T_{n-s} = (-1)^{n-1} \left(A_{2n} - \frac{A_{2n-1}}{2p} \right),$$

$$(19) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} (2p)^{2s+2} T_{n-s} = (-1)^{n-1} (A_{2n+1} - 2p A_{2n}).$$

3° $p=2$; nous aurons :

$$A_m = 4^m - 2^{m+1}, \quad A_m - 4A_{m-1} = 2^{m+1},$$

ce qui donnera les formules 45, 46, 47 et 48 de la Table; la formule 45 est due à G.-F. MEYER¹⁾.

Les suites parfaites qui correspondent aux formules régulières (12) et (16), (13) et (17) ont respectivement les éléments généraux

$$(20) \quad \frac{\sigma_n(x, p) + (-1)^n \sigma_n(-x-1, p)}{n!},$$

$$(21) \quad \frac{\sigma_{n+1}(x, p) + (-1)^n \sigma_{n+1}(-x-1, p)}{(n+1)!}.$$

Troisième application. Supposons dans la formule (3)

$$x = -\frac{a}{3}, \quad a = 1 \text{ ou } a = 2,$$

puis posons :

$$(22) \quad A_m = 6^m \sigma_{m-1} \left(-\frac{a}{3}, p \right) - 3(6p-2a)^{m-1},$$

nous aurons pour $m=2n-1$, la formule récursive

$$(24) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2n-2s} (6p-2a)^{2s} T_{n-s} = (-1)^{n-1} A_{2n} - \frac{(-1)^{a+p} (3^{2n}-3)}{2} T_n,$$

ce qui donnera pour $p=1$, $p=2$ les formules 53, 54, 55 et 56 de la Table.

La suite parfaite qui correspond à la formule régulière (24) a l'élément général

$$(25) \quad (-1)^{a+p} \left(\frac{\chi_n(3x)}{3^n} - \chi_n(x) \right) + \frac{\sigma_n \left(x - \frac{a}{3}, p \right) + (-1)^n \sigma_n \left(-x - \frac{a}{3}, p-1 \right)}{n!}.$$

§ 12. Formules pour les E_n .

Nos recherches sur les nombres d'EULER sont fondées sur la formule (10) du paragraphe 5, savoir :

$$(1) \quad 2\chi_m(x) = \frac{\left(x + \frac{1}{2}\right)^m}{m!} + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^s E_s}{(2s)! 2^{2s}} \cdot \frac{\left(x + \frac{1}{2}\right)^{m-2s}}{(m-2s)!};$$

dans ce qui suit nous désignons toujours par p un positif entier.

¹⁾ Die Bernoullischen Zahlen. Göttingue 1859.

Première application. Supposons, dans (1), $x = p$, puis posons pour abrégé:

$$(2) \quad A_m = (2p+1)^m - 2^{m+1} \sigma_m(p),$$

le procédé ordinaire nous conduira à ces quatre formules récurrentes:

$$(3) \quad \sum_{s=0}^{n-1} (-1)^s \binom{2n}{2s} (2p+1)^{2s} E_{n-s} = (-1)^{n-1} A_{2n},$$

$$(4) \quad \sum_{s=1}^{n-1} (-1)^s (-1) \binom{2n-1}{2s-1} (2p+1)^{2s-1} E_{n-s} = (-1)^p T_n + (-1)^n A_{2n-1},$$

$$(5) \quad \sum_{s=0}^{n-1} (-1)^s \binom{2n-1}{2s} (2p+1)^{2s} E_{n-s} = (-1)^p (2p+1) T_n + (-1)^n (A_{2n} - (2p+1) A_{2n-1}),$$

$$(6) \quad \sum_{s=0}^{n-1} (-1)^s \binom{2n}{2s+1} (2p+1)^{2s+1} E_{n-s} = (-1)^p T_{n+1} + (-1)^{n-1} (A_{2n+1} - (2p+1) A_{2n}),$$

1° $p = 0$, ce qui donnera $\sigma_m(0) = 0$, d'où:

$$A_m = 1, \quad A_m = A_{m-1} = 0;$$

c'est-à-dire que nous aurons les formules classiques 57, 58, 59 et 60 de la Table.

Les formules 57 et 59 sont dues respectivement à EULER¹⁾ et à SCHERK²⁾.

2° $p = 1$, nous aurons dans ce cas:

$$A_m = 3^m - 2^{m+1}, \quad A_m = 3A_{m-1} - 2^m,$$

ce qui donnera les formules 65, 66, 67 et 68 de la Table.

3° $p = 2$; les formules (5) et (6) donnent respectivement les formules 71 et 72 de la Table.

Deuxième application. Introduisons, dans la formule (1), $x = p - \frac{1}{2}$, puis posons pour abrégé:

$$(7) \quad A_m = (2p)^m - 2^m \sigma_m(p),$$

nous aurons pour p impair:

$$(8) \quad 2E_n + \sum_{s=1}^{n-1} (-1)^s \binom{2n}{2s} (2p)^{2s} E_{n-s} = (-1)^{n-1} A_{2n},$$

$$(9) \quad \sum_{s=0}^{n-1} (-1)^s \binom{2n}{2s+1} (2p)^{2s+1} E_{n-s} = (-1)^{n-1} A_{2n+1},$$

$$(10) \quad (2n-1)E_n + \sum_{s=1}^{n-1} (-1)^s \binom{2n}{2s+1} (2p)^{2s} E_{n-s} = (-1)^{n-1} \left(\frac{A_{2n+1}}{2p} - A_{2n} \right),$$

¹⁾ Institutiones calculi differentialis, p. 545; Saint-Petersbourg 1755. Opuscula analytica, t. II, p. 269-70; Saint-Petersbourg 1785.

²⁾ Mathematische Abhandlungen; Berlin 1825.

$$(11) \quad 2E_n + \sum_{s=1}^{s=2n-1} (-1)^s \binom{2n-1}{2s} (2p)^{2s} E_{n-s} = (-1)^{n-1} (A_{2n} - 2p A_{2n-1}).$$

1° $p = 1$; nous aurons:

$$A_m = 2^m - 2, \quad A_m - 2A_{m-1} = 2,$$

ce qui donnera les formules 61, 62, 63 et 64 de la Table.

Soit ensuite p un nombre pair, nous aurons:

$$(12) \quad \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n}{2s} (2p)^{2s} E_{n-s} = (-1)^n A_{2n},$$

$$(13) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} (2p)^{2s+1} E_{n-s} = (-1)^{n-1} A_{2n+1},$$

$$(14) \quad (2n+1)E_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} (2p)^{2s} E_{n-s} = (-1)^{n-1} \begin{pmatrix} A_{2n+1} & A_{2n} \\ 2p & \end{pmatrix},$$

$$(15) \quad \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} (2p)^{2s+2} E_{n-s} = (-1)^n (A_{2n+2} - 2p A_{2n+1}).$$

2° $p = 2$; les formules (14) et (15) donnent respectivement les formules 69 et 70 de la Table.

Troisième application. Introduisons dans la formule (1):

$$x = p - \frac{a}{3}, \quad a = 1 \text{ ou } a = 2,$$

puis posons pour abréger:

$$(16) \quad A_m = 2 \cdot 6^{m-1} \sigma_{m-1} \left(-\frac{a}{3}, p \right) = (6p+3-2a)^{m-1},$$

nous aurons pour $m = 2n-1$:

$$(17) \quad \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s-1} 3^{2n-2s} (6p+3-2a)^{2s+1} E_{n-s} = (-1)^{n-1} A_{2n} \frac{(-1)^{p+a}(3^{2n}-3)}{6} T_n.$$

Posons particulièrement $p=0$, $a=1$; $p=1$, $a=2$; $p=1$, $a=1$, nous aurons respectivement les formules 73, 74 et 75 de la Table.

§ 13. Formules contenant les $s_n(p)$ et les $\sigma_n(p)$.¹⁾

Les formules de BERNOULLI et d'EULER montrent clairement que les B_n et les T_n sont intimement liés avec les deux sommes de puissances numériques $s_n(p)$ et $\sigma_n(p)$.

¹⁾ Dans un Mémoire qui paraîtra dans les *Annali di matematica* j'ai donné des généralisations très étendues et très curieuses des formules du § 13.

De plus, il est très facile d'évaluer, pour les B_n et les T_n , une suite de formules récursives dont les coefficients contiennent les deux sommes susdites. Nous nous bornerons à développer les formules de ce genre qui sont essentielles dans les recherches qui nous occupent ici.

Remarquons tout d'abord que les développements pour les $s_n(x, p)$ et les $\sigma_n(x, p)$ que nous avons donnés dans le paragraphe 4, nous donnent des formules du genre susdit.

En effet, posons, dans la formule (12) du paragraphe 4, $x = 0$, $x = -1$, puis soustrayons les deux résultats ainsi obtenus, nous aurons la formule inverse de celle de BERNOULLI, savoir :

$$(1) \quad p^n = \sum_{r=0}^{\lfloor \frac{n-1}{2} \rfloor} (-1)^r \binom{n}{2r+1} (2s_{n-2r-1}(p) - p^{n-2r-1}) \frac{T_{r+1}}{2^{r+1}} + n! 4p \chi_n(0).$$

Traisons de la même manière les formules (14) et (16) du paragraphe 4, nous aurons :

$$(2) \quad \frac{n+1}{2} p^n = 2\sigma_{n+1}(p) - p^{n+1} + \sum_{r=1}^{\lfloor \frac{n+1}{2} \rfloor} (-1)^{r-1} \binom{n+1}{2r} B_r (2\sigma_{n-2r+1}(p) - p^{n-2r+1}),$$

ce qui est l'inversion de la formule d'EULER.

Posons maintenant, dans la formule de BERNOULLI,

$$ms_{m-1}(r) = r^m - \frac{m}{2} r^{m-1} = \sum_{s=1}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{s-1} \binom{m}{2s} B_s r^{m-2s},$$

successivement

$$r = 1, 2, 3, \dots, p-1,$$

puis additionnons les $p-1$ équations ainsi obtenues, nous avons à étudier la somme

$$A = s_{m-1}(1) + s_{m-1}(2) + \dots + s_{m-1}(p-1).$$

A cet effet, ordonnons le second membre selon les puissances

$$1^{m-1}, 2^{m-1}, 3^{m-1}, \dots, (p-1)^{m-1},$$

nous aurons :

$$A = (p-1) \cdot 1^{m-1} + (p-2) \cdot 2^{m-1} + \dots + (p-(p-1)) \cdot (p-1)^{m-1},$$

ce qui donnera immédiatement :

$$A = p s_{m-1}(p-1) - s_m(p-1).$$

Cela posé, nous aurons finalement :

$$(3) \quad \sum_{r=1}^{\lfloor \frac{m-1}{2} \rfloor} (-1)^{r-1} \binom{m}{2r} s_{m-2r}(p-1) B_r = m \binom{p-1}{2} s_{m-1}(p-1) - (m+1) s_m(p-1).$$

D'autres formules récursives de ce genre sont des conséquences immédiates des développements de $\varphi_n(px)$ et $\chi_n(px)$ donnés dans le paragraphe 5, savoir les formules (21), (22) et (23).

Le premier de ces développements:

$$\varphi_m(px) = p^m \varphi_m(x) + \sum_{r=1}^{r=m} \frac{(-1)^r p^{m-r-1}}{r!} s_r(p-1) \varphi_{m-r}(x)$$

donnera pour $x = 0$ et $m = 2n$ respectivement $m = 2n+1$ ces deux formules récursives:

$$(4) \quad (p^{2n+1} - p) B_n + \sum_{r=1}^{r=n-1} (-1)^r \binom{2n}{2r} p^{2n-2r} s_{2r}(p-1) B_{n-r} = (-1)^n (s_{2n}(p-1) - np s_{2n-1}(p-1)),$$

$$(5) \quad \sum_{r=0}^{r=n-1} (-1)^r \binom{2n+1}{2r+1} p^{2n-2r} s_{2r+1}(p-1) B_{n-r} = (-1)^n \left(s_{2n+1}(p-1) - \left(n + \frac{1}{2}\right) p s_{2n}(p-1) \right),$$

formules qui peuvent être traitées d'après la méthode appliquée dans les paragraphes précédents.

Les deux formules (4) et (5) sont dues à M. A. RADICKE¹⁾; posons dans les formules en question $p = 2$, nous retrouvons les formules 9 et 10 de la Table.

Remarquons encore qu'il est permis de remplacer, dans (4) et (5), les $s_m(p-1)$ par les $s_m(p)$ correspondantes.

Posons, dans (3), $m = 2n$, puis soustrayons de (4) le résultat ainsi obtenu, nous aurons:

$$(6) \quad \left\{ \begin{aligned} p(p^{2n}-1) B_n + \sum_{r=1}^{r=n-1} (-1)^r \binom{2n}{2r} s_{2r}(p-1) (p^{2n-2r}-1) B_{n-r} = \\ = (-1)^n (n(p-1) s_{2n-1}(p-1) - 2n s_{2n}(p-1)), \end{aligned} \right.$$

formule qui est essentielle dans la théorie des nombres de BERNOULLI.

Quant à la formule (22) du paragraphe 9, savoir:

$$\chi_m(px) = \sum_{r=0}^{r=m} \frac{(-1)^r p^{m-r-1}}{(r+1)!} \sigma_{r+1}(p-1) \varphi_{m-r}(x),$$

où p désigne un nombre pair, nous aurons en posant $x = 0$ et $m = 2n$ respectivement $m = 2n+1$, les formules récursives:

$$(7) \quad \sum_{r=0}^{r=n-1} (-1)^r \binom{2n+1}{2r+1} p^{2n-2r} \sigma_{2r+1}(p-1) B_{n-r} = (-1)^n \left(\sigma_{2n+1}(p-1) - \left(n + \frac{1}{2}\right) p \sigma_{2n}(p-1) \right),$$

¹⁾ Die Recursionsformeln für die Berechnung der Bernoullischen und Eulerschen Zahlen, p. 7; Halle a. S. 1880.

$$(8) \quad (2^{2n}-1)pB_n + \sum_{r=1}^{n-1} (-1)^r \binom{2n}{2r} p^{2n-2r} \sigma_{2r}(p-1) B_{n-r} - (-1)^n (\sigma_{2n}(p-1) - np \sigma_{2n-1}(p-1)),$$

dont la dernière est très curieuse en comparaison avec la formule (4).

Posons $p=2$, nous retrouvons de nouveau les formules 9 et 10 de la Table.

Du reste, on pourrait, dans (7) et (8), remplacer les $\sigma_m(p-1)$ par les $\sigma_m(p)$ correspondantes.

La troisième des formules susdites, savoir la formule (23) du paragraphe 5 :

$$\chi_m(px) = p^m \chi_m(x) + \sum_{r=1}^{m-1} \frac{(-1)^r p^{m-r}}{r!} \sigma_r(p-1) \chi_{m-r}(x),$$

où p doit être un nombre impair, donnera de la même manière les formules récursives

$$(9) \quad (p^{2n-1}-1)T_n + \sum_{r=1}^{n-1} (-1)^r \binom{2n-1}{2r} p^{2n-2r-1} 2^{2r} \sigma_{2r}(p-1) T_{n-r} = (-1)^{n-1} 2^{2n-1} \sigma_{2n-1}(p-1),$$

$$(10) \quad \sum_{r=0}^{n-1} (-1)^r \binom{2n}{2r+1} p^{2n-2r-1} 2^{2r} \sigma_{2r+1}(p-1) T_{n-r} = (-1)^{n-1} 2^{2n-1} \sigma_{2n}(p-1).$$

Dans ces deux formules on pourrait remplacer les $\sigma_m(p-1)$ par les $\sigma_m(p)$ correspondantes, pourvu que nous remplaçons en même temps, dans (9), $p^{2n-1}-1$ par $p^{2n-1}+1$.

Pour mettre en pleine lumière la grande flexibilité de nos méthodes générales, nous avons encore à étudier les deux suites parfaites formées des deux polynômes :

$$(11) \quad f_n(x) = \frac{1}{n!} \cdot \sum_{s=1}^{s=p-1} \left(x + \frac{s}{p}\right)^n, \quad G_n(x) = \frac{1}{n!} \cdot \sum_{s=1}^{s=p-1} (-1)^{s-1} \left(x + \frac{p-s}{p}\right)^n$$

qui figurent à la fin du paragraphe 9.

Nous aurons immédiatement, pour le premier de ces deux polynômes, les deux développements :

$$(12) \quad \frac{1}{2} f_n(x) = \sum_{r=0}^{\leq \frac{n}{2}} \frac{s_{2r+1}(p-1)}{(2r+1)!} p^{2r+1} \varphi_{n-2r}(x),$$

$$(13) \quad \frac{1}{2} f_n(x) = (p-1) \chi_n(x) + \sum_{r=1}^{\leq \frac{n}{2}} \frac{s_{2r}(p-1)}{(2r)!} p^{2r} \chi_{n-2r}(x);$$

posons $x=0$, puis remplaçons n par $2n$ respectivement par $2n+1$, nous aurons respectivement la formule (5) et la formule nouvelle :

$$(14) \quad s_{2n+1}(p-1) = \frac{(-1)^n T_{n+1}(p-1)}{2^{2n+1}} p^{2n+1} + \sum_{r=0}^{r=\frac{n-1}{2}} (-1)^r \binom{2n+1}{2r+1} s_{2n-2r}(p-1) \left(\frac{p}{2}\right)^{2r+1} T_{r+1},$$

dans laquelle on pourrait remplacer les $s_m(p-1)$ par les $s_m(p)$ correspondantes, pourvu que nous remplacions en même temps le facteur $p-1$ du premier terme au second membre par $p+1$.

Quant au polynome $G_n(x)$, nous aurons pour p pair :

$$(15) \quad \frac{1}{2} G_n(x) = \chi_n(x) + \sum_{r=1}^{\leq \frac{n}{2}} \frac{\sigma_{2r}(p-1)}{(2r)! p^{2r}} \chi_{n-2r}(x),$$

$$(16) \quad \frac{1}{2} G_n(x) = \sum_{r=0}^{\leq \frac{n}{2}} \frac{\sigma_{2r+1}(p-1)}{(2r+1)! p^{2r+1}} \varphi_{n-2r}(x),$$

tandis que nous trouvons pour p impair :

$$(17) \quad \frac{1}{2} G_n(x) = \sum_{r=0}^{< \frac{n-1}{2}} \frac{\sigma_{2r+1}(p-1)}{(2r+1)! p^{2r+1}} \chi_{n-2r-1}(x),$$

$$(18) \quad \frac{1}{2} G_n(x) = \sum_{r=0}^{\leq \frac{n-1}{2}} \frac{\sigma_{2r+2}(p-1)}{(2r+2)! p^{2r+2}} \varphi_{n-2r-1}(x).$$

Posons dans les quatre dernières formules $x=0$, nous aurons, pourvu que n et p soient de parité différente :

$$(19) \quad \sigma_n(p-1) = \frac{np}{2} \sigma_{n-1}(p-1) + \sum_{r=1}^{< \frac{n-1}{2}} (-1)^r \binom{n}{2r} p^{2r} \sigma_{n-2r}(p-1) B_r,$$

$$(20) \quad \sigma_n(p-1) = n! 2p^n \chi_n(0) + \sum_{r=0}^{< \frac{n-1}{2}} (-1)^r \binom{n}{2r+1} \left(\frac{p}{2}\right)^{2r+1} \sigma_{n-2r-1}(p-1) T_{r+1};$$

dans ces deux formules il est permis de remplacer les $\sigma_m(p-1)$ par les $\sigma_m(p)$ correspondantes.

§ 14. Exemples des formules irrégulières.

Pour obtenir des formules récurrentes d'une nature entièrement différente des précédentes, nous avons à étudier le polynome

$$(1) \quad \psi_n(x) = \sum_{s=0}^{s=n} \frac{2n}{2n-s} \binom{2n-s}{s} \frac{x^{n-s}}{2^{2s}}, \quad \psi_0(x) = 2;$$

nous aurons pour $n=1$ et $n=2$ respectivement :

$$(2) \quad \psi_1(x) = x + \frac{1}{2}, \quad \psi_2(x) = x^2 - x + \frac{1}{8}.$$

De plus, il est très facile de vérifier la formule récursive

$$(3) \quad \left(x + \frac{1}{2}\right) \psi_p(x) = \psi_{p+1}(x) + \frac{1}{2^4} \psi_{p-1}(x), \quad p \geq 1,$$

ou, ce qui est la même chose,

$$\psi_1(x) \psi_p(x) = \psi_{p+1}(x) + \frac{1}{2^4} \psi_{p-1}(x), \quad p \geq 1,$$

d'où, par la conclusion ordinaire de n à $n+1$, la formule plus générale

$$(4) \quad \psi_q(x) \psi_p(x) = \psi_{p+q}(x) + \frac{1}{2^{4q}} \psi_{p-q}(x), \quad p \geq q;$$

c'est-à-dire que nous aurons particulièrement:

$$(5) \quad \left(x^2 + x + \frac{1}{8}\right) \psi_p(x) = \psi_{p+2}(x) + \frac{1}{2^8} \psi_{p-2}(x), \quad p \geq 2.$$

Cela posé, nous aurons immédiatement la formule curieuse:

$$(6) \quad \psi_n(4x^2 + 4x) = 2^{2n} \psi_{2n}(x),$$

d'où, en vertu de la définition de $\psi_n(x)$:

$$(7) \quad \psi_{2n}(x) = \sum_{s=0}^{s=n} \frac{2n}{2n-s} \binom{2n-s}{s} \frac{(x^2+x)^{n-s}}{2^{4s}};$$

appliquons ensuite la formule (3), nous aurons de même:

$$(8) \quad \psi_{2n+1}(x) = \left(x + \frac{1}{2}\right) \cdot \sum_{s=0}^{s=n} \binom{2n-s}{s} \frac{(x^2+x)^{n-s}}{2^{4s}}.$$

De plus, nous aurons, en vertu de (3), par la conclusion ordinaire de n à $n+1$, cet autre développement:

$$(9) \quad \psi_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} \frac{(-1)^s n}{n-s} \binom{n-s}{s} \frac{\left(x - \frac{1}{2}\right)^{n-2s}}{2^{4s}}.$$

Les formules que nous venons de développer donnent immédiatement les valeurs numériques

$$(10) \quad \psi_n(0) = \frac{1}{2^{2n-1}}, \quad \psi'_n(0) = \frac{n^2}{2^{2n-2}}, \quad \psi_{2n}\left(-\frac{1}{2}\right) = \frac{(-1)^n}{2^{4n-1}}.$$

Nous avons encore à déterminer cette autre valeur numérique:

$$(11) \quad \psi_n\left(-\frac{1}{4}\right) = \frac{a_n}{2^{2n}};$$

les développements (1) et (9) donnent immédiatement pour a_n ces deux expressions:

$$(12) \quad a_n = \sum_{s=0}^{\leq \frac{n}{2}} \frac{(-1)^s n}{n-s} \binom{n-s}{s} = (-1)^n \sum_{s=0}^{s=n} \frac{(-1)^s 2n}{2n-s} \binom{2n-s}{s},$$

tandis que l'équation fonctionnelle (3) donnera la formule récursive :

$$(13) \quad a_{n+1} = a_n + a_{n-2}, \quad n \geq 2.$$

Appliquons les valeurs initiales :

$$a_0 = 2, \quad a_1 = 1,$$

nous aurons généralement :

$$(14) \quad a_{6n} = 2, \quad a_{6n+1} = 1, \quad a_{6n+3} = -2, \quad a_{6n+5} = -1.$$

Posons pour abréger

$$2 \cos n\theta = \phi_n(2 \cos \theta),$$

nous aurons :

$$\psi_n(x) = \frac{(-1)^n}{2^{2n}} \phi_{2n}(2i\sqrt{x});$$

c'est-à-dire que l'équation algébrique du degré n

$$(15) \quad \psi_n(x) = 0$$

a les n racines négatives inégales.

$$(16) \quad \alpha_p = -\cos^2 \frac{(2p+1)\pi}{4n}, \quad 0 \leq p < n-1,$$

de sorte que nous aurons

$$(17) \quad \alpha_p + \alpha_{n-p-1} = -1, \quad p > n-p-1.$$

Cela posé, il est évident que $\psi_n(x)$ représente une solution très intéressante de l'équation fonctionnelle

$$(-1)^n f(-x-1) = f(x),$$

où $f(x)$ est un polynome du degré n par rapport à x .

Quant aux applications de $\psi_n(x)$ dans la théorie des nombres de BERNOULLI, nous aurons les deux développements :

$$(18) \quad \frac{1}{2} \psi_m(x) = K_m + \sum_{s=0}^{\leq \frac{m-1}{2}} \frac{2m}{2m-2s-1} \binom{2m-2s-1}{2s+1} \frac{(m-2s-1)!}{2^{4s+2}} \zeta_{m-2s}(x),$$

$$(19) \quad \frac{1}{2} \psi_m(x) = \sum_{s=0}^{\leq \frac{m}{2}} \frac{m}{m-s} \binom{2m-2s}{2s} \frac{(m-2s)!}{2^{4s}} \chi_{m-2s}(x).$$

Dans (18) nous aurons par conséquent :

$$K_{2n+1} = 0;$$

mais je n'ai pas réussi à déterminer sous une simple formule la valeur de K_{2n} .

Posons dans (18) $m = 2n+1$, $x = -\frac{1}{4}$, nous aurons, en vertu de (11) et (14), la formule récursive

$$(20) \quad \sum_{s=0}^{s=n-1} \frac{(-1)^s (2n-1)}{4n-2s-1} \binom{4n-2s-1}{2s+1} E_{n-s} = (-1)^n \omega_n,$$

où nous avons posé pour abréger :

$$\omega_{3n} = 0, \quad \omega_{3n+1} = -3, \quad \omega_{3n+2} = 0.$$

Posons dans la même formule $m = 2n$ et $x = 0$, $x = -\frac{1}{2}$, puis soustrayons les deux équations ainsi obtenues, nous aurons :

$$(21) \quad \sum_{s=0}^{s=n-1} \frac{(-1)^s 2n}{4n-2s-1} \binom{4n-2s-1}{2s+1} T_{n-s} = 1 - (-1)^n.$$

Enfin, différencions par rapport à x la formule (18), puis posons $m = 2n+1$, $x = 0$, il en résulte :

$$(22) \quad \sum_{s=0}^{s=n-1} \frac{(-1)^s (2n+1)}{(4n-2s+1) 2^{4s}} \binom{4n-2s+1}{2s+1} B_{n-s} = \frac{(-1)^{n-1} n(n+1)}{2^{4n-2}}.$$

Quant à la formule (19), posons $m = 2n-1$, $x = 0$, nous aurons :

$$(23) \quad \sum_{s=0}^{s=n-1} \frac{(-1)^s (2n-1)}{(2n-s-1) 2^{2s}} \binom{4n-2s-2}{2s} T_{n-s} = \frac{(-1)^{n-1}}{2^{2n-2}},$$

tandis que les hypothèses $m = 2n$, $x = -\frac{1}{2}$ donnent :

$$(24) \quad \sum_{s=0}^{s=n-1} \frac{(-1)^s n}{(2n-s) 2^{2s}} \binom{4n-2s}{2s} E_{n-s} = \frac{1 - (-1)^n}{2^{2n-1}}.$$

On voit que plusieurs des formules récursives que nous venons de trouver sont homogènes.

Différencions plusieurs fois par rapport à x les formules (18) et (19), nous trouverons des formules récursives irrégulières d'une forme plus compliquée.

CHAPITRE IV.

Formules linéaires incomplètes.

§ 15. Étude d'un polynôme entier.

Désignons par x et a deux variables complexes, par n et p deux nombres entiers non négatifs, nous avons à étudier le polynôme entier du degré $n+p$:

$$(1) \quad f(x) = (x+a)^n (x+1-a)^p.$$

Écrivons

$$f(x) = (x+a)^n \left((x+1-a) + (1-2a) \right)^p,$$

nous aurons, en appliquant la formule binomiale :

$$(2) \quad f(x) = \sum_{s=0}^{s=p} \binom{p}{s} (1-2a)^s (x+a)^{n+p-s},$$

et il est évident que l'identité

$$f(x-1) = (x-a)^p [(x-a) - (1-2a)]^n$$

donnera de même:

$$(3) \quad f(x-1) = \sum_{s=0}^{s=p} (-1)^s \binom{n}{s} (1-2a)^s (x-a)^{n+p-s}.$$

Cela posé, additionnons, puis soustrayons les formules (2) et (3), nous aurons ces deux développements:

$$(4) \quad \begin{cases} f(x) = \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! (1-2a)^s \chi_{n+p-s}(x+a) + \\ + \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! (1-2a)^s \chi_{n+p-s}(x-a), \end{cases}$$

$$(5) \quad \begin{cases} f(x) = K_{n,p}(a) + \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-s+1}(x+a) - \\ - \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-s+1}(x-a), \end{cases}$$

où le terme $K_{n,p}(a)$ qui figure au second membre de (5) est indépendant de la variable x .

Pour déterminer la valeur de $K_{n,p}(a)$ nous différencions par rapport à x la formule (5), ce qui donnera:

$$(6) \quad \begin{cases} f'(x) = \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-s}(x+a) - \\ - \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-s}(x-a), \end{cases}$$

et nous aurons évidemment:

$$(7) \quad f'(x) = n(x+a)^{n-1}(x+1-a)^p + p(x-a)^n(x+1-a)^{p-1}.$$

Développons maintenant, en vertu de (5), les deux fonctions qui figurent au second membre de (7), nous trouvons précisément tous les termes du second membre de la formule (6), mais nous trouvons en outre un terme qui est indépendant de x ; c'est-à-dire que ce terme constant par rapport à x s'évanouira, de sorte que nous aurons:

$$nK_{n-1,p}(a) + pK_{n,p-1}(a) = 0,$$

ou, ce qui est la même chose :

$$K_{n,p}(a) = \frac{p}{n+1} K_{n+1,p-1}(a);$$

c'est-à-dire que nous aurons :

$$(8) \quad K_{n,p}(a) = \frac{(-1)^p n! p!}{(n+p)!} K_{n+p,0}(a).$$

Or, l'identité évidente

$$\frac{(x+a)^m}{m!} - \frac{(x+a-1)^m}{m!} = \frac{(x+a)^m}{m!} - \sum_{s=0}^{s=m} \frac{(-1)^s (1-2a)^s (x-a)^{m-s}}{s! (m-s)!}$$

donnera :

$$(9) \quad \frac{(x+a)^m}{m!} = \varphi_{m+1}(x+a) - \sum_{s=0}^{s=m+1} \frac{(-1)^s (1-2a)^s}{s!} \varphi_{m-s+1}(x-a);$$

car les deux membres de la formule (9) sont des éléments de deux suites harmoniques.

Remarquons ensuite que la formule (9) peut être obtenue directement de (5) si nous posons $p=0$, $n=m$ et si nous divisons ensuite par $m!$.

Cela posé, nous aurons :

$$K_{m,0}(x) = \frac{(-1)^m (1-2a)^{m+1}}{m+1},$$

d'où, en vertu de (8), généralement :

$$(10) \quad K_{n,p}(x) = \frac{(-1)^n n! p!}{(n+p+1)!} (1-2a)^{n+p+1}.$$

Soit ensuite $q < n+p$ un positif entier, la formule (4) donnera :

$$(11) \quad \left\{ \begin{aligned} D_x^q [(x+a)^n (x+1-a)^p] &= \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! (1-2a)^s \chi_{n+p-q-s}(x+a) + \\ &+ \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! (1-2a)^s \chi_{n+p-q-s}(x-a), \end{aligned} \right.$$

où il faut supprimer les termes contenant des fonctions d'EULER à indice négatif.

Remarquons que la formule (4) est un cas particulier de (11), c'est-à-dire correspond à $q=0$.

La formule (5) donnera de même :

$$(12) \quad \left\{ \begin{aligned} D_x^{q+1} [(x+a)^n (x+1-a)^p] &= \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-q-s}(x+a) - \\ &- \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! (1-2a)^s \varphi_{n+p-q-s}(x-a), \end{aligned} \right.$$

où il faut supprimer les termes contenant des fonctions de BERNOULLI à indice négatif.

Il est évident que la formule (5) n'est pas un cas particulier de (12); c'est-à-dire qu'il faut étudier séparément la seule formule (5) et l'ensemble des formules (12).

Il est très intéressant, ce me semble, que nos identités algébriques précédentes nous donnent, d'un seul coup, toutes les formules incomplètes de première espèce connues et un grand nombre d'autres. On sait que les formules connues de ce genre ont été trouvées par des méthodes différentes.

§ 16. Formules de M. Saalschütz.

Posons, dans notre formule générale (5) du paragraphe 15, $\alpha = 0$, la fonction $\varphi_{n+p+1}(x)$ disparaîtra, de sorte que nous aurons, après une légère modification, le développement suivant:

$$(1) \quad \left\{ \begin{aligned} x^n(x+1)^p &= \frac{(-1)^n n! p!}{(n+p+1)!} + \sum_{s=0}^{s=p-1} \binom{p}{s-1} (n+p-s-1)! \varphi_{n+p-s}(x) + \\ &+ \sum_{s=0}^{s=n-1} (-1)^s \binom{n}{s+1} (n+p-s-1)! \varphi_{n+p-s}(x), \end{aligned} \right.$$

ce qui nous donnera les formules incomplètes que M. SAALSCHÜTZ¹⁾ a trouvées à l'aide de la formule sommatoire d'EULER et de MAC LAURIN.

1° Supposons tout d'abord $n+p$ pair, savoir:

$$(2) \quad n+p = 2m,$$

puis posons $x=0$, nous aurons la formule incomplète:

$$(3) \quad \frac{(-1)^{m+n} n! p!}{(2m+1)!} = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \frac{B_{m-s}}{2m-2s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \frac{B_{m-s}}{2m-2s},$$

d'où particulièrement pour $p=n$, ce qui donnera également $m=n$, la formule plus élégante

$$(4) \quad \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \frac{B_{n-s}}{n-2s} = \frac{n! n!}{(2n+1)!}.$$

Supposons dans cette formule $n=2q$ respectivement $n=2q+1$, nous verrons qu'elle contient respectivement l'ensemble des nombres de BERNOULLI:

$$\begin{aligned} B_{2q}, B_{2q-1}, \dots, B_{q+1}, \\ B_{2q+1}, B_{2q}, \dots, B_{q+1}. \end{aligned}$$

¹⁾ Zeitschrift für Mathematik und Physik, Bd. 37, p. 374—378; 1892. Vorlesungen über die Bernoullischen Zahlen, p. 185—189; Berlin 1893.

Posons, au contraire, dans (1), $x = -\frac{1}{2}$, puis ajoutons l'équation ainsi obtenue à la formule (3), puis multiplions par 2^{4m-1} , nous aurons :

$$(5) \quad (-1)^{m+n} 2^{2m-1} = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} 2^{4s} T_{m-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} 2^{4s} T_{m-s},$$

ce qui donnera particulièrement pour $p = n = m$:

$$(6) \quad 2^{2n-2} = \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} 2^{4s} T_{n-s},$$

formule qui est du même caractère que (4).

2° Supposons ensuite $n+p$ impair, savoir :

$$(7) \quad n+p = 2m+1,$$

la formule (1) donnera pour $x=0$:

$$(8) \quad \frac{(-1)^{m+n} n! p!}{(2m+2)!} = \sum_{s=0}^{\leq \frac{p-2}{2}} (-1)^s \binom{p}{2s+2} \frac{B_{m-s}}{2m-2s} - \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} \frac{B_{m-s}}{2m-2s},$$

d'où, pour $p = n+1$, après un simple calcul, la formule (4).

Posons encore, dans (1), $x = -\frac{1}{2}$, nous aurons par le procédé ordinaire :

$$(9) \quad (-1)^{m+n} 2^{2m-2} = \sum_{s=0}^{\leq \frac{p-2}{2}} (-1)^s \binom{p}{2s+2} 2^{4s} T_{m-s} - \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} 2^{4s} T_{m-s};$$

soit $p = n+1$, ce qui donnera $m = n$, nous retrouvons, après un simple calcul, la formule (6).

§ 17. Généralisations des formules de Stern.

Supposons, dans la formule générale (12) du paragraphe 15, $\alpha = 0$, remplaçons q par $q-1$, où

$$1 < q < n-1,$$

puis posons pour abréger :

$$(1) \quad n+p = m+q,$$

l'hypothèse $x=0$ donnera :

$$(2) \quad 0 = \sum_{s=0}^{s=p-1} \binom{p}{s+1} (m+q-s-1)! \varphi_{m-s}(0) + \sum_{s=0}^{s=n-1} (-1)^s \binom{n}{s+1} (m+q-s-1)! \varphi_{m-s}(0).$$

1° Soit m un nombre pair; nous posons $2m$ au lieu de m ; divisons ensuite par $(q-1)!$ les deux membres de (2), nous aurons:

$$(3) \quad 0 = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \binom{2m+q-2s-1}{q-1} B_{m-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \binom{2m+q-2s-1}{q-1} B_{m-s},$$

formule, dont le cas particulier $q=1$ appartient à STERN¹⁾. Posons par conséquent

$$n+p = 2m+1,$$

nous aurons

$$(4) \quad 0 = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} B_{m-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} B_{m-s}.$$

On voit que la plus simple de ces formules correspond à $p=n+1$, ce qui donnera $m=n$, et la formule récursive correspondante deviendra:

$$(5) \quad 0 = \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n+1}{2s+1} B_{n-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} B_{n-s}, \quad n \geq 2.$$

Supposons $n=2q$, respectivement $n=2q+1$, la formule (5) contiendra respectivement l'ensemble des nombres de BERNOULLI:

$$B_{2q}, B_{2q-1}, \dots, B_q, \\ B_{2q+1}, B_{2q}, \dots, B_{q+1};$$

c'est-à-dire que la formule (5) est, pour $n=2q$, moins avantageuse que la formule analogue (4) du paragraphe 16.

Soit ensuite $q=2$, ce qui donnera:

$$n+p = 2m+2;$$

nous aurons:

$$(6) \quad 0 = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} (2m-2s+1) B_{m-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} (2m-2s+1) B_{m-s};$$

remplaçons dans cette formule n et p par $n+1$, ce qui donnera $m=n$; nous aurons la formule la plus simple de ce genre, savoir la formule de SEIDEL²⁾:

$$(7) \quad 0 = \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n+1}{2s+1} (2n-2s+1) B_{n-s}, \quad n \geq 2.$$

¹⁾ Beiträge zur Theorie der Bernoullischen und Eulerschen Zahlen, p. 7—16. Mémoires de la Société de Göttingue 1878.

²⁾ Sitzungsberichte der Münchener Akademie 1877, p. 164—165.

On voit que la formule de SEIDEL contient précisément les mêmes nombres de BERNOULLI que la formule (5).

2° Soit, dans (1) et (2), m un nombre impair, nous remplaçons m par $2m+1$, ce qui donnera :

$$(8) \sum_{s=0}^{\leq \frac{p-2}{2}} (-1)^s \binom{p}{2s+2} \binom{2m+q-2s-1}{q-1} B_{m-s} = \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} \binom{2m+q-2s-1}{q-1} B_{m-s};$$

posons particulièrement $q=1$, ce qui donnera :

$$n+p = 2m+2;$$

nous aurons la formule de STERN :

$$(9) \sum_{s=0}^{\leq \frac{p-2}{2}} (-1)^s \binom{p}{2s+2} B_{m-s} = \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} B_m.$$

La formule la plus simple de cette catégorie correspond à $p=n+2$, ce qui donnera $m=n$.

Posons, dans (8), $q=2$, ce qui donnera :

$$n+p = 2m+3;$$

nous aurons la formule

$$(10) \sum_{s=0}^{\leq \frac{p-2}{2}} (-1)^s \binom{p}{2s+2} (2m-2s-1) B_{m-s} = \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} (2m-2s-1) B_m,$$

dont le cas le plus simple correspond à $p=n+1$, ce qui nous conduira à la formule (7) de SEIDEL.

Quant à la formule générale (11) du paragraphe 15, nous posons $a=0$; supposons ensuite

$$0 \leq q \leq n-1,$$

l'hypothèse $x=0$ donnera, avec la définition (1) du nombre m :

$$(11) \sum_{s=0}^{s=p} \binom{p}{s} (m+q-s)! \chi_{m-s}(0) + \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (m+q-s)! \chi_{m-s}(0) = 0.$$

1° Soit m un nombre impair, nous remplaçons m par $2m+1$, ce qui donnera, après une division par $q!$:

$$(12) \sum_{s=0}^{\leq \frac{p}{2}} (-1)^s \binom{p}{2s} \binom{2m+q-2s+1}{q} 2^{2s} T_{m-s+1} + \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} \binom{2m+q-2s+1}{q} 2^{2s} T_{m-s+1} = 0;$$

le cas particulier correspondant à $q=0$ appartient à STERN¹⁾; posons encore

¹⁾ loc. cit.

$p = n + 1$, ce qui donnera $m = n$, nous trouvons la formule la plus simple de ce genre.

Posons encore $q = 1$, nous obtenons la formule la plus simple en supposant $p = n$, ce qui donnera $m = n - 1$. De cette manière nous trouvons la formule de SEIDEL¹⁾:

$$(13) \quad \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} (n-s) 2^{2s} T_{n-s} = 0, \quad n \geq 2,$$

ou, ce qui est évidemment la même chose:

$$(14) \quad \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} (2^{2n-2s} - 1) B_{n-s} = 0, \quad n \geq 2.$$

2° Soit ensuite, dans (11), m un nombre pair, nous aurons, en remplaçant m par $2m$:

$$(15) \quad \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \binom{2m+q-2s-1}{q} 2^{2s} T_{m-s} = \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \binom{2m+q-2s-1}{q} 2^{2s} T_{m-s},$$

dont le cas particulier $q = 0$ est dû à STERN; posons $p = n + 2$, ce qui donnera $m = n + 1$, nous aurons la formule la plus simple de ce genre.

Posons encore $q = 1$, $p = n + 1$, ce qui donnera $m = n$, nous retrouvons la formule (13) de SEIDEL.

§ 18. Formules contenant les nombres d'Euler.

Il est évident que les formules générales (11) et (12) du paragraphe 15 ne nous donnent, pour $\alpha = 0$, de formules élégantes que dans le cas où $x = 0$; car les dérivées qui figurent aux premiers membres des formules susdites seront très compliquées pour d'autres valeurs de x .

Quant aux nombres d'EULER, nous avons par conséquent à prendre pour point de départ les formules (4) et (5) du paragraphe 15. Posons, dans la première de ces deux formules, $\alpha = 0$, nous aurons:

$$(1) \quad x^n (x+1)^p = \sum_{s=0}^{s=p} \binom{p}{s} (n+p-s)! \chi_{n+p-s}(x) + \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} (n+p-s)! \chi_{n+p-s}(x).$$

Supposons ensuite $x = -\frac{1}{2}$, nous aurons selon que $n+p = 2m$ ou $n+p = 2m+1$ respectivement:

¹⁾ loc. cit. p. 172.

$$(8) \quad 3^n = \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n+1}{2s+1} 2^{4s} E_{n-s} + \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} 2^{4s} E_{n-s},$$

$$(9) \quad 3^n = \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n+2}{2s+2} 2^{4s} E_{n-s} - \sum_{s=0}^{\leq \frac{n-2}{2}} (-1)^s \binom{n}{2s+2} 2^{4s} E_{n-s};$$

ces deux formules particulières sont également dues à M. RADICKE.

En second lieu, différenciations par rapport à x la formule (1), puis posons $x = -\frac{1}{2}$, nous aurons, selon que $n+p = 2m+1$ ou $n+p = 2m+2$ respectivement :

$$(10) \quad \left\{ \begin{aligned} (-1)^{m+n} (2p-2n) &= \sum_{s=0}^{\leq \frac{p}{2}} (-1)^s \binom{p}{2s} (2m-2s+1) 2^{2s} E_{m-s} + \\ &+ \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} (2m-2s+1) 2^{2s} E_{m-s}, \end{aligned} \right.$$

$$(11) \quad \left\{ \begin{aligned} (-1)^{m+n} (p-n) &= \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} (2m-2s+1) 2^{2s} E_{m-s} - \\ &- \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} (2m-2s+1) 2^{2s} E_{m-s}, \end{aligned} \right.$$

formules dont les cas les plus simples correspondent à $p = n+1$ respectivement $p = n+2$, ce qui donnera $m = n$.

§ 19. Autres formules incomplètes.

Posons dans la formule générale (5) du paragraphe 15

$$a = \frac{1}{4}, \quad x = -\frac{1}{4},$$

nous aurons :

$$(1) \quad \left\{ \begin{aligned} 0 &= \frac{(-1)^n n! p!}{(n+p+1)! 2^{n+p+1}} + \sum_{s=0}^{s=p} \frac{(n+p-s)!}{2^s} \binom{p}{s} \varphi_{n+p-s+1}(0) - \\ &- \sum_{s=0}^{s=n} \frac{(-1)^s (n+p-s)!}{2^s} \binom{n}{s} \varphi_{n+p-s+1} \left(-\frac{1}{2} \right). \end{aligned} \right.$$

1° Soit, dans cette formule, $n + p = 2m - 1$, nous aurons :

$$(2) \quad \frac{(-1)^{m+n} n! p!}{(2m)!} = \sum_{s=0}^{\leq \frac{p}{2}} (-1)^s \binom{p}{2s} \frac{2^{2m-2s} B_{m-s}}{2m-2s} + \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} \frac{2^{2m-2s-2} B_{m-s}}{2m-2s},$$

d'où pour $p = n - 1$, ce qui donnera $m = n$, la formule la plus élégante de ce genre :

$$(3) \quad \frac{n! n-1!}{(2n)!} = \sum_{s=1}^{\leq \frac{n}{2}} (-1)^{s-1} \binom{n-1}{2s-1} \frac{2^{2n-2s} B_{n-s}}{2n-2s} + \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} \frac{T_{n-s}}{2^{2n-2s-1}}.$$

2° Supposons, dans (2), $n + p = 2m$, nous aurons de même :

$$(4) \quad \frac{(-1)^{m+n} n! p!}{(2m+1)!} = \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \frac{2^{2m-2s} B_{m-s}}{2m-2s} - \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \frac{2^{2m-2s-2} B_{m-s}}{2m-2s},$$

ce qui donnera pour $p = n = m$ la formule de M. SAALSCHÜTZ, savoir la formule (4) du paragraphe 16.

Posons ensuite, dans la formule (12) du paragraphe 15,

$$x = -\frac{1}{4}, \quad a = \frac{1}{4}, \quad n + p = m + q; \quad 0 \leq q < n-2,$$

nous aurons :

$$(5) \quad \sum_{s=0}^{s=p} \binom{p}{s} \binom{m+q-s}{2s}! \varphi_{m-s}(0) - \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} \binom{m+q-s}{2s}! \varphi_{m-s}\left(-\frac{1}{2}\right).$$

1° Soit m un nombre pair, nous remplaçons m par $2m$, ce qui donnera :

$$(6) \quad \sum_{s=0}^{\leq \frac{p}{2}} (-1)^s \binom{p}{2s} \binom{2m+q-2s}{q} 2^{2m-2s} B_{m-s} + \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} \binom{2m+q-2s}{q} (2^{2m-2s-2} B_{m-s}) = 0;$$

posons dans cette formule $q = 0$ et $p = n = m$, nous retrouvons la formule de SEIDEL, savoir la formule (14) du paragraphe 17.

2° Supposons m impair, posons $2m + 1$ au lieu de m , nous aurons, en vertu de (5) :

$$(7) \quad \left\{ \begin{array}{l} \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \binom{2m+q-2s}{q} 2^{2m-2s} B_{m-s} \\ \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \binom{2m+q-2s}{q} (2^{2m-2s-2} B_{m-s}) \end{array} \right.$$

d'où pour $q=0$, $p=n+1$, ce qui donnera $m=n$:

$$(8) \quad \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} 2^{2n-2s} B_{n-s} + 2 \cdot \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} B_{n-s} = 0, \quad n > 2.$$

En dernier lieu nous avons à étudier la formule (11) du paragraphe 15; posons:

$$x = -\frac{1}{4}, \quad \alpha = \frac{1}{4}, \quad n+p = m+q; \quad 0 \leq q \leq n-1,$$

nous aurons:

$$(9) \quad 0 = \sum_{s=0}^{s=p} \binom{p}{s} \frac{(m+q-s)!}{2^s} \chi_{m-s}(0) + \sum_{s=0}^{s=n} (-1)^s \binom{n}{s} \frac{(m+q-s)!}{2^s} \chi_{m-s}\left(-\frac{1}{2}\right);$$

remplaçons ensuite m par $2m$ respectivement par $2m+1$, nous aurons:

$$(10) \quad \sum_{s=0}^{\leq \frac{p-1}{2}} (-1)^s \binom{p}{2s+1} \binom{2m+q-2s-1}{q} T_{m-s} = \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} \binom{2m+q-2s}{q} E_{m-s},$$

$$(11) \quad \sum_{s=0}^{\leq \frac{p}{2}} (-1)^s \binom{p}{2s} \binom{2m+q-2s+1}{q} T_{m-s+1} = \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \binom{2m+q-2s}{q} E_{m-s},$$

formules dont le cas particulier $q=0$ appartient à STERN¹⁾.

CHAPITRE V.

Formules récursives non linéaires.

§ 20. Développements divers.

Remarquons que les $\varphi_n(x)$ et les $\chi_n(x)$ forment des suites parfaites, les expressions explicites de ces deux fonctions donnent les développements suivants:

$$(1) \quad \frac{1}{2} \varphi_m(x) = \chi_m(x) + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \chi_{m-2s}(x),$$

$$(2) \quad \chi_m(x) = \sum_{s=0}^{\leq \frac{m}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+1}} \varphi_{m-2s}(x).$$

¹⁾ Beiträge etc. p. 31–34.

Supposons ensuite $x = 0$ et $m = 2n - 1$ respectivement $m = 2n$, nous aurons :

$$(3) \quad T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{2s} B_s T_{n-s},$$

tandis que les hypothèses $x = -\frac{1}{2}$, $m = 2n$ donnent ces deux autres formules :

$$(4) \quad E_n = 2(2^{2n} - 1)B_n + \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{2s} B_s E_{n-s},$$

$$(5) \quad (2n+1)E_n - T_{n+1} = \sum_{s=0}^{s=n-1} \binom{2n+1}{2s+1} (2^{2n-2s} - 2) B_{n-s} T_{s+1}.$$

Appliquons ensuite les identités :

$$2^{m-1} \varphi_m \left(\frac{x}{2} \right) + 2^{m-1} \varphi_m \left(\frac{x-1}{2} \right) = \varphi_m(x),$$

$$2^{m-1} \varphi_m \left(\frac{x}{2} \right) - 2^{m-1} \varphi_m \left(\frac{x-1}{2} \right) = \chi_{m-1}(x),$$

nous aurons de même :

$$(6) \quad 2^{m-1} \varphi_m \left(\frac{x}{2} \right) = \chi_m(x) + \frac{1}{2} \chi_{m-1}(x) + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \chi_{m-2s}(x),$$

$$(7) \quad 2^{m-1} \varphi_m \left(\frac{x}{2} \right) = \frac{1}{2} \varphi_m(x) + \sum_{s=1}^{\leq \frac{m+1}{2}} \frac{(-1)^{s-1} T_s}{(2s-1)! 2^{2s}} \varphi_{m-2s+1}(x),$$

tandis que les formules :

$$2^{m+1} \chi_m \left(\frac{x}{2} \right) = \frac{x^m}{m!} + \sum_{s=1}^{\leq \frac{m+1}{2}} \frac{(-1)^{s-1} T_s}{(2s-1)!} \cdot \frac{x^{m-2s+1}}{(m-2s+1)!},$$

$$2^{m+1} \chi_m \left(\frac{x-1}{2} \right) = \frac{x^m}{m!} + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^s E_s}{(2s)!} \cdot \frac{x^{m-2s}}{(m-2s)!},$$

tirées directement des formules (18) du paragraphe 3 et (10) du paragraphe 5, donnent ces deux autres développements :

$$(8) \quad 2^{m+1} \chi_m \left(\frac{x}{2} \right) = \sum_{s=0}^{\leq \frac{m}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)!} \varphi_{m-2s}(x) + \sum_{s=0}^{\leq \frac{m-1}{2}} \frac{(-1)^s E_{s+1}}{(2s+2)!} \varphi_{m-2s-1}(x),$$

$$(9) \quad 2^{m+1} \chi_m \left(\frac{x}{2} \right) = 2 \chi_m(x) + \sum_{s=0}^{\leq \frac{m-1}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)!} \chi_{m-2s-1}(x) - \sum_{s=0}^{\leq \frac{m-2}{2}} \frac{(-1)^s E_{s+1}}{(2s+2)!} \chi_{m-2s-2}(x).$$

On voit que les développements (6) et (7) nous conduiront aux formules (3), (4) et (5). Posons, au contraire, dans (8), $x = 0$ et $m = 2n$ respectivement $m = 2n-1$, nous aurons :

$$(10) \quad T_{n+1} - \left(n + \frac{1}{2}\right) E_n = \sum_{s=1}^{s=n} \binom{2n+1}{2s-1} T_s B_{n-s+1},$$

$$(11) \quad E_n - n T_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} E_s B_{n-s}.$$

Quant à la formule (9), nous aurons pour les mêmes valeurs de x et m

$$(12) \quad (2^{2n-1} - 2) T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{2s} E_s T_{n-s},$$

$$(13) \quad 2^{2n-1} E_n = \sum_{s=0}^{s=n-1} \binom{2n}{2s+1} 2^{2s} T_{s+1} T_{n-s}.$$

Revenons maintenant aux formules (18) du paragraphe 2 et (20) du paragraphe 3, savoir

$$(14) \quad \left(x + \frac{1}{2}\right) \varphi_{m-1}(x) = m \varphi_m(x) + \sum_{s=1}^{\leq \frac{m}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \varphi_{m-2s}(x),$$

$$(15) \quad \left(x + \frac{1}{2}\right) \chi_m(x) = (m+1) \chi_{m+1}(x) + \sum_{s=0}^{\leq \frac{m-1}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+2}} \chi_{m-2s-1}(x);$$

posons, dans (14), $m = 2n$ et $x = 0$, $x = -\frac{1}{2}$, nous aurons respectivement :

$$(16) \quad (2n+1) B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} B_s B_{n-s},$$

$$(17) \quad ((2n-1) 2^{2n-4n}) B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2n} - 2^{2s+1}) B_s B_{n-s},$$

d'où, en multipliant par 2^{2n} la formule (16), puis soustrayant de (17) l'équation ainsi obtenue :

$$(18) \quad (2^{2n} + 2n) B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{2s} B_s B_{n-s},$$

tandis que nous aurons en additionnant les formules susdites, puis introduisant les T_n :

$$(19) \quad 2n T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{4s} B_s T_{n-s}.$$

La formule (16) appartient à EULER¹⁾.

Posons ensuite, dans (14), $x = -\frac{1}{4}$, $m = 2n+1$, respectivement $m = 2n$, nous aurons :

$$(20) \quad (2n+1)E_n - (2^{2n}+2)(2^{2n}-1)B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{4s} B_s E_{n-s},$$

$$(21) \quad 2n(2n-1)E_{n-1} + (2n(2^{2n}-2) - 2^{4n})B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2n-2s}-2) 2^{4s} B_s B_{n-s};$$

soustrayons les formules (4) et (20), puis, en introduisant les T_n , nous aurons la formule de SCHERK²⁾:

$$(22) \quad E_n - T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s-1} T_s E_{n-s},$$

tandis que nous retrouvons la formule (5) en soustrayant (17) de (21).

Quant à la formule (15), les hypothèses $m = 2n$, $x = 0$ respectivement $x = -\frac{1}{2}$ donnent :

$$(23) \quad T_{n+1} = \sum_{s=0}^{s=n-1} \binom{2n}{2s+1} T_{s+1} T_{n-s}$$

respectivement la formule (22) de SCHERK; la formule (23) est due à EULER³⁾.

§ 21. Formules d'addition.

Il est très facile de généraliser beaucoup les formules (6) et (7), (14) et (15) du paragraphe 20.

En effet, écrivons comme suit les formules en question:

$$(1) \quad (x+1)\varphi_{n-1}(x) - (n-1)\varphi_n(x) = \sum_{s=0}^{s=n} \varphi_s(0)\varphi_{n-s}(x),$$

$$(2) \quad (x+1)\chi_n(x) - (n+1)\chi_{n+1}(x) = \sum_{s=0}^{s=n} \chi_s(0)\chi_{n-s}(x),$$

$$(3) \quad 2^{n-1}\varphi_n\left(\frac{x}{2}\right) = \sum_{s=0}^{s=n} \chi_s(0)\varphi_{n-s}(x) = \sum_{s=0}^{s=n} \varphi_s(0)\chi_{n-s}(x),$$

¹⁾ Institutiones calculi differentialis, p. 416; Saint-Petersbourg 1755. Opuscula analytica, t. II, p. 266; Saint-Petersbourg 1785.

²⁾ Mathematische Abhandlungen; Berlin 1825.

³⁾ Institutiones calculi differentialis, p. 495.

je dis, que nous aurons les formules générales :

$$(4) \quad (x + y + 1) \varphi_{n-1}(x + y) - (n-1) \varphi_n(x + y) = \sum_{s=0}^{s=n} \varphi_{n-s}(x) \varphi_s(y),$$

$$(5) \quad (x + y + 1) \chi_n(x + y) - (n+1) \chi_{n+1}(x + y) = \sum_{s=0}^{s=n} \chi_{n-s}(x) \chi_s(y),$$

$$(6) \quad 2^{n-1} \varphi_n\left(\frac{x+y}{2}\right) = \sum_{s=0}^{s=n} \varphi_{n-s}(x) \chi_s(y).$$

En effet, remarquons que les deux membres de chacune des trois formules générales forment des suites harmoniques qui deviennent égales deux à deux pour $y=0$; le théorème II du paragraphe 1 nous conduira immédiatement au but.

Posons maintenant dans nos trois formules générales $y = -x-1$, puis remplaçons n par $2n$, les équations fonctionnelles (1) du paragraphe 5 donnent immédiatement les formules remarquables

$$(7) \quad \frac{(-1)^n (2n-1) B_n}{(2n)!} = \sum_{s=0}^{s=2n} (-1)^s \varphi_s(x) \varphi_{2n-s}(x),$$

$$(8) \quad \frac{(-1)^n T_{n+1}}{(2n)! 2^{2n+2}} = \sum_{s=0}^{s=2n} (-1)^s \chi_s(x) \chi_{2n-s}(x),$$

$$(9) \quad \frac{(-1)^n (2^{2n-1}-1) B_n}{(2n)!} = \sum_{s=0}^{s=2n} (-1)^s \chi_s(x) \varphi_{2n-s}(x).$$

Remarquons que les hypothèses $x=0$ ou $y=0$ sont étudiées dans le paragraphe précédent, il nous reste encore à considérer les deux cas suivants :

1° Posons dans (7), (8) et (9) $x=y=-\frac{1}{2}$, nous aurons respectivement :

$$(10) \quad ((2n-3) 2^{2n} + 4) B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2s}-2) (2^{2n-2s}-2) B_s B_{n-s},$$

$$(11) \quad T_{n+1} - 2 E_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} E_s E_{n-s},$$

$$(12) \quad (2^{2n}-1) (2^{2n}-2) B_n - E_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2s}-2) B_s E_{n-s};$$

soustrayons, puis additionnons les formules (12) et (4) du paragraphe 20, nous aurons la formule (11) du paragraphe 20 respectivement

$$(13) \quad (2^{2n-1}-2) T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s-1} 2^{2n-2s} T_s E_{n-s}.$$

2° Posons, dans (1), $x = -\frac{1}{2}$, $y = -\frac{1}{4}$, puis remplaçons n par $2n$ respectivement par $2n+1$, nous aurons:

$$(14) \quad (2n-1)E_n - (2^{2n}-1)(2^{2n}-2)B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2s}-2) 2^{2s} B_s E_{n-s},$$

$$(15) \quad 2n(2n-1)E_{n-1} - (2n+2^{2n})(2^{2n}-2)B_n = \sum_{s=1}^{s=n-1} \binom{2n}{2s} (2^{2n-2s}-2)(2^{2s}-2) 2^{2s} B_s B_{n-s},$$

d'où en soustrayant (12) de (14)

$$(16) \quad 2^{2n} E_n - (2^{2n+1}-4) T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s-1} (2^{2s}-2) 2^{2n-2s} T_s E_{n-s},$$

tandis que nous aurons en soustrayant (10) de (15):

$$(17) \quad (2n-1) 2^{2n} E_{n-1} - (2^{2n}+4n-4) T_n = \sum_{s=1}^{s=n-1} \binom{2n-1}{2s-1} (2^{2s}-2)(2^{2n-2s}-2) 2^{2n-2s} T_s B_{n-s}.$$

§ 22. Sur les produits $\varphi_n(x) \varphi_p(x)$, $\chi_n(x) \chi_p(x)$ et $\varphi_n(x) \chi_p(x)$.

Il est très facile de généraliser à un autre point de vue les formules (1) et (2) du paragraphe 21.

A cet effet, posons pour abréger:

$$f_n(x) = \frac{x^n}{n!} + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s x^{n-2s}}{(2s)! (n-2s)!},$$

$$g_n(x) = \sum_{s=0}^{\leq \frac{n-1}{2}} \frac{(-1)^s T_{s+1} x^{n-2s-1}}{(2s+1)! (n-2s-1)! 2^{2s+2}},$$

nous aurons pour les $\varphi_n(x)$:

$$(1) \quad \varphi_n(x) = f_n(x) - \frac{x^{n-1}}{(n-1)! 2}, \quad \varphi_n(x-1) = f_n(x) - \frac{x^{n-1}}{(n-1)! 2}$$

et pour les $\chi_n(x)$:

$$(2) \quad \chi_n(x) = \frac{x^n}{n! 2} - g_n(x), \quad \chi_n(x-1) = \frac{x^n}{n! 2} - g_n(x),$$

ce qui donnera immédiatement les trois équations aux différences finies

$$(3) \quad \varphi_n(x) \varphi_p(x) - \varphi_n(x-1) \varphi_p(x-1) = \frac{x^{p-1} f_n(x)}{(p-1)!} - \frac{x^{n-1} f_p(x)}{(n-1)!},$$

$$(4) \quad \chi_n(x) \chi_p(x) - \chi_n(x-1) \chi_p(x-1) = \frac{x^n g_p(x)}{n!} + \frac{x^p g_n(x)}{p!},$$

$$(6) \quad \varphi_n(x) \chi_p(x) - \varphi_n(x-1) \chi_p(x-1) = \frac{x^p f_n(x)}{p!} + \frac{x^{n-1} g_p(x)}{(n-1)!}.$$

Cela posé, nous avons à étudier les trois développements suivants:

1° L'équation (3) donnera un développement de la forme

$$(7) \quad \left\{ \begin{aligned} \varphi_n(x) \varphi_p(x) &= K_{n,p} + \binom{n+p}{n} \varphi_{n+p}(x) + \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \binom{n+p-2s-1}{p-1} \varphi_{n+p-2s}(x) + \\ &+ \sum_{s=1}^{\leq \frac{p}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \binom{n+p-2s-1}{n-1} \varphi_{n+p-2s}(x), \end{aligned} \right.$$

où la constante $K_{n,p}$ se détermine comme suit:

Différentions par rapport à x la formule (7), puis appliquons l'identité

$$D_x(\varphi_n(x) \varphi_p(x)) = \varphi_{n-1}(x) \varphi_p(x) + \varphi_n(x) \varphi_{p-1}(x),$$

nous aurons en développement, d'après la formule (7), les deux fonctions qui figurent au second membre

$$K_{n-1,p} + K_{n,p-1} = 0,$$

d'où immédiatement:

$$K_{n,p} = -K_{n+1,p-1} = (-1)^{p-1} K_{n+p-1,1}.$$

Or, nous aurons en posant dans (7) $p=1$, ce qui nous conduira à la formule (1) du paragraphe 21:

$$K_{2r-1,1} = \frac{(-1)^{r+1} B_r}{(2r)!}, \quad K_{2r,1} = 0,$$

de sorte que nous aurons pour $K_{n,p}$ la détermination suivante:

$$(8) \quad \begin{cases} K_{n,p} = 0, & n+p = 2k+1, \\ K_{n,p} = \frac{(-1)^{n+k} B_k}{(2k)!}, & n+p = 2k, \end{cases}$$

où k désigne un positif entier.

Posons dans (7) particulièrement $p=n$, ce qui donnera $k=n$, nous aurons:

$$(9) \quad (\varphi_n(x))^2 = \frac{B_n}{(2n)!} + \binom{2n}{n} \varphi_{2n}(x) + 2 \cdot \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \binom{2n-2s-1}{n-1} \varphi_{2n-2s}(x),$$

tandis que l'hypothèse $p=n+1$ donnera:

$$(10) \quad \varphi_n(x) \varphi_{n+1}(x) = \binom{2n+1}{n} \varphi_{2n+1}(x) - \sum_{s=1}^{\leq \frac{n+1}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \binom{2n-2s+1}{n} \varphi_{2n-2s+1}(x).$$

Remplaçons, dans (7), x par le positif entier q , nous retrouvons un cas particulier d'une formule plus générale de M. E. LAMPE¹⁾.

2° L'équation (4) donnera de même:

²⁾ Journal de Crelle, t. 84, p. 270—272; 1878.

$$(11) \quad \left\{ \begin{aligned} \chi_n(x) \chi_p(x) &= K_{n,p} + \sum_{s=0}^{\leq \frac{p-1}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+2}} \binom{n+p-2s-1}{n} \varphi_{n+p-2s}(x) + \\ &+ \sum_{s=0}^{\leq \frac{n-1}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+2}} \binom{n+p-2s-1}{p} \varphi_{n+p-2s}(x), \end{aligned} \right.$$

où la constante $K_{n,p}$ peut être déterminée par la méthode appliquée dans le cas précédent; car en posant dans (8) $p=0$, on retrouve la formule (2) du paragraphe 20.

De cette manière nous trouvons ici:

$$(12) \quad \begin{cases} K_{n,p} = 0, & n+p = 2k-1, \\ K_{n,p} = \frac{(-1)^{n+k} T_{k+1}}{(2k+1)! 2^{2k+2}}, & n+p = 2k, \end{cases}$$

où k désigne par conséquent un positif entier.

Soit particulièrement $p=n$, $p=n+1$, nous aurons respectivement:

$$(13) \quad (\chi_n(x))^2 = \frac{T_{n+1}}{(2n+1)! 2^{2n+2}} + 2 \cdot \sum_{s=0}^{\leq \frac{n-1}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+2}} \binom{2n-2s-1}{n} \varphi_{2n-2s}(x),$$

$$(14) \quad \chi_n(x) \chi_{n+1}(x) = \sum_{s=0}^{\leq \frac{n}{2}} \frac{(-1)^s T_{s+1}}{(2s+1)! 2^{2s+2}} \binom{2n-2s+1}{n+1} \varphi_{2n-2s+1}(x).$$

Quant à la formule (6), nous aurons:

$$(15) \quad \left\{ \begin{aligned} \varphi_n(x) \chi_p(x) &= \binom{n+p}{p} \chi_{n+p}(x) - \sum_{s=1}^{\leq \frac{n}{2}} \frac{(-1)^{s-1} B_s}{(2s)!} \binom{n+p-2s}{p} \chi_{n+p-2s}(x) + \\ &+ \sum_{s=1}^{\leq \frac{p+1}{2}} \frac{(-1)^{s-1} T_s}{(2s-1)! 2^{2s}} \binom{n+p-2s}{n-1} \chi_{n+p-2s}(x), \end{aligned} \right.$$

d'où en posant particulièrement $n=p+1$, puis remplaçant p par n :

$$(16) \quad \varphi_{n+1}(x) \chi_n(x) = \binom{2n+1}{n} \chi_{2n+1}(x) - \sum_{s=1}^{\leq \frac{n+1}{2}} \frac{(-1)^{s-1} 2^{2s} B_s}{(2s)!} \binom{2n-2s+1}{n} \chi_{2n-2s+1}(x);$$

car nous aurons évidemment:

$$\frac{T_s}{2^{2s}} = \frac{B_s}{2s} = \frac{2^{2s} B_s}{2s}.$$

Il est évident que les formules précédentes nous permettent de déduire un très grand nombre de formules récursives non linéaires, formules qui sont à con-

sidérer comme des inversions des formules contenues dans les deux paragraphes précédents.

Nous nous bornerons à citer les plus simples des formules en question :

1° Posons, dans (9), $x = 0$, puis remplaçons n par $2n$ respectivement par $2n+1$, nous trouvons les formules 100 et 101 de la Table.

2° La formule (13) donnera de même pour $x = 0$ les formules 102 et 103 de la Table.

3° Posons, dans (10), $x = -\frac{1}{4}$, puis remplaçons n par $2n$ respectivement par $2n+1$, nous trouvons les formules 104 et 105 de la Table.

4° En dernier lieu, la formule (16) donnera pour $x = 0$ les formules 106 et 107 de la Table.

On voit que les formules 104 et 105 sont à considérer comme des formules récurrentes incomplètes pour les E_n .

CHAPITRE VI.

Théorèmes sur des nombres entiers.

§ 23. Applications des factorielles.

Je me suis proposé de développer, dans ce chapitre final, quelques résultats concernant la théorie des nombres, résultats qui se présentent comme des conséquences immédiates de nos recherches précédentes. Cependant je me réserve de revenir, à une autre occasion, à mes recherches ultérieures de ce sujet.

Nous commençons par une étude de la factorielle du rang n :

$$(1) \quad \omega_n(x) = x(x+1) \dots (x+n-1);$$

posons

$$(2) \quad \omega_n(x) = C_n^0 x^n + C_n^1 x^{n-1} + \dots + C_n^{n-1} x,$$

les coefficients C_n^p sont les coefficients de la factorielle du rang n ; nous aurons particulièrement :

$$(3) \quad C_n^0 = 1, \quad C_n^{n-1} = (n-1)!$$

Posons plus généralement :

$$(4) \quad \omega_n(x+a) = C_n^0(a) x^n + C_n^1(a) x^{n-1} + \dots + C_n^{n-1}(a) x + C_n^n(a),$$

nous verrons que $C_n^p(a)$ est, pour $1 \leq p \leq n-1$, la somme de tous les produits formés de p facteurs différents pris parmi les n expressions

$$(5) \quad a, a+1, a+2, \dots, a+n-1,$$

tandis que nous aurons particulièrement:

$$(6) \quad C_n^0(a) = 1, \quad C_n^n(a) = \omega_n(a).$$

Cela posé, l'identité évidente

$$\omega_n(x+a) - \omega_n(x+a-1) = n \omega_{n-1}(x+a)$$

donnera, en vertu du théorème I du paragraphe 2, le développement suivant:

$$(7) \quad \omega_n(x+a) = f_n(a) + n \cdot \sum_{s=0}^{n-1} (n-s-1)! C_{n-1}^s(a) \varphi_{n-s}(x),$$

où $f_n(a)$ est un polynome entier de a .

Différentions maintenant $n-p$ fois par rapport à x la formule (7), puis posons $x=0$, nous aurons la formule récursive pour les B_n :

$$(8) \quad \frac{n-p}{n} C_n^p(a) = C_{n-1}^p(a) + \frac{n-p}{2} C_{n-1}^{p-1}(a) + \sum_{s=1}^{\leq \frac{p}{2}} (-1)^{s-1} \binom{n-p+2s-1}{2s} B_s C_{n-1}^{p-2s}(a),$$

d'où particulièrement, en posant $a=0$, $p=n-1$, la formule de SCHLÖMILCH¹⁾

$$(9) \quad \frac{(n-2)(n-2)!}{2n} = \sum_{s=1}^{\leq \frac{n-1}{2}} (-1)^{s-1} B_s C_{n-1}^{n-2s-1},$$

formule qui est retrouvée par M. A. RADICKE²⁾.

Nous obtenons d'autres formules récursives de ce genre en appliquant le polynome du degré n par rapport à x :

$$(10) \quad \mathcal{Q}_n(x) = \left(x + \frac{1}{n+1}\right) \left(x + \frac{2}{n+1}\right) \cdots \left(x + \frac{n}{n+1}\right), \quad n \geq 1,$$

qui satisfait évidemment à l'équation fonctionnelle:

$$(-1)^n \mathcal{Q}_n(-x-1) = \mathcal{Q}_n(x),$$

de sorte que nous aurons, en vertu du théorème I du paragraphe 8, ces deux développements:

$$(11) \quad \frac{1}{2} \mathcal{Q}_n(x) = K_n \cdot \sum_{s=0}^{\leq \frac{n-1}{2}} \frac{(n-2s-1)!}{(n+1)^{2s+1}} C_{n+1}^{2s+1} \varphi_{n-2s}(x),$$

$$(12) \quad \frac{1}{2} \mathcal{Q}_n(x) = \sum_{s=0}^{\leq \frac{n}{2}} \frac{(n-2s)!}{(n+1)^{2s}} C_{n+1}^{2s} \varphi_{n-2s}(x),$$

¹⁾ Archiv de Grunert, t. 9, p. 334; 1847.

²⁾ Die Recursionsformeln für die Berechnung der Bernoullischen und Eulerschen Zahlen, p. 15; Halle a. S. 1880.

où la constante K_n qui figure au second membre de (11) satisfait à la condition

$$K_{2n+1} = 0.$$

Appliquons maintenant les formules (10) et (11) du paragraphe 8, nous aurons, en vertu de (11) et (12) ces deux formules récursives:

$$(13) \quad (n+1)C_{n+1}^{2p} = \frac{2}{n-2p} \left(C_{n+1}^{2p+1} + \sum_{s=1}^{s=p} (-1)^{s-1} \binom{n-2p+2s-1}{2s} (n+1)^{2s} B_s C_{n+1}^{2p-2s+1} \right),$$

$$(14) \quad 2^{2p} C_{n+1}^{2p+1} = (n+1) \cdot \sum_{s=0}^{s=p} (-1)^s \binom{n-2p+2s}{2s+1} (n+1)^{2s} 2^{2p-2s} T_{s+1} C_{n+1}^{2p-2s},$$

où il faut admettre, dans (13), $p \geq 1$.

Posons encore dans (12) $x = -\frac{1}{2}$, puis remplaçons n par $2n$, nous aurons, après une légère modification, la formule curieuse:

$$(15) \quad [1 \cdot 3 \cdot 5 \dots (2n-1)]^2 = (-1)^n (2n)! 2^{2n} + \sum_{s=0}^{s=n-1} (-1)^s (2n+1)^{2n-2s} 2^{2s} E_n {}_s C_{2n+1}^{2s}.$$

§ 24. Généralisation d'un théorème de Lipschitz.

Démontrons maintenant comment nos formules récursives nous conduisent immédiatement à des résultats intéressants concernant la nature algébrique des nombres de BERNOULLI.

A cet effet, prenons pour point de départ la formule de G.-F. MEYER, savoir la formule 8 de la Table:

$$(1) \quad (2n+1)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2s} B_{n-s} = (-1)^{n-1} \left(2^{2n-2} - \binom{2n-1}{2} \right),$$

je dis que l'expression

$$(2) \quad a_n = 3 \cdot 5 \cdot 7 \dots (2n+1) \cdot 2 B_n$$

est, pour tous les n , un nombre entier impair.

En effet, la formule (1) donnera, en vertu de (2), une formule résursive de la forme

$$a_n + 4 \cdot \sum_{s=1}^{s=n-1} A_s a_{n-s} = 2A' + 1,$$

où A' et les A_s sont des nombres entiers.

Remarquons ensuite que nous aurons $a_1 = 1$; la conclusion ordinaire de n à $n+1$ nous conduira immédiatement au but.

Cela posé, nous aurons une expression de la forme

$$(3) \quad B_n = \frac{a_n}{2b_n},$$

où a_n et b_n sont des positifs entiers impairs, premiers entre eux.

De plus, nous aurons, en vertu de (2):

$$(4) \quad 3 \cdot 5 \cdot 7 \dots (2n+1) \equiv 0 \pmod{b_n}.$$

Désignons ensuite par $2G_n$ le dénominateur général des n premiers nombres de BERNOULLI, savoir

$$B_1, B_2, B_3, \dots, B_n,$$

G_n est un nombre impair qui n'est jamais divisible par un nombre premier plus grand que $2n+1$.

Or, il est très intéressant que la formule de SCHLÖMILCH, savoir la formule (9) du paragraphe 23, nous détermine immédiatement la valeur exacte de G_n .

En effet, remplaçons dans la formule susdite n par $2n+2$, nous aurons, après une légère modification:

$$(5) \quad \frac{(2n)! n}{2n+1} = (-1)^{n-1} B_n + \sum_{s=1}^{s=n-1} (-1)^{s-1} B_s C_{2n+1}^{2n-2s}.$$

Soit maintenant, dans (5), $2n+1$ un nombre composé: le premier membre de cette formule est un nombre entier, de sorte que nous aurons dans ce cas:

$$G_n = G_{n-1};$$

soit, au contraire, $2n+1$ un nombre premier, nous aurons:

$$G_n = (2n+1) G_{n-1},$$

d'où la proposition suivante:

I. Le nombre G_n est précisément le produit de tous les nombres premiers qui ne dépassent pas $2n+1$, de sorte que le nombre impair b_n qui figure dans le dénominateur de B_n ne peut jamais être divisible par un nombre carré plus grand que l'unité.

Cela posé, je dis que la formule (6) du paragraphe 13, savoir

$$(6) \quad \left\{ \begin{aligned} p(p^{2n-1}) B_n + \sum_{r=1}^{r=n-1} (-1)^r \binom{2n}{2r} s_{2r}(p-1) (p^{2n-2r}-1) B_{n-r} = \\ = (-1)^n (n(p-1) s_{2n-1}(p-1) - 2n s_{2n}(p-1)), \end{aligned} \right.$$

nous conduira au théorème suivant:

II. Désignons par n et p deux positifs entiers quelconques, l'expression

$$(7) \quad c_n(p) = \frac{p^{n+1} (p^{2n}-1) B_n}{2n}$$

est toujours un nombre entier.

Il est évident qu'il suffit de considérer le cas où $p \neq 1$. Remarquons ensuite que

$$c_1(p) = \frac{(p-1)p^2(p+1)}{12}$$

est un nombre entier, puis supposons que les expressions

$$c_2(p), c_3(p), \dots, c_{n-1}(p)$$

aient la même propriété: nous aurons, en multipliant par p^n les deux membres de (6) et en appliquant l'identité évidente

$$(2n-2r) \binom{2n}{2r} = 2n \binom{2n-1}{2r},$$

pour les $c_n(p)$ la formule récursive

$$\begin{aligned} c_n(p) + \sum_{r=1}^{r=n-1} (-1)^r \binom{2n-1}{2r} s_{2r-1}(p-1) p^{r-1} c_{n-r}(p) = \\ = (-1)^n \binom{p^n(p-1)}{2} s_{2n-1}(p-1) - p^n s_{2n}(p-1), \end{aligned}$$

ce qui nous conduira immédiatement au but.

On sait que LIPSCHITZ¹⁾ a démontré que l'expression

$$\frac{p^{2n}(p^{2n}-1)B_n}{2n}$$

est toujours un nombre entier; ce théorème de LIPSCHITZ appartient à ceux sur lesquels M. P. BACHMANN²⁾ remarque qu'ils „bisher auf rein arithmetische Weise nicht gewonnen werden konnten.“

Soit p de la forme $6m \pm 1$, nous verrons par le même procédé que l'expression

$$(8) \quad \frac{p^{n-1}(p^{2n}-1)B_n}{2n}$$

est un nombre entier.

Remarquons que le dénominateur de B_n , dans la formule (3), est le produit de certains nombres premiers différents entre eux, le théorème II montrera que l'expression

$$(9) \quad d_n(p) = p(p^{2n}-1)B_n$$

est toujours un nombre entier, pourvu que p le soit.

Cela posé, on voit que le nombre impair b_n qui figure dans le dénominateur du second membre de (3) n'est divisible que par des nombres premiers de la forme $2\lambda + 1$, où λ est diviseur de n . Le théorème de v. STAUDT et de THOMAS CLAUSEN montrera que b_n est précisément le produit des nombres premiers susdits.

Appliquons maintenant la formule d'EULER³⁾

$$(10) \quad T_n = \frac{2^{2n}(2^{2n}-1)B_n}{2n},$$

nous aurons, en vertu de (9), la proposition suivante due à WORPITZKY⁴⁾:

¹⁾ Journal de Crelle, t. 96, p. 3; 1884.

²⁾ Niedere Zahlentheorie, t. II, p. 31; Leipsic 1910.

³⁾ Opuscula analytica, t. II, p. 273; Saint-Petersbourg 1785.

⁴⁾ Journal de Crelle, t. 94, p. 231; 1883.

III. Soit l'indice n de la forme $n = 2^p(2q+1)$, le n -ième coefficient des tangentes T_n est de la forme

$$(11) \quad T_n = 2^{2n-p-2}(2Q+1).$$

EULER¹⁾ indique que le nombre (9) pour $p=2$, savoir $d(2)$, est un nombre entier. Cependant, j'ignore s'il a connu le théorème III.

Soit enfin, dans (5), $2n+1$ un nombre premier, nous aurons, en multipliant par $2G_n$, la formule en question, une égalité de la forme

$$(2n)! 2n G_{n-1} = (-1)^{n-1} a_n + (2n+1)A,$$

où A est un nombre entier; c'est-à-dire que le théorème de WILSON donnera la proposition suivante:

IV. Soit $2n+1$ un nombre premier, nous aurons toujours

$$(12) \quad a_n \equiv (-1)^{n-1} G_{n-1} \pmod{2n+1}.$$

§ 25. Théorèmes sur les nombres $s_n(p)$, $\sigma_n(p)$ et C_n^p .

Les résultats du paragraphe 13 nous conduiront en outre à une suite de résultats intéressants concernant les sommes $s_n(p)$ et $\sigma_n(p)$.

En effet, appliquons les formules (4) et (5) du paragraphe susdit, savoir

$$(p^{2n+1}-p)B_n + \sum_{r=1}^{r=n-1} (-1)^r \binom{2n}{2r} p^{2n-2r} s_{2r}(p-1) B_{n-r} = (-1)^n (s_{2n}(p-1) - np s_{2n-1}(p-1)),$$

$$\sum_{r=0}^{r=n-1} (-1)^r \binom{2n+1}{2r+1} p^{2n-2r} s_{2r+1}(p-1) B_{n-r} = (-1)^n \left(s_{2n+1}(p-1) - \left(n + \frac{1}{2}\right) p s_{2n}(p-1) \right),$$

puis appliquons les formules élémentaires

$$s_1(p-1) = \frac{(p-1)p}{2}, \quad s_2(p-1) = \frac{(p-1)p(2p-1)}{6},$$

la conclusion ordinaire de m à $m+1$ donnera immédiatement le théorème suivant:

I. Soit n et p deux positifs entiers quelconques, et soit $2G_n$ le dénominateur général des n premiers nombres de Bernoulli, nous aurons toujours:

$$(1) \quad 2G_n s_{2n}(p-1) \equiv 0 \pmod{p},$$

$$(2) \quad 2G_n s_{2n+1}(p-1) \equiv 0 \pmod{p^2}.$$

Supposons que p soit un nombre impair non divisible par les nombres premiers égaux à $2n+1$ au plus, le facteur $2G_n$ qui figure aux premiers membres des congruences (1) et (2) peut être supprimé.

⁵⁾ Institutiones calculi differentialis, p. 495—496; Saint-Petersbourg 1755.

Ces remarques faites, nous aurons la proposition suivante:

II. Soit n un positif entier quelconque, et soit $p > 2n+1$ un nombre premier, nous aurons toujours:

$$(3) \quad s_{2n}(p-1) \equiv 0 \pmod{p},$$

$$(4) \quad s_{2n+1}(p-1) \equiv 0 \pmod{p^2}.$$

Quant aux sommes $\sigma_m(p-1)$, la formule d'EULER du paragraphe 4 donnera immédiatement le théorème analogue à I:

III. Soient n et p des positifs entiers quelconques, nous aurons toujours:

$$(5) \quad 2^{2n-1} \sigma_{2n}(p-1) \equiv 0 \pmod{p},$$

$$(6) \quad 2^{2n+1} \sigma_{2n+1}(p-1) \equiv (-1)^{n+1} T_{n+1} \pmod{p}.$$

On voit du reste que la congruence (5) est évidente pour une valeur impaire de p .

Discutons maintenant les résultats obtenus dans le paragraphe 23, la formule (8) donnera immédiatement:

IV. Soit n un nombre premier impair, et soit p un entier tel que $1 \leq p \leq n-2$, tous les coefficients du polynôme $C_n^p(a)$ sont divisibles par n .

Posons $a = 0$, nous aurons, en vertu de IV, le théorème de LAGRANGE¹⁾:

V. Soit n un nombre premier impair, et soit p un entier tel que $1 \leq p \leq n-2$, nous aurons toujours:

$$(7) \quad C_n^p \equiv 0 \pmod{p}.$$

C'est un fait bien connu que LAGRANGE, en appliquant ses congruences (7), a démontré en même temps les théorèmes de FERMAT et de WILSON.

Appliquons le théorème de LAGRANGE, nous aurons, en vertu de la formule (14) du paragraphe 23, l'autre théorème:

VI. Soit $2n+1 > 3$ un nombre premier, et soit p un nombre entier tel que $1 \leq p \leq n-1$, nous aurons toujours:

$$(8) \quad C_{2n+1}^{2p+1} \equiv 0 \pmod{(2p+1)^2}.$$

Supposons $p = n-1$, le cas particulier correspondant de la congruence (8) est bien connu²⁾. Dans une Note datant de ma jeunesse, j'ai démontré, il y a trente ans à peu près, la congruence générale (8)³⁾; cependant je ne veux pas prétendre que la formule générale (8) m'appartienne⁴⁾.

Comme une autre conséquence de la congruence de LAGRANGE nous aurons, en vertu de la formule récursive (15) du paragraphe 23, la proposition suivante:

VII. Soit $2n+1 > 3$ un nombre premier, nous aurons toujours:

$$(9) \quad 1 \cdot 3 \cdot 5 \dots (2n-1) \equiv (-1)^n n! 2^{3n} \pmod{(2n+1)^3}.$$

¹⁾ Mémoires de l'Académie de Berlin t. 2 (1771), p. 125—137; 1773.

²⁾ E. RIEKE: Zeitschrift für Mathematik und Physik, Bd. 34, p. 190—191; 1889. C. LEUDESdorf: Proceedings of the London Math. Society, t. 20, p. 199—212; 1889. R. E. ALLARDICE: Edinburgh math. Soc. Proceedings, t. 8, p. 16—19; 1890.

³⁾ Voir ma Note insérée dans: Nyt Tidsskrift for Matematik t. 4, p. 1—10; 1893.

⁴⁾ Comparer ma Note dans: Nyt Tidsskrift for Matematik, t. 21, p. 8—10; 1910.

TABLE

des

simples formules récursives.

Il faut remarquer que cette Table ne contient que des formules récursives d'une forme simple. La plupart de ces formules sont obtenues en donnant, dans des formules beaucoup plus générales, à un paramètre des valeurs particulières.

I. Formules récursives linéaires.

1. Formules contenant les B_n .

- *1.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} B_{n-s} = (-1)^{n-1} \binom{n-1}{2}.$$
- *2.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+2}{2s+2} B_{n-s} = (-1)^{n-1} n.$$
- *3.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} B_{n-s} = \frac{(-1)^{n-1}}{2}.$$
- *4.
$$(2n+1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} B_{n-s} = \frac{(-1)^{n-1}}{2}.$$
- *5.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 2^{2s} B_{n-s} = (-1)^{n-1} \left((2n-3) 2^{2n-2} + \frac{2n+1}{2} \right).$$
- *6.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+2}{2s+2} 2^{2s} B_{n-s} = (-1)^{n-1} \left((n-1) 2^{2n-1} + \frac{n+1}{2} \right).$$
- *7.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} 2^{2s} B_{n-s} = (-1)^{n-1} \left(2^{2n-2} - \frac{n}{2} \right).$$
- *8.
$$(2n+1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2s} B_{n-s} = (-1)^{n-1} \left(2^{2n-2} - \frac{2n-1}{2} \right).$$
- *9.
$$(2^{2n+1} - 2) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1} (2n-1).$$
- *10.
$$\sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 2^{2n-2s} B_{n-s} = (-1)^{n-1} 2n.$$

$$*11. ((2n-1)2^{2n}+2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2n-2s} B_{n-s} = (-1)^{n-1}.$$

$$*12. (2^{2n+1}-2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1}.$$

$$13. (2^{2n+1}-2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1} ((2n-3)3^{2n-1} + 4n).$$

$$14. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 3^{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1} \left((2n-2)3^{2n-1} + \frac{4n+2}{3} \right).$$

$$15. ((2n-1)2^{2n}+2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 3^{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1} \left(3^{2n-1} - \frac{8n-2}{3} \right).$$

$$16. (2^{2n+1}-2)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2s} 2^{2n-2s} B_{n-s} = (-1)^{n-1} (3^{2n-2} - (8n-6)).$$

$$*17. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 4^{2n-2s} B_{n-s} = (-1)^{n-1} (4n+1) + (2n+1)E_n.$$

$$18. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 4^{2n-2s} 3^{2s+1} B_{n-s} = (-1)^{n-1} (4n-1)3^{2n} - (2n+1)E_n.$$

$$*19. (2^{2n}+2)(2^{2n}-1)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 4^{2n-2s} B_{n-s} = (-1)^{n-1} (4n-1).$$

$$*20. (2^{2n}+2)(2^{2n}-1)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 4^{2n-2s} 3^{2s} B_{n-s} = (-1)^{n-1} (4n-3)3^{2n-1}.$$

$$21. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 4^{2n-2s} 5^{2s+1} B_{n-s} = (-1)^{n-1} ((4n-3)5^{2n} + (8n+4)) + (2n+1)E_n.$$

$$22. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 4^{2n-2s} 7^{2s+1} B_{n-s} = (-1)^{n-1} ((4n-5)7^{2n} + (8n+4)3^{2n}) - (2n+1)E_n.$$

$$23. (2^{2n}+2)(2^{2n}-1)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 4^{2n-2s} 5^{2s} B_{n-s} = (-1)^{n-1} ((4n-5)5^{2n-1} + 8n).$$

$$24. (2^{2n}+2)(2^{2n}-1)B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 4^{2n-2s} 7^{2s} B_{n-s} = (-1)^{n-1} ((4n-7)7^{2n-1} + 8n \cdot 3^{2n-1}).$$

25. $\frac{3}{2} (3^{2n}-1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2n-2s} B_{n-s} = (-1)^{n-1} (3n-1).$
26. $\frac{3}{2} (3^{2n}-1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2n-2s} 2^{2s} B_{n-s} = (-1)^{n-1} (3n-2) 2^{2n-1}.$
27. $\frac{3}{2} (3^{2n}-1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2n-2s} 4^{2s} B_{n-s} = (-1)^{n-1} ((3n-4) 4^{2n-1} + 6n).$
28. $\frac{3}{2} (3^{2n}-1) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2n-2s} 5^{2s} B_{n-s} = (-1)^{n-1} ((3n-5) 5^{2n-1} + 6n \cdot 2^{2n-1}).$
29. $\left(\frac{(3^{2n}+3)(2^{2n}+2)-12}{2} \right) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 6^{2n-2s} B_{n-s} = (-1)^{n-1} (6n-1).$
30. $((3^{2n}+3)(2^{2n-1}+1)-6) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 6^{2n-2s} 5^{2s} B_{n-s} = (-1)^{n-1} (6n-5) 5^{2n-1}.$
31. $((3^{2n}+3)(2^{2n-1}+1)-6) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 6^{2n-2s} 7^{2s} B_{n-s} =$
 $= (-1)^{n-1} ((6n-7) 7^{2n-1} + 12n).$
32. $((3^{2n}+3)(2^{2n-1}+1)-6) B_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 6^{2n-2s} 11^{2s} B_{n-s} =$
 $= (-1)^{n-1} ((6n-11) 11^{2n-1} + 12n \cdot 5^{2n-1}).$

2. Formules contenant les T_n .

- *33. $\sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} T_{n-s} = E_n + (-1)^{n-1}.$
- *34. $\sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} T_{n-s} = (-1)^{n-1}.$
- *35. $\sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} T_{n-s} = E_n.$
- *36. $T_{n+1} - \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} T_{n-s} = E_n.$
- *37. $\sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2s} T_{n-s} = (-1)^{n-1} 2^{2n-1}.$

$$*38. 2T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s} 2^{2s} T_{n-s+1} = (-1)^n 2^{2n+1}.$$

$$*39. 2n T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s+1} 2^{2s} T_{n-s+1} = 0.$$

$$*40. 2T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n}{2s} 2^{2s} T_{n-s+1} = 0.$$

$$41. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 3^{2s+1} T_{n-s} = -E_n - (-1)^n (3^{2n} - 2).$$

$$42. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2s} T_{n-s} = (-1)^{n-1} (3^{2n-1} - 2).$$

$$43. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} 3^{2s+1} T_{n-s} = -E_n + (-1)^{n-1} 4.$$

$$44. T_{n+1} - \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} 3^{2s+2} T_{n-s} = -3E_n + (-1)^n 4.$$

$$45. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 4^{2s} T_{n-s} = (-1)^{n-1} (4^{2n-1} - 2^{2n-1}).$$

$$46. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} 4^{2s} T_{n-s} = (-1)^{n-1} (4^{2n-1} - 2^{2n-2}).$$

$$47. 2n T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} 4^{2s} T_{n-s} = (-1)^{n-1} 2^{2n-1}.$$

$$48. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} 4^{2s} T_{n-s} = (-1)^{n-1} 2^{2n-2}.$$

$$49. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s+1} 5^{2s+1} T_{n-s} = E_n + (-1)^{n-1} 4 (3^{2n-1} - 2).$$

$$50. T_{n+1} - \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+2} 5^{2s+2} T_{n-s} = 5E_n + (-1)^n 4 (3^{2n} - 2).$$

$$51. 2n T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n+1}{2s+1} 6^{2s} T_{n-s+1} = \frac{(-1)^n 8}{3} (4^{2n} - 2^{2n}).$$

$$52. 2 T_{n+1} + \sum_{s=1}^{s=n} (-1)^s \binom{2n}{2s} 6^{2s} T_{n-s+1} = (-1)^n (4^{2n+1} - 2^{2n+3}).$$

$$\begin{aligned}
53. \quad & \frac{3^{2n+1}}{2} T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2n-2s} 2^{2s} T_{n-s} = (-1)^{n-1} 3 \cdot 2^{2n-1}. \\
54. \quad & \frac{3^{2n+1}-3}{2} T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2n-2s} 4^{2s} T_{n-s} = (-1)^{n-1} 3 \cdot 4^{2n-1}. \\
55. \quad & \frac{3^{2n+1}-3}{2} T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2n-2s} 8^{2s} T_{n-s} = (-1)^{n-1} 3 (8^{2n-1} - 2^{2n}). \\
56. \quad & \frac{3^{2n+1}-3}{2} T_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2n-2s} 10^{2s} T_{n-s} = (-1)^{n-1} 3 (10^{2n-1} - 2^{4n-1}).
\end{aligned}$$

3. Formules contenant les E_n .

$$\begin{aligned}
*57. \quad & \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} E_{n-s} = (-1)^{n-1}. \\
*58. \quad & \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n-1}{2s-1} E_{n-s} = T_n + (-1)^n. \\
*59. \quad & \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} E_{n-s} = T_n. \\
*60. \quad & \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} E_{n-s} = T_{n+1}. \\
*61. \quad & 2E_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s} 2^{2s} E_{n-s} = (-1)^{n-1} (2^{2n} - 2). \\
*62. \quad & \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+1} 2^{2s} E_{n-s} = (-1)^{n-1} (2^{2n} - 1). \\
*63. \quad & (2n-1)E_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 2^{2s} E_{n-s} = (-1)^{n-1}. \\
*64. \quad & 2E_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n-1}{2s} 2^{2s} E_{n-s} = (-1)^{n-1} \cdot 2. \\
*65. \quad & \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s} 3^{2s} E_{n-s} = (-1)^n (2^{2n+1} - 3^{2n}).
\end{aligned}$$

$$66. \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n-1}{2s-1} 3^{2s-1} E_{n-s} = -T_n + (-1)^n (3^{2n-1} - 2^{2n}).$$

$$67. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} 3^{2s} E_{n-s} = -3T_n - (-1)^n 2^{2n}.$$

$$68. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 3^{2s+1} E_{n-s} = -T_{n+1} - (-1)^n 2^{2n+1}.$$

$$69. (2n+1)E_n + \sum_{s=1}^{s=n-1} (-1)^s \binom{2n}{2s+1} 4^{2s} E_{n-s} = \frac{(-1)^{n-1}}{2} (3^{2n} - 3).$$

$$70. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n+1}{2s+2} 4^{2s} E_{n-s} = \frac{(-1)^{n-1}}{8} (3^{2n+1} - 3).$$

$$71. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n-1}{2s} 5^{2s} E_{n-s} = 5T_n + (-1)^n (2^{2n-1} - 3) 2^{2n}.$$

$$72. \sum_{s=0}^{s=n-1} (-1)^s \binom{2n}{2s+1} 5^{2s+1} E_{n-s} = T_{n+1} + (-1)^n (2^{2n} - 3) 2^{2n+1}.$$

$$73. \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n-1}{2s-1} 3^{2n-2s} E_{n-s} = \frac{3^{2n}-3}{6} T_n + (-1)^n.$$

$$74. \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n-1}{2s-1} 3^{2n-2s} 5^{2s-1} E_{n-s} = \frac{3^{2n}-3}{6} T_n + (-1)^n (5^{2n-1} - 2^{2n}).$$

$$75. \sum_{s=1}^{s=n-1} (-1)^{s-1} \binom{2n-1}{2s-1} 3^{2n-2s} 7^{2s-1} E_{n-s} = \frac{3^{2n}-3}{6} T_n + (-1)^n (7^{2n-1} - 2^{4n-1}).$$

4. Exemples des formules irrégulières.

$$76. \sum_{s=0}^{s=n-1} (-1)^s (2n+1) \binom{4n-2s-1}{2s+1} B_{n-s} = \frac{(-1)^{n-1} n(n+1)}{2^{4n-2}}.$$

$$77. \sum_{s=0}^{s=n-1} \frac{(-1)^s 2n}{4n-2s-1} \binom{4n-2s-1}{2s+1} T_{n-s} = 1 - (-1)^n.$$

$$78. \sum_{s=0}^{s=n-1} \frac{(-1)^s (2n-1)}{(2n-s-1) 2^{2s}} \binom{4n-2s-2}{2s} T_{n-s} = \frac{(-1)^{n-1}}{2^{2n-2}}.$$

$$79. \sum_{s=0}^{s=n-1} \frac{(-1)^s n}{(2n-s) 2^{2s}} \binom{4n-2s}{2s} E_{n-s} = \frac{1 - (-1)^n}{2^{2n-1}}.$$

$$80. \sum_{s=0}^{s=n-1} \frac{(-1)^s (2n+1)}{4n-2s+1} \binom{4n-2s+1}{2s+1} E_{n-s} = (-1)^n \omega_n,$$

$$\omega_{3n} = 0, \quad \omega_{3n+1} = -3, \quad \omega_{3n+2} = 0.$$

5. Formules linéaires incomplètes.

$$*81. \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} \frac{B_{n-s}}{n-s} = \frac{n! n!}{(2n+1)!}.$$

$$*82. \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n+1}{2s+1} (2n-2s+1) B_{n-s} = 0, \quad (n > 2).$$

$$*83. \sum_{s=0}^{\leq \frac{n-1}{2}} (-1)^s \binom{n}{2s+1} 2^{4s} T_{n-s} = 2^{2n-2}.$$

$$*84. \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} (n-s) 2^{2s} T_{n-s} = 0, \quad (n \geq 2).$$

$$*85. \sum_{s=0}^{\leq \frac{n}{2}} (-1)^s \binom{n}{2s} 2^{2s} E_{n-s} = 1.$$

II. Formules récursives non linéaires.

6. Formules de première espèce.

$$*86. \sum_{s=1}^{s=n-1} \binom{2n}{2s} B_s B_{n-s} = (2n+1) B_n.$$

$$87. \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{2s} B_s B_{n-s} = (2^{2n} + 2n) B_n.$$

$$88. \sum_{s=1}^{s=n} \binom{2n+1}{2s-1} T_s B_{n-s+1} = T_{n+1} - \left(n + \frac{1}{2}\right) E_n.$$

$$*89. \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{2s} B_s T_{n-s} = T_n.$$

$$*90. \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{4s} B_s T_{n-s} = 2n T_n.$$

$$*91. \sum_{s=0}^{s=n-1} \binom{2n}{2s+1} T_{s+1} T_{n-s} = T_{n+1}.$$

$$*92. \sum_{s=1}^{s=n} \binom{2n}{2s-1} 2^{2s} T_s T_{n-s+1} = 2^{2n+1} E_n.$$

$$*93. \sum_{s=1}^{s=n-1} \binom{2n}{2s} E_s B_{n-s} = E_n - n T_n.$$

$$94. \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{2s} B_s E_{n-s} = E_n - 2(2^{2n}-1) B_n.$$

$$95. \sum_{s=1}^{s=n-1} \binom{2n}{2s} 2^{4s} B_s E_{n-s} = (2n+1) E_n - (2^{2n}-1)(2^{2n}+2) B_n.$$

$$*96. \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} E_s T_{n-s} = E_n - T_n.$$

$$97. \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{2s} E_s T_{n-s} = (2^{2n-1}-2) T_n.$$

$$98. \sum_{s=1}^{s=n-1} \binom{2n-1}{2s} 2^{2n-2s} T_s E_{n-s} = \frac{2^{2n-1}-2}{2n} T_n.$$

$$*99. \sum_{s=1}^{s=n-1} \binom{2n}{2s} E_s E_{n-s} = T_{n+1} - 2 E_n.$$

7. Formules de seconde espèce.

$$100. \left(\binom{4n}{2n} - 1 \right) B_{2n} + \binom{4n}{2n} B_n^2 = 2 \cdot \sum_{s=1}^{s=n} \binom{4n}{2s} \binom{4n-2s-1}{2n-1} B_s B_{2n-s}.$$

$$101. \left(\binom{4n-2}{2n+1} + 1 \right) B_{2n+1} = 2 \cdot \sum_{s=1}^{s=n} \binom{4n+2}{2s} \binom{4n-2s-1}{2n} B_s B_{2n-s+1}.$$

$$102. T_{2n} = (4n-1) \binom{4n-2}{2n-1} T_n^2 - 2 \cdot \sum_{s=1}^{s=n} \binom{4n-1}{2s-1} \binom{4n-2s-1}{2n-1} 2^{4n-2s} T_s B_{2n-s}.$$

$$103. T_{2n+1} = 2 \cdot \sum_{s=0}^{s=n-1} \binom{4n+1}{2s+1} \binom{4n-2s-1}{2n} 2^{4n-2s} T_{s+1} B_{2n-s}.$$

$$104. \binom{4n-1}{2n} E_{2n} = \binom{4n}{2n} (2^{2n}-2) B_n E_n - \sum_{s=1}^{s=n} \binom{4n}{2s} \binom{4n-2s-1}{2n} 2^{4s} B_s E_{2n-s}.$$

$$105. \binom{4n+3}{2n+1} E_{2n+1} =$$

$$= \binom{4n+2}{2n} (2^{2n+2} - 2) B_{n+1} E_n + \sum_{s=1}^{s=n+1} \binom{4n+2}{2s} \binom{4n-2s+3}{2n+1} 2^{4s} B_s E_{2n-s+1}.$$

$$106. \binom{4n-1}{2n-1} T_{2n} = - \binom{4n-1}{2n-1} 2^{2n} B_n T_n + \sum_{s=1}^{s=n} \binom{4n-1}{2s} \binom{4n-2s-1}{2n-1} 2^{4s} B_s T_{2n-s}.$$

$$107. \binom{4n+1}{2n} T_{2n+1} = \sum_{s=1}^{s=n} \binom{4n+1}{2s} \binom{4n-2s+1}{2n} 2^{4s} B_s T_{2n-s+1}.$$

Corrections et remarques historiques.

Quoique mes citations précédentes sont conformes aux indications données généralement dans des Mémoires récents, dans des Monographies, dans le *Jahrbuch über die Fortschritte der Mathematik* etc., mes études plus approfondies de la littérature très riche et très étendue sur les nombres de BERNOULLI entraînent néanmoins, nous le verrons, beaucoup de corrections.

C'est-à-dire qu'il est nécessaire de revoir profondément, à ce point de vue aussi, la théorie des nombres de BERNOULLI et d'EULER.

Page 287 (5): Les premières formules récursives pour les B_n , dont les indices forment une série arithmétique ayant une différence plus grande que l'unité, savoir la valeur 2, semblent être dues à KNAR¹⁾.

Contemporainement KRONECKER²⁾ a publié des formules récursives pour les B_n et les T_n qui donnent comme des cas particuliers des formules récursives pour les B_n et T_n , dont les indices sont divisibles par le positif entier quelconque k . Cependant, KRONECKER ne mentionne pas les formules particulières de ce genre.³⁾

F.-J. VAN DEN BERG⁴⁾ a trouvé une formule récursive pour les B_n , dont les indices forment une série arithmétique quelconque, et il a indiqué des exemples de formules analogues pour les T_n et les E_n , dont les indices forment des séries arithmétiques ayant la différence 2 ou 3.

Les frères MM. J.-C. KAPTEYN et W. KAPTEYN⁵⁾ ont retrouvé les formules particulières de VAN DEN BERG, tandis que M. HAUSSNER a retrouvé les formules générales concernant les B_n et trouvé les formules générales correspondantes pour les T_n et les E_n .

Les belles découvertes de KNAR et de VAN DEN BERG semblent être inaperçues jusqu'ici.⁶⁾

Page 295 (13): La fonction considérée par JACOBI

$$\varphi_{2n}(x) = \frac{(-1)^n B_n}{(2n)!}$$

est essentielle dans les recherches de KUMMER⁷⁾ sur l'équation indéterminée de FERMAT:

$$x^\lambda + y^\lambda = z^\lambda.$$

¹⁾ Archiv de Grunert, t. 27, p. 455—456; 1856.

²⁾ Journal de Liouville (2) t. 1, p. 385—391; 1856.

³⁾ Ni dans la Note originelle ni dans ses remarques insérées dans le Journal de Crelle, t. 94, p. 268—269; 1883. Comparez du reste mon Mémoire récent dans: *Berichte der kgl. sächs. Gesellschaft der Wissenschaften*, t. 65, p. 25; 1913.

⁴⁾ Verslagen en mededeelingen der koninklijke Akademie Amsterdam (2) t. 16, p. 74—176; 1881.

⁵⁾ Wiener Sitzungsberichte t. 93 II, p. 836; 1886.

⁶⁾ Voir par exemple: Correspondance d'HERMITE et de STIELTJES, t. II, p. 432; 1905. Il est très regrettable que le collaborateur du *Jahrbuch über die Fortschritte der Mathematik* ne dit rien sur la forme singulière des formules de VAN DEN BERG; voir t. 13, p. 193.

⁷⁾ Journal de Crelle, t. 40, p. 119; 1850.

Page 298 (16): La formule (7) doit être:

$$E_n = (-1)^n (2n)! 2^{2n+1} \chi_{2n} \left(-\frac{1}{2} \right), \quad n \geq 1,$$

comme le montrent clairement les deux formules suivantes (8) et (9).

Page 301 (19): Posons dans (1) $x=0$, puis remplaçons n par $2n$, il résulte

$$\sum_{s=1}^{s=p-1} \left(\varphi_{2n}(0) - \varphi_{2n} \left(-\frac{s}{p} \right) \right) = \frac{(-1)^n (p^{2n}-1) B_n}{(2n)! p^{\frac{2n-1}{2}}};$$

cette formule, trouvée par KUMMER¹⁾, est essentielle dans ses recherches susdites.

Page 315 (33): La formule 10 d'EULER joue aussi un rôle dans les recherches de KUMMER²⁾. La formule eulérienne susdite est souvent attribuée³⁾ à GRUNERT⁴⁾, qui a démontré la formule à l'aide de celle de MOIVRE, savoir la formule 1 de la Table.

Page 331 (49): La formule (4) est trouvée aussi par HERMITE⁵⁾.

Page 332 (50) § 17: Il semble être complètement inaperçu jusqu'ici qu'ANDREAS V. ETTINGSHAUSEN⁶⁾ a découvert, déjà en 1827, c'est-à-dire un demi-siècle avant SEIDEL et STERN, les formules récursives incomplètes pour les B_n qui correspondent à $q=1$.

La démonstration de V. ETTINGSHAUSEN est entièrement élémentaire; car elle n'applique que les fondements du calcul des différences finies.

Page 345 (63): Déjà en 1875 LUCAS⁷⁾ a développé le produit $s_m s_n$ selon des s_q , savoir le produit $\varphi_m(x) \varphi_n(x)$ selon des $\varphi_q(x)$.

Page 350 (68): Il est très intéressant, ce me semble, que la formule essentielle (6) est la même que celle de KUMMER mentionnée dans la Note à la page 301.

Page 351 (69): Le théorème concernant le produit

$$\frac{p^{2n}(p^{2n}-1)B_n}{2n}$$

est dû à SYLVESTER⁸⁾; mais quoique LIPSCHITZ⁹⁾ a proclamé la priorité de l'éminent géomètre anglais, on¹⁰⁾ désigne néanmoins généralement le théorème en question comme appartenant à LIPSCHITZ.

Page 352 (70): Le théorème III appartient à STERN¹¹⁾.

Page 353 (71): Le théorème VI se trouve dans un Mémoire de M. J.-W.-L. GLAISHER¹²⁾; il attribue à WOLSTENHOLME¹³⁾ le cas particulier $p=n-1$. Curieusement M. GLAISHER désigne le théorème V de LAGRANGE comme appartenant à FERRERS!

Page 361 (79): La formule 98 doit être biffée; elle n'est qu'une forme inexacte de 97.

¹⁾ loc. cit. p. 120—121.

²⁾ loc. cit. p. 121.

³⁾ Voir par exemple GÖPEL dans Grunert Archiv t. 3, p. 66; 1843.

⁴⁾ Mathematische Abhandlungen p. 57—59; Altona 1822.

⁵⁾ Comptes rendus du troisième Congrès Scientifique, Bruxelles 1894.

⁶⁾ Vorlesungen über die höhere Mathematik, t. I, p. 284—285; Vienne 1827.

⁷⁾ Nouvelles Annales (2) t. 14, p. 487—494; 1875.

⁸⁾ Philosophical Magazine, février 1861.

⁹⁾ Bulletin de Darboux (2) t. 10, p. 141; 1886.

¹⁰⁾ Voir par exemple M. P. BACHMANN: Niedere Zahlentheorie, t. II, p. 33; Leipsic 1910.

¹¹⁾ Journal de Crelle, t. 88. p. 92; 1880.

¹²⁾ Quarterly Journal of Mathematics, t. 31, pp. 1—35, 321—353; 1899—1900.

¹³⁾ Ibid. t. 5, p. 35—39; 1862.

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UNDERSØGELSE
AF NOGLE
GRÆSSERS OG KLØVERARTERS
KEMISKE SAMMENSÆTNING
PAA FORSKELLIGE MODNINGSSTADIER
AF
K. RØRDAM

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER. 7. RÆKKE, NATURV. OG MATHEMATISK AFD. X. 4



KØBENHAVN
HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL
BIANCO LUNOS BOGTRYKKERI
1913

Skønt der siden LIEBIG's Dage i snart sagt utallige Tilfælde rundt om i Kulturlandene er foretaget kemiske Undersøgelser af de Planter og Produkter fra Planteriget, der bliver benyttet til Foder for Husdyr og til Menneskers Ernæring, er det en kendt Sag, at den nærmere Kendskab til mange Landbrugsplanters kemiske Sammensætning endnu lader adskilligt tilbage at ønske.

Dette beror paa forskellige Forhold dels af kemisk dels af ren praktisk Art, som her kun kortelig skal berøres. De kemiske Metoder til kvantitativ Analyse, der har været benyttet og i mange Tilfælde endnu benyttes i Foderstofanalysen, har været og er behæftet med meget store Mangler, som det først i den nyeste Tid og kun til en vis Grad er lykkedes at raade Bod paa. En stor Mængde kemiske Analyser af Plantedele eller Produkter fra Planteriget, som er foretaget i ældre Tid, maa derfor betragtes som værende af ringere Værdi som Oplysningsmiddel om Stofferne Sammensætning.

I de fleste Tilfælde, hvor der er foretaget saadanne kvantitative kemiske Bestemmelser, har man desuden af let forstaaelige Grunde mere beskæftiget sig med ved Benyttelsen af de „konventionelle Metoder“ at finde Tal for „Næringsstofindholdet“ i et vist givet Foder eller Foderblanding til Benyttelse ved foreliggende Fodringsforsøg, da dette var, hvad Praxis i Øjeblikket krævede, end man har søgt at udrede de enkelte Plantearters nærmere Sammensætning under de forskellige Omstændigheder, hvorunder de benyttes i Landbrugets Praxis.

Noget af den anførte Uensartethed beror for en Del paa de noget forskellige Fordringer fra Agrikulturkemiens Side, der stilles til kvantitativ kemisk Analyse i Sammenligning med den rent videnskabelige Kemis Fordringer. Medens for denne Videnskab den kemiske Analyses Beviskraft er saa stor, at en enkelt eller nogle faa vel udførte kvantitative Analyser ofte er tilstrækkeligt til at begrunde Slutninger af en meget stor videnskabelig Rækkevidde, er dette kun sjældent Tilfældet for Agrikulturkemiens Vedkommende. Paa Grund af den store Mængde Tilfældigheder, der kan være ved det undersøgte Materiales Herkomst og Art og yderligere ved

Indhøstningen og Opbevaringstilstanden o. s. v., Tilfældigheder, der atter kan faa væsentlig og uberegnelig Indflydelse paa den kemiske Sammensætning, betyder en enkelt eller nogle faa noksaa vel udførte kvantitative kemiske Analyser i Reglen ikke ret meget i Agrikulturmekemien. Man maa have Rækker af Bestemmelser for at kunne eliminere Tilfældighederne, og Metoderne, der bliver benyttet, maa saa vidt muligt tillade hurtig Udførelse af mange Bestemmelser samtidigt.

Man har derfor paa Foderstofanalysens Omraade som paa mange andre Punkter i Agrikulturmekemien søgt, tvunget af Omstændighederne, ved en eller anden analytisk Metode at naa et Talresultat, der kunde benyttes i Praksis uden altid at have fuld Sikkerhed for, at de benyttede Metoder gav et korrekt Udtryk for det man søgte at bestemme.

I den senere Tid er der dog, takket være mange forskellige Forskeres Arbejde, sket betydelig Fremgang, navnlig hvad angaar Kendskaben til Beskaffenheden af Æggehvidestofferne og Kulhydraterne særlig i Kornsorterne og Roer. I 1905 blev der af V. STORCH i 58de Beretning fra d. kgl. Veterinær- og Landbohøjskoles Laboratorium for landøkonomiske Forsøg: „Den kemiske Analyse af Foderstoffer“, givet en udførlig Oversigt over disse Undersøgelseres daværende Standpunkt, hvortil kan henvises. Siden den Tid er Undersøgelserne baade i Udlandet og herhjemme i betydelig Grad blevet forøget i Antal, uden at der dog kan siges at være sket væsentlige Forandringer, hvad Principerne angaar.

I landøkonomisk Henseende er der i Løbet af de seneste Aar her i Danmark til en vis Grad sket en Svingning i nogle Sider af Plantedyrkningen, som forarsager, at ny Undersøgelserækker blev ønskelige. Avl af Frø af Græsarterne og forskellige Bælgplanter kommer økonomisk set til at spille en større og større Rolle i Danmarks Landbrug og Spørgsmaalet om, hvad den udtærskede Halm af disse Planter indeholder og er værd som Foderstof i Sammenligning med paagældende Planter høstet i grøn Tilstand og tørret som Hø, kan derfor siges i de seneste Aar mere og mere at træde i Forgrunden. „Halm“ og „Hø“ har mangfoldige Gange i Løbet af det sidste Aarhundrede været Genstand for Undersøgelse efter mer eller mindre brugbare Metoder, og skønt man vel for de vigtigste Kornsorters Vedkommende kender Sammensætningen af den Halm, der vindes af disse Planter, er Græsarternes Halm kun i meget ringe Grad undersøgt og i hvert Fald slet ikke af dansk Avl. For Høets Vedkommende har man ogsaa talrige Analyser dels af „Agerhø“ og „Enghø“ og tildels ogsaa „Kløverhø“, men hvilke Planter disse Produkter bestod af og i hvilket indbyrdes Mængdeforhold de forekom i Høsorterne er i Reglen ikke angivet eller ikke angivet med tilbørlig Nøjagtighed. Analyserne mister derved en stor Del af den Betydning, de ellers i videnskabelig Henseende kunde have faaet m. H. t. Studiet af paagældende Foderplanters Sammensætning.

Prøvernes Tilvejebringelse.

Fra de samvirkende Landboforeninger i Sjællands Stift gennem daværende Konsulent i Planteavl O. H. LARSEN blev i Foraaret 1911 rettet en Opfordring til mig om muligt at bøde noget paa dette Forhold ved at paatage mig Undersøgelserne af et Antal botanisk bestemte Foderurter dels Græsarter, dels Bælgplanter, der i Sommeren 1911 blev dyrket i Renkultur. Læreren i Landbrugets Planteavl ved Landbohøjskolen min Kollega Prof. WESTERMANN sluttede sig tillige til Landboforeningernes Opfordring og stillede til Disposition et Antal udsøgte Planteprøver, som under hans Opsyn var dyrket og høstet dels i grøn Tilstand og tørret til Hø dels efter Modning og Udtærskning som Halm. Skønt jeg straks var klar paa, at den foreliggende Opgave vilde kræve et meget betydeligt analytisk kemisk Arbejde, hvortil jeg næppe kunde paaregne Hjælp ude fra, og at de kemiske Undersøgelser, som der kunde være Tale om at foretage, hvis Arbejdet skulde tilendebringes i rimelig Tid, vanskelig kunde blive udtømmende nok, syntes en Løsning af Opgaven, selv om den til en Begyndelse kun blev partiel, dog at have saa stor baade praktisk og videnskabelig Interesse, at jeg har paataget mig Arbejdets Udførelse. Hertil blev jeg i høj Grad opmuntret ved det fortræffelige rene og rigelige Undersøgelsesmateriale, som Prof. WESTERMANN og O. H. LARSEN stillede til min Raadighed. Sidstnævnte meddeler følgende om Prøvernes Tilvejebringelse og Art:

„Hvad Prøvernes Tilvejebringelse angaar, stammer de allesammen fra Landbohøjskolens Undervisningsmark, hvor de enkelte Græsser og Kløverarter hvert Aar udsaaes i Renbestand bl. a. for at demonstrere Saatidens Betydning. Til hver Høst er benyttet en Parcel paa 2 m². Høsttiderne for de enkelte Arter har forøvrigt været som nedenfor anført“:

Tabel 1.

1911.	Til Høslæt.		Til Frøavl.	
	Slaaet.	Hjemkørt.	Slaaet.	Hjemkørt.
<i>Italiensk Rajgræs</i> (<i>Lolium italicum</i>).	8/6	13 ¹ / ₆	10 7	28 ¹ / ₇
<i>Draphavre</i> (<i>Avena elatior</i>).	8/6	13 ¹ / ₆	10 7	28 ¹ / ₇
<i>Ager-Hejre</i> (<i>Bromus arvensis</i>).	8/6	13 ¹ / ₆	23 7	9 ¹ / ₈
<i>Hundegræs</i> (<i>Dactylis glomerata</i>).	8/6	13 6	10/7	28/7
<i>Engsvingel</i> (<i>Festuca pratensis</i>).	8 6	13 6	10 ¹ / ₇	28 ¹ / ₇
<i>Sildig Rødkløver</i> (<i>Trifolium rubrum</i>).	8/6	13/6	7 8	28/8
<i>Kællingeland</i> (<i>Lotus corniculata</i>).	8/6	13/6	7/8	28 8
<i>Gul Rundbælg</i> (<i>Anthyllis vulneraria</i>).	8/6	16 6	7 8	28 8

Høhøsten, der strakte sig fra den 8de--16de Juni, faldt i en noget urolig Vejrperiode med jævnlig Nedbør, som nedenstaaende Tabel viser:

Tabel 2.

Dato.	Temp. Kl. 8 Morgen paa Landbohøjskolen.	Døgnet Nedbør
8de Juni 1911	16°	1 mm.
9de —	11°	7 —
10de —	9°	16 —
11te —	13°	0 —
12te —	12°	1 —
13de —	13°	2 —
14de —	13°	0 —
15de —	14°	13 —
16de —	14°	0 —

Af Periodens 9 Dage var altsaa kun 3 Dage helt fri for Regn. Det er muligt, at Nedbørsforholdene har faaet nogen Indflydelse paa Høets Beskaffenhed, skønt det efter Øjesyn og Lugten at dømme ikke havde lidt Skade, men gjorde et ganske normalt Indtryk som velbjerget Hø.

De 49 Dage i Løbet af hvilke Indhøstningen af Græsserne og Kløverarterne til Frøavl foregik („Halmhøsten“) havde mere stadigt Vejr. Denne Periode strakte sig fra 10de Juli til 28de August og havde Nedbør i følgende Døgn:

Tabel 3.

Den 17de Juli	24 mm.	Den 15de August	4 mm.
19de —	7 —	16de —	4 —
20de —	1 —	22de —	1 —
21de —	4 —	23de —	7 —
		26de —	3 —
		27de —	8 —

Af Periodens 49 Dage var der altsaa kun Nedbør i de 10 Dage og blandt disse har Regnen kun i de fire Nedbørsdage 17de og 19de Juli samt 23de og 27de August været af større Udstrækning. Den aftærskede Halm var af ganske normalt Udseende. Selv om der altsaa baade under Høhøsten og under Halmhøsten har været Regnvejr, er det næppe mer end hvad der i Almindelighed tør ventes i Praksis under vore urolige Vejrforhold.

I. Forberedelse til Analyse.

Hø- og Halmprøverne blev skaaret i Hakkelse paa 2—3 cm. Længde. I Laboratoriet blev de derefter malet meget fint paa en Kværn med riflede Staalmaleplader. Kværnen blev drevet af en Motor paa 1.5 H. K. Det viste sig herved nødvendigt først at tørre Prøverne noget ved at lade dem henligge i c. 8 Døgn i et støvfrit Rum, hvor Temperaturen var c. 30°. Malingen kunde derefter let foretages i tre Gange gentagne Operationer, medens Kværnen ved at benytte de ikke

tørrede Prøver løb varm og Prøverne var udsatte for at blive brankede. Prøverne dannede efter vel gennemført Maling et ret fint ensartet Pulver. Høprøverne er mat græsgrønne med meget karakteristisk behagelig Hølugt. Halmpulveret er gulgraat med svagere Lugt, omtrent som moden Rug. Af hver Prøve blev malet 1—1,5 Kilo, saa at der var rigeligt ganske ensartet Materiale til Analyse. Endnu efter et Aars Forløb viser Pulveret ikke at have forandret sig ved at opbevares i Mørke i et veltillukket Glas paa et tørt Sted. I Lyset bleges derimod særligt Høprøverne meget hurtigt. Selv i et Glas lukket med en veltilslebet Glasprop kan man, naar en malet Høprøve henstaar blot en Dags Tid i Solen, se Forskel i Farven paa Prøvens ydre Lag nærmest Glasset og de andre ikke paavirkede Dele.

Den forholdsvis stærkt drevne Lufttørring af Prøverne for at faa dem sprøde nok til Malingen har naturligvis foraarsaget, at Indholdet af hygroskopisk Vand er noget mindre, altsaa Indholdet af Tørstof noget større, end det vil være i Hø og Halm, der opbevares frit i Luften, hvor Tørstofprocenten stiger og falder med Luftens mindre eller større Fugtighedsgrad. Men da Hø- og Halmprøverne af de forskellige Planter er blevet behandlede ganske paa samme Maade, maa Forholdene mellem de enkelte Prøvers Tørstofmængder vist nok være vedbleven at være det samme før og efter Tørringen til Maling. Alle Bestemmelser er udført ved Hjælp af de malede Prøver.

II. Bestemmelse af Tørretab og Tørstof.

I et almindeligt Vejglas af flad Form af kendt Vægt blev udbredt en passende Mængde af Stoffet (1—3 grm.), som blev vejjet. Efter Henstand i et Tørreskab i 12—18 Timer ved 105° blev Glasset atter vejjet efter Afkøling i Exsikkator. Man kan opnaa konstant Vægt efter blot 3 Timers Tørring ved 105°, men det er bekvemt at lade Prøverne som nævnt henstaa fra den ene Dag til den næste. Prøverne er efter Tørring noget hygroskopiske og maa vejes i det tillukkede Glas. Vægttabet ved Tørring er hygroskopisk Vand. Resten i Glasset efter Tørring er Tørstof. Resultaterne var følgende i Procent af det lufttørre Stof.

Tabel 4.

Græsarterne:			Hø.	Halm.
		Tørretab $\frac{0}{100}$	Tørstof $\frac{0}{100}$	Tørstof $\frac{0}{100}$
A ₁	Italiensk Rajgræs	Hø	7,39	—
A ₂	—	Halm	5,52	94,48
B ₁	Draphavre	Hø	2,34	—
B ₂	—	Halm	1,45	98,55
C ₁	Agerhejre	Hø	3,06	96,94
C ₂	—	Halm	2,49	97,51
D ₁	Hundegræs	Hø	7,17	—
D ₂	—	Halm	8,95	91,05
C ₁	Engsvingel	Hø	8,37	—
C ₂	—	Halm	4,29	95,71
Middeltal . . .			94,34 $\frac{0}{100}$	95,56 $\frac{0}{100}$

Tabel 5.

Kløverarterne:			Hø.	Halm.
		Torretab $\frac{0}{10}$	Torstof $\frac{0}{10}$	Torstof $\frac{0}{10}$
F ₁	Sildig Rødkløver	Hø	8,38	—
F ₂	—	Halm	8,01	91,99
G ₁	Kællingetand	Hø	2,76	97,24
G ₂	—	Halm	1,25	98,75
H ₁	Gul Rundbælg	Hø	5,81	94,19
H ₂	—	Halm	2,13	97,87
Middeltal			94,35 $\frac{0}{10}$	96,20 $\frac{0}{10}$

Der er altsaa lidt mindre Tørstof i Høet end i Halmen af de forskellige Græsser og Kløverarter. Hundegræs alene danner en dog ikke meget væsentlig Undtagelse. Ved Analyse c. 9 Maanedere senere fandtes omtrent det samme Forhold, nemlig 92,19 $\frac{0}{100}$ Tørstof i Hundegræs Hø, 91,14 $\frac{0}{100}$ Tørstof i Hundegræs Halm og 93,81 $\frac{0}{100}$ Tørstof i Rundbælg Hø.

III. Askebestemmelser.

A. Bestemmelser af den samlede Askemængde.

Ved Askebestemmelser af Halm og Hø som i det hele taget ved saadanne Bestemmelser i Rester eller Dele af Organismer, er der visse Vanskeligheder baade i teoretisk og praktisk Henseende at tage i Betragtning.

Hvad skal der overhovedet forstaaes ved Askebestanddele? I første Linje maa det være de Bestanddele, der direkte indgaar i det organiske Væv, hvoraf Planten eller Plantedelen bestaar og som ved Forbrænding bliver omdannet til uorganisk Stof (Kiselsyren fra Straaet, Fosfor og Svovl fra Æggehviden, Jern fra Klorofyllet m. fl.). Dernæst maa ogsaa til Asken henregnes de Bestanddele, der i opløst eller udskilt Form findes i Saften (Calciumoxalat o. lgn.). Kun Stoffer fra disse to Grupper kan henregnes til Renasken. Dog maa til denne ogsaa henregnes Ilten, som disse Stoffer har optaget ved Forbrændingen. Hvorvidt Kulsyren, som Baserne er blevet bundet til ved Forbrændingen, egentlig bør henregnes under Bestanddelene i Renasken, kan derimod være Tvivl underkastet.

I Asken kan yderligere ofte rent mekanisk indblandes Støv og Jordpartikler, der har sat sig fast uden paa Planten eller er blevet indesluttet i Plantevævet under Væksten. Nogen fuldstændig Sikkerhed for at skaffe de undersøgte Plantedele fri for Stoffer af sidstnævnte Art kan naturligvis ikke faas, men det tør siges, at de foreliggende Prøver i hvert Fald kun har indeholdt meget lidt Forureninger af denne Art. Til Bestemmelse af Aske i de foreliggende Prøver blev benyttet følgende Fremgangsmaade efter en Række forudgaaende Forsøg:

En vejet Mængde af det lufttørre malede Stof blev anbragt i en vejet Porcellænsdigel og forkullet ved svag Varme over Flammespreder. Det er af Vigtighed

at lade Forkulningen foregaa meget langsomt ved saa svag Varme som muligt. Naar der ikke længere udvikles Dampe, stilles Diglen paa skraa over en lille Bunsensk Brænder, og nu lykkes det i Reglen let at faa Kullet bortbrændt, saa at der efterlades en løs hvid eller hvidgraa pulverformig Aske, i hvilken Calcium- og Magniumsaltene er til Stede som Karbonater. Hvis man opvarmer for stærkt i Begyndelsen smelter Alkalisaltene i Asken og indhyller Kullet, der i saa Fald vanskelig brænder bort¹⁾.

Naar ovennævnte Fremgangsmaade følges, har det ved de foreliggende Prøver været let at faa forholdsvis vel overensstemmende Resultater ved en Række nedenfor angivne Parallelforsøg. Senere skal angives, hvorledes man, ved Bestemmelsen af de enkelte Bestanddele i Asken fremstillet paa en noget anden Maade, til en vis Grad har et Middel til at kontrollere de her udførte Askebestemmelser. Resultaterne af de enkelte Bestemmelser var følgende:

Tabel 6.

Stoffets Art	I Arbejde		Fundet Askemængde		Askeprocent af Torstofs Middeltal
	Lufttørt Stof Grm.	Torstof Grm.	Aske i Grm.	Aske i %	
A ₁ Italiensk Rajgræs Hø	1,4135	1,3090	0,0915	6,99	} 7,01
	0,8645	0,8004	0,0563	7,03	
A ₂ Italiensk Rajgræs Halm	1,4400	1,3605	0,0680	5,00	} 5,20
	0,6165	0,5826	0,0315	5,40	
B ₁ Draphavre Hø	1,5625	1,5260	0,0885	5,79	} 5,71
	0,5580	0,5449	0,0300	5,51	
	0,7970	0,7783	0,0452	5,80	
	0,6940	0,6777	0,0390	5,75	
B ₂ Draphavre Halm	0,6120	1,5885	0,0910	5,73	} 5,68
	0,6355	0,6263	0,0353	5,64	
C ₁ Agerhejre Hø	1,0470	1,0150	0,0750	7,39	} 7,28
	0,5965	0,5783	0,0415	7,18	
C ₂ Agerhejre Halm	1,2820	1,2500	0,0590	4,72	} 4,51
	0,5128	0,5001	0,0215	4,30	
D ₁ Hundegræs Hø	3,0575	2,8385	0,1355	7,02	} 7,38
	0,8480	0,7872	0,0585	7,43	
	1,2065	1,1200	0,0860	7,68	

¹⁾ Efter at denne Undersøgelse var udført, er Forf. gennem den i September 1912 udkomne Jahresbericht d. Agrikultur-Chemie (3te Folge Bd. XIV 1911 S. 623 blevet opmærksom paa et Arbejde af K. STOLTE i „Biochem. Ztschr.“ 35, 1911, S. 104—107, „Eine einfache und zuverlässige Methodik der Aschenanalyse“. Principet i denne Fremgangsmaade er i det væsentlige det samme som ovenfor omtalt, nemlig at lade Forkulningen af Stofferne foregaa ved den lavest mulige Temperatur og derefter ved noget højere Varme forbrænde Kullet uden nogen Tilsætning. Forf. lader Forbrændingen foregaa i en Platinskaal, der stilles inden i en Porcellænsskaal adskilt fra denne ved smaa Porcellænsstykker altsaa i et Luftbad. Ved den langsomme Forkulning og den derpaa følgende Forbrænding naas meget tilfredsstillende Resultater.

Tabel 6 (fortsat).

Stoffets Art	I Arbejde		Fundet Askemængde		Askeprocent af Tørstof Middeltal
	Lufttørt Stof Grm.	Tørstof Grm.	Aske i Grm.	Aske i %	
D ₂ Hundegræs Halm	1,7939	1,6325	0,1150	7,04	7,15
	0,8070	0,7347	0,0535	7,26	
E ₁ Engsvingel Hø	1,2121	1,1103	0,0958	8,61	8,65
	1,0420	0,9548	0,0830	8,69	
E ₂ Engsvingel Halm	2,8095	2,1995	0,1385	6,30	6,36
	1,1860	1,1351	0,0730	6,43	
F ₁ Sildig Rødkløver Hø	2,2680	2,0680	0,1895	9,12	8,90
	0,7795	0,7140	0,0625	8,69	
F ₂ Sildig Rødkløver Halm	1,5410	1,4185	0,0740	5,22	5,20
	0,4295	0,3951	0,0180	5,19	
G ₁ Kællingetand Hø	2,3005	2,2370	0,1400	6,26	6,15
	0,7730	0,7517	0,0455	6,05	
G ₂ Kællingetand Halm	1,4085	1,3910	0,0420	3,01	3,08
	0,6260	0,6181	0,0195	3,16	
H ₁ Gul Rundbælg Hø	1,9705	1,8560	0,2080	11,20	10,94
	2,5915	2,4409	0,2630	10,78	
	3,0810	2,9019	0,3145	10,84	
H ₂ Gul Rundbælg Halm	1,2405	1,2135	0,0410	3,38	3,19
	2,4670	2,2595	0,0805	3,00	

Sammenligner man de her fundne Askemængder for Hø og Halm af samme Plante ser man at Askeprocenten i Tørstoffet er desto større jo yngre paagældende Plante eller Plantedel er. Denne Regel, som man blandt andre Steder har paavist i udpræget Grad hos Skovtræerne¹⁾, bekræftes altsaa ogsaa ved de her foreliggende Undersøgelser. For Rødkløverens Vedkommende er Forskellen i Askeindholdet af Tørstoffet paa Plantens forskellige Udviklingstrin allerede forlængst konstateret af E. WOLFF. Han angiver saaledes²⁾:

Tabel 7.

	Analysen Antal	Aske i % af Tørstof	K. R. fandt
Rødkløver, ganske ung	11	9,98	—
— i Knop	13	8,07	8,90 % (F ₁)
— i Blomstring	98	6,83	
— helt moden	6	5,28	5,20 % (F ₂)

De fundne Tal for Rødkløvers Askeindhold paa forskellig Tid (F₁ Hø, F₂ Halm) for denne Plante af dansk Avl stemmer forbausende godt med de Wolff'ske Middeltal, der er tilvejebragt med et mange Gange større Materiale af tysk Avl.

¹⁾ E. EBERMAYER: „Physiologische Chemie d. Pflanzen“. Berlin 1882, S. 730.

²⁾ E. WOLFF: „Aschen-Analysen von landwirtschaftlichen Produkten“. Berlin 1871, S. 153—154.

Beregner man Middeltal af Tallene i Tab. 6, finder man at:

Græsarterne indeholder i Høtørstoffet	6,00 %	Aske
— — i Halmtørstoffet	5,78	—
Kløverarterne — i Høtørstoffet	8,66	
— — i Halmtørstoffet	3,82	—

Forskellen i Askeindholdet i unge og gamle Plantedele er altsaa langt større hos Kløverarterne end hos Græsarterne. Særlig ringe Forskel er der mellem Draphavre Hø (B_1) og Draphavre Halm (B_2), samt mellem Hundegræs Hø (D_1) og Hundegræs Halm (D_2). Andre Græsarter viser dog betydelig større Forskel. L. D. HAIGH og P. F. TROWBRIDGE fandt saaledes for Timothégræs¹⁾:

	For Skridning	Efter Skridning	I Blomstring	Modnet
Aske	7,50 %	6,80 %	5,21 %	4,58 %

B. Bestemmelser af nogle af Askebestanddelene.

Herved er benyttet følgende Fremgangsmaade! 30,00 grm lufttørt Stof afvejes paa en god Tareringsvægt, der med 100 grm's Belastning giver Udslag for 0,005 grm. Stoffet forkulles i Platinskaal over Flammespreder, hvorefter det meste Kul bliver borthrændt ved lidt stærkere Varme. Den undertiden endnu noget kulholdige Rest blev udkogt med saltsyreholdigt Vand. Resten blev frafiltreret, glødet i Platindigel og bragt over i den saltsure Opløsning, der blev inddampet til Tørhed. Efter skarp Tørring fugtes med Saltsyre og kogende Vand. Kiselsyre frafiltreres; Filtratet fra Kiselsyren deles i tre ligestore Dele og anvendes paa følgende Maade:

1) En Del til Bestemmelse af Fosforsyre med Ammoniummolybdat ved direkte Vejning af det gule Bundfald efter ganske svag Glødning.

2) I en anden Portion blev Calcium og Magnium bestemt, efter at Fosforsyren var udfældet som Ferrifosfat i eddikesur Væske. Der blev tilsat lidt Ferriklorid, Ammoniumacetat og Eddikesyre og kogt til Bundfaldet var udskilt. Af Filtratet udfældedes om fornødent den sidste Rest af Jernilte med lidt Ammoniak og frafiltreredes. I Filtratet udfældes Calcium som Oxalat og Magnium som Magniumammoniumfosfat. Man maa dog paa Grund af det ringe Magniumindhold og den store Mængde Ammoniumsalte helst inddampe Filtratet fra Calciumoxalatet til Tørhed og bortgløde Ammoniumsaltene. Resten i Platinskaalen opløses i lidt Saltsyre og filtreres fra lidt Platin, der altid dannes i findelt Tilstand, naar Ammoniumklorid afdampes og bortglødes i Platinskaal. Af Opløsningen fældes Magnium paa sædvanlig Maade som Magniumammoniumfosfat.

3) I en tredie Portion blev Kalium bestemt som Kaliumplatinklorid. Fosforsyre, Svovlsyre, Magnium og Jern blev udfældet med en Opløsning af Bariumklorid og Bariumhydroxyd. Overskud af Bariumsalt og tilstedeværende Calciumsalt blev

¹⁾ I „Americ. Chemical Society in Indianapolis“ 1911 efter Jahresb. der Agrikultur-Chemie f. 1911, Berlin 1912, S. 291.

fældet med en Opløsning af Ammoniumkarbonat. Filtratet inddampes, glødes, opløses i saltsyreholdigt Vand, filtreres fra Platinstøv, og Kalium bestemmes paa sædvanlig Maade ved Inddampning med Brintplatinchloridopløsning og Filtrering i en Gooch-Porcellænsdigel; Udvaskning med 92 % Alkohol, Tørring og Vejning.

Ved Udførelsen af disse Undersøgelser er jeg for en meget væsentlig Grad blevet assisteret af min Søn stud. polyt. SVEND RØRDAM. Som man vil se er der ikke i Asken bestemt Ferrioxyd, Aluminiumoxyd eller Svovlsyre, skønt disse Stoffer forekommer, men deres Bestemmelse vilde kræve Fremstillingen af Asken af endnu større Stofmængder, end de anvendte 30 grm lufttørt Stof, og Bestemmelsen turde ikke anses for at være af saa stor Vigtighed, at den vilde lønne det meget betydelige analytiske Arbejde, der maatte anvendes herpaa.

Tabel 8.

Græsarterne:

	Askebestanddele i Procent af Tørstoffet.									
	A ₁	A ₂	B ₁	B ₂	C ₁	C ₂	D ₁	D ₂	E ₁	E ₂
Si O ₂	3,01 %	2,60 %	2,10 %	2,74 %	2,41 %	1,94 %	3,21 %	2,83 %	3,18 %	2,64 %
Ca O	0,62	0,54	0,12	0,47	0,66	0,52	0,51	0,51	0,68	0,64
Mg O	0,16	0,12	0,19	0,19	0,20	0,16	0,26	0,23	0,11	0,12
K ₂ O	1,81	1,38	1,63	1,39	2,19	0,87	1,47	1,48	2,21	1,82
P ₂ O ₅	0,51	0,11	0,12	0,23	0,44	0,10	0,42	0,17	0,58	0,12
Beregnet C O ₂	0,77	0,55	0,54	0,58	0,72	0,59	0,68	0,65	0,64	0,63
I alt	6,87	5,30	5,60	5,60	6,62	4,18	6,55	5,87	7,40	5,97

Tabel 9.

Kløverarterne:

	Askebestanddele i Procent af Tørstoffet.					
	F ₁	F ₂	G ₁	G ₂	H ₁	H ₂
Si O ₂	1,40 %	0,42 %	0,78 %	0,37 %	1,42 %	0,41 %
Ca O	3,35	2,11	1,91	0,97	4,73	1,60
Mg O	0,61	0,70	0,40	0,30	0,39	0,17
K ₂ O	1,01	0,38	0,87	0,37	1,39	0,22
P ₂ O ₅	0,49	0,14	0,38	0,23	0,49	0,16
Beregnet C O ₂	2,91	2,13	1,93	1,09	4,09	1,44
I alt	9,77	6,18	6,27	3,32	12,49	4,00

I disse Analyser er som meddelt ikke gjort særskilte Bestemmelser af Ferrioxyd, Aluminiumoxyd og Svovlsyre, og som en Følge heraf er, som man vil se ved Sammenligning af Tab. 8 med Tab. 6, Summen af de enkelte bestemte Bestanddele i Tab. 8 lidt mindre end Resultaterne af de direkte Askebestemmelser i Tab. 6¹⁾. For Kløverarterne er Forholdet derimod noget anderledes.

¹⁾ A₂ giver dog direkte 5,20 % og ved Summering af Bestanddelene 5,30 %, men som angivet S. 9 under A₂ er der her to Bestemmelser, den ene paa 5,00 %, den anden paa 5,40 %. Den første har sandsynligvis mistet Kulsyre.

Tabel 10.
Askebestemmelse i Kløverarterne.

Hø		Halm	
direkte	ved Summering	direkte	ved Summering
$F_1 = 8,90 \text{ } ^0\text{/}_0$	$9,77 \text{ } ^0\text{/}_0$	$F_2 = 5,20 \text{ } ^0\text{/}_0$	$6,18 \text{ } ^0\text{/}_0$
$G_1 = 6,15$	$6,27$	$G_2 = 3,08$	$3,32$
$H_1 = 10,94$	$12,49$	$H_2 = 3,19$	$4,00$

Ved alle disse Bestemmelser er altsaa den direkte fundne Askemængde mindre end den, der findes ved Summering af de enkelte Bestanddele, skønt der blandt disse mangler Tal for Ferrioxyd, Aluminiumoxyd og Svovlsyre. Dette tilsyneladende anomale Forhold beror paa, at Kløverarternes Aske ved Glødning langt lettere end Græsarternes taber Kulsyre, hvad der direkte kan vises ved nedenstaaende Forsøg.

Kløverhø F_1 , hvor der ved Vejning af den paa sædvanlig Maade under tidligere beskrevne Forsigtighedsregler fremstillede Aske er omtrent 1 $\text{ } ^0\text{/}_0$ mindre end Summen af de enkelte Bestanddele, blev indasket paa sædvanlig Maade og gav en let hvid Aske, som atter blev glødet svagt sammen med Ammoniumkarbonat i lukket Digel. I Asken blev efter Vejning Kulsyren bestemt med SCHEIBLER's Apparat.

Resultaterne var følgende:

F_1 Kløverhø (Sildig Rødkløver).

- 1) 1,2688 grm lufttørt Stof = 1,1624 grm Tørstof gav med Ammoniumkarbonat 0,1203 grm Aske = 10,35 $\text{ } ^0\text{/}_0$ af Tørstof.

Ved Kulsyrebestemmelse i Asken: 2,39 $\text{ } ^0\text{/}_0$ C O_2 af Tørstof.

- 2) 1,3547 grm lufttørt Stof = 1,2412 grm Tørstof gav 0,1265 grm Aske = 10,19 $\text{ } ^0\text{/}_0$ af Tørstof.

Ved Kulsyrebestemmelse i Asken: 2,15 $\text{ } ^0\text{/}_0$ C O_2 af Tørstof.

I Middeltal altsaa fundet $10,27 \text{ } ^0\text{/}_0$ Aske } af Tørstof.
 $2,27 \text{ } ^0\text{/}_0$ C O_2 }

Ved Summering af Bestanddele (Tab. 9), hvori mangler Bestemmelse af $\text{Fe}_2 \text{ O}_3$, $\text{Al}_2 \text{ O}_3$, S O_3 : 9,77 $\text{ } ^0\text{/}_0$ Aske.

Beregnet; 2,19 $\text{ } ^0\text{/}_0$ C O_2 .

Hvilket altsaa viser, at i Asken er noget af Baserne bundet til Kulsyre eller Fosforsyre, som det kunde forudsættes.

Det samme fremgaar af følgende:

H_1 Kløverhø (Gul Rundbælg).

1,5360 grm lufttørt Stof = 1,4467 grm Tørstof gav med Ammoniumkarbonat 0,1842 grm Aske = 12,74 $\text{ } ^5\text{/}_0$ Aske af Tørstof.

Ved Kulsyrebestemmelse i Asken 3,52 $\text{ } ^0\text{/}_0$ C O_2 af Tørstof.

Ved Summering af Bestanddelene, hvori ligeledes mangler Bestemmelse af $\text{Fe}_2 \text{ O}_3$, $\text{Al}_2 \text{ O}_3$, S O_3 12,49 $\text{ } ^0\text{/}_0$ Aske.

Beregnet 4,07 $\text{ } ^0\text{/}_0$ C O_2 .

Hvorfor Kløverarternes Aske lettere taber noget af Kulsyren end Græsarternes Aske, er det næppe muligt at afgøre med Sikkerhed, men det tør maaske formodes, at det beror paa Formen, hvorunder Calcium og Magnium er udskilt i paagældende Planter. Beregner man af Tabellerne 8—9 Middeltallene for S sammensætningen af Hø og Halm henholdsvis af Græsarter og Kløverarter i Almindelighed faas følgende Tal:

Tabel 11.

	Græsarternes Aske		Kløverarternes Aske	
	Procent af Tørstof Hø	Halm	Procent af Tørstof Hø	Halm
Si O ₂	2,81 ⁰ / ₁₀	2,55 ⁰ / ₁₀	1,20 ⁰ / ₁₀	0,43 ⁰ / ₁₀
Ca O	0,58	0,51	3,33	1,56
Mg O	0,18	0,16	0,46	0,39
K ₂ O	1,86	1,39	1,09	0,32
P ₂ O ₅	0,17	0,15	0,12	0,18

Afgrøden af Græs og af Kløver bortfører altsaa ret forskellige Mængder af Plantenæringsstoffer fra Jordbunden, men saavel for Græsserne som for Kløverarterne relativt betydeligt mindre, naar Modningen er indtraadt end tidligere ¹⁾. Beregner man Tørstofindholdet i en Høafgrøde til 6000 kg pr. Hektar, indeholdes der heri:

Tabel 12.

	Græsarternes Hø	Kløverarternes Hø
Si O ₂	170 kg.	72 kg.
Ca O	35 —	200 —
Mg O	11 —	28 —
K ₂ O	112 —	65 —
P ₂ O ₅	28 —	25 —

Foruden den almindelig bekendte Ejendommelighed at Græsarterne er forholdsvis kiselsyrerige, tildrager Forholdet mellem Magnesia og Kalk sig Opmærksomhed:

I Græsarterne	Mg O : Ca O = 1 : 3,2
I Kløverarterne	Mg O : Ca O = 1 : 7,1

Disse Tal kan suppleres fra WOLFF'ske Askeanalyser med følgende ²⁾:

I Vinterhvede	Straa	Mg O : Ca O = 1 : 2,3
I Vinterrug	—	Mg O : Ca O = 1 : 3,1
I Byg	—	Mg O : Ca O = 1 : 3,0
I Havre	—	Mg O : Ca O = 1 : 2,2
I Ærter	—	Mg O : Ca O = 1 : 4,6
I Lupiner	—	Mg O : Ca O = 1 : 4,0

Naar man nu ved, at saagodt som alle Karbonater i dansk Jordbund hidrører fra de under Istiden fint malede Kalksten for største Delen af dansk Oprindelse,

¹⁾ Herved er dog ikke taget Hensyn til de gennem Frøet bortførte Stofmængder.

²⁾ E. WOLFF: Anf. St. Side 155.

vil det være af Interesse at betragte Forholdet mellem Magnesia og Kalk i saadanne Stenarter. Forf. har andetsteds givet Beretning om en Række Undersøgelser af danske Kalksten¹⁾. Forholdet er der beregnet som $MgCO_3 : CaCO_3$, men omregnes det til $MgO : CaO$, vil man se, at i det magnesiafattigste rene hvide Skrivekridt er Forholdet:

$$MgO : CaO = 1 : 151.$$

I den forholdsvis magnesiariige Bryozokalk derimod:

$$MgO : CaO = 1 : 43.$$

I Middeltal er Forholdet mellem Magnesia og Kalk i danske Kalksten omtrent:

$$MgO : CaO = 1 : 100,$$

medens det altsaa i Græsarterne og Kornsorternes Straa er omtrent som $1 : 3^2)$. Det er bekendt, at meget af den dyrkede Jord i Danmark er fattig paa Karbonater. Man vil af ovenstaaende Betragtninger se, at Magnesiaforraadet i Jordbunden er langt snarere udsat for at blive opbrugt end Kalkforraadet, hvis der da ikke tilføres Magnesiumsalte ved Tilførsel af de magnesiumholdige Kaligødninger³⁾.

IV. Bestemmelse af Raafedt.

Bestemmelse af de i Æter opløselige Bestanddele blev udført paa sædvanlig Maade som angivet i „Fælles Arbejdsmetoder“⁴⁾. Herved fremkom følgende Resultater:

Tabel 13.

Græsarterne:		Anvendt Stofmængde		Fundet Fedtmængde		Fedt $\frac{\%}{100}$ af Torstof Middeltal
Stoffets Art		Lufttørt Stof Grm.	Torstof Grm.	Grm.	$\frac{\%}{100}$ af Torstof	
A ₁ Italiensk Rajgræs Hø		12,00	11,110	0,3165	2,85	} 2,97
		12,00	11,110	0,3265	3,08	
A ₂ Italiensk Rajgræs Halm		10,00	9,448	0,1760	1,86	} 1,90
		5,000	4,724	0,0910	1,93	
B ₁ Draphavre Hø		11,00	10,743	0,2475	2,31	} 2,24
		5,000	4,883	0,1058	2,17	

¹⁾ K. RØRDAM: „Geologi og Jordbundslære. II Bd. Danmarks Geologi“. Kbhvn. 1909, S. 440 o. flg.

²⁾ E. SCHULZE & CH. GODET har vist (Zeitschft. f. physiol. Chemie 58 1908, S. 156–161) at Kærnen af Frøet af en Del Træer og urteagtige Planter indeholder betydelig mere Magnesia end Kalk. Forfatterne slutter deraf, at Magnesia maa spille en væsentlig Rolle under Kimplanternes Spiring og Udvikling (Jahresb. d. Agrikultur-Chemie 3. Folge XII 1909, S. 230).

³⁾ Dette Spørgsmaal er i den nyeste Tid udførligt behandlet i et Arbejde af OSCAR LÖEW: „Chaux et Magnésie, dans la plante et dans le sol“, Annales de Gembloux Bruxelles 1912, S. 357 o. flg.

⁴⁾ „Fælles Arbejdsmetoder for Undersøgelser foretaget i Medfør af Lov om Handel med Gødnings- og Foderstoffer af 26de Marts 1898“. Fastsat af Landbrugsministeriet. Kbhvn. 1908.

Tabel 13 (fortsat).

	Anvendt Stofmængde		Fundet Fedtmængde Grm.	Fedtmængde % af Fedt	Fedt % af Tørstof Middeltal
	Lufttørt	Tørstof			
B ₂ Draphavre Halm	10,00	9,855	0,1585	1,61	1,52
	5,000	4,927	0,0701	1,42	
C ₁ Agerhejre Hø	9,000	8,725	0,2430	2,72	2,79
	5,000	4,347	0,1385	2,86	
C ₂ Agerhejre Halm	9,000	8,776	0,1525	1,74	1,70
	5,000	4,875	0,0810	1,66	
D ₁ Hundegræs Hø	7,000	6,498	0,1870	2,81	2,81
	7,000	6,498	0,1955	2,93	
	5,000	4,646	0,1282	2,76	
	5,000	4,646	0,1315	2,83	
D ₂ Hundegræs Halm	7,000	6,374	0,1795	2,97	2,97
	5,000	4,553	0,1345	2,95	
	5,000	4,553	0,1360	2,98	
E ₁ Engsvingel Hø	9,000	8,247	0,1920	2,33	2,22
	5,000	4,581	0,0966	2,11	
E ₂ Engsvingel Halm	10,00	9,571	0,1660	1,73	1,72
	5,000	4,785	0,0815	1,70	

Som det fremgaar af Tab. 13 er Fedtindholdet i Høprøverne gennemgaaende 0,5—1,0 % højere end i Halmen af samme Plante. Hundegræs-Hø og Halm danner dog i saa Henseende en dog ikke meget betydelig Undtagelse.

For at undersøge om dette er et konstant Træk eller maaske beroede paa en Tilfældighed ved Høet og Halmen af Hundegræs af Høst 1911, blev der fra samme Kilde fremskaffet vel-indhøstede Hø- og Halmprøver af Hundegræs af Høst 1912. Disse Prøver viste følgende Forhold:

Tørstofbestemmelse.

Hundegræs-Hø Høst 1912:

1,9845 grm lufttørt Stof gav 1,9394 grm Tørstof = 97,73 %.

Hundegræs-Halm Høst 1912:

2,0200 grm lufttørt Stof gav 1,9570 grm Tørstof = 96,88 %.

Fedtbestemmelse.

Tabel 14.

Stoffets Art	Anvendt Stofmængde		Fundet Fedtmængde Grm.	Fedtmængde % af Tørstof	Fedt % af Tørstof Middeltal
	Lufttørt Stof grm.	Tørstof beregnet grm.			
Hundegræs-Hø Høst 1912	5,000	4,887	0,1055	2,16	2,14
	5,000	4,887	0,1040	2,13	
	5,000	4,887	0,1045	2,14	
Hundegræs-Halm Høst 1912	5,000	4,891	0,0835	1,72	1,70
	5,000	4,891	0,0820	1,69	
	5,000	4,891	0,0825	1,70	

Disse Prøver viser altsaa det normale Forhold — mere Fedt i Hø end i Halm — hvorfor den forholdsvis store Mængde Æterekstrakt i Halmprøverne fra 1911 (D_2) maa bero paa en eller anden Tilfældighed, som det ikke er let at udrede. Beregner man af Tab. 13 Middeltallene af Fedtindholdet for Hø- og Halmprøverne, faas

Middeltal af Raafedtprocent af Tørstof Høst 1911:

Hø 2,61 $\frac{0}{0}$, Halm 1,71 $\frac{0}{0}$ ¹⁾)

Kløverarterne blev behandlet paa samme Maade som Græsarterne og gav følgende analytiske Resultater:

Tabel 15.

Kløverarterne:		Anvendt Stofmængde		Fundet Fedtmængde		Fedt $\frac{0}{0}$ af Tørstof Middeltal
Stoffets Art		Lufttørt Stof Grm.	Tørstof beregnet Grm.	Grm.	$\frac{0}{0}$ af Tørstof	
F ₁ Sildig Rødkløver Hø		12,00	10,990	0,1565	1,29	} 4,40
		12,00	10,990	0,1835	1,52	
F ₂ Sildig Rødkløver Halm		8,00	7,359	0,1825	2,48	} 2,30
		5,00	4,599	0,0980	2,13	
G ₁ Kællingetand Hø		10,00	9,724	0,4165	4,42	} 4,46
		5,00	4,862	0,2285	4,70	
		5,00	4,862	0,3075	4,27	
G ₂ Kællingetand Halm		9,00	8,888	0,1805	2,03	} 1,97
		5,00	4,938	0,0945	1,91	
H ₁ Gul Rundbælg Hø		5,00	4,709	0,1220	2,59	} 2,55
		5,00	4,709	0,1185	2,51	
H ₂ Gul Rundbælg Halm		5,00	4,894	0,0570	1,17	} 1,14
		5,05	4,894	0,0510	1,12	

I Kløverarterne er der altsaa omtrent dobbelt saa meget Fedt i Høet som i Halmen. Sildig Rødkløver-Hø og Kællingetand-Hø er langt fedtrigere (i Middeltal 4,43 $\frac{0}{0}$ mod 2,61 $\frac{0}{0}$) end Græsarternes Hø, medens der ikke er stor Forskel i Fedtmængde mellem disse Planters Halm (i Middeltal 2,13 $\frac{0}{0}$ mod 1,71 $\frac{0}{0}$) og Græsarternes Halm. For Gul Rundbælgs Vedkommende er baade Hø og Halm fedtfattigere end Middeltallet af Græsarternes Hø og Halm.

De angivne Mængder af Fedt er som det vil vides „Raafedt“, det vil sige Resten, der tilbagebliver ved Fordampning af Æteren, der har været anvendt til Extraktion af Prøverne i SOXLETH's Apparat. Det udvundne Raafedt er altsaa stærkt grønt farvet af Klorofyl og følgelig ikke rent og ligeledes blandet med, hvad Planterne har indeholdt af voksagtige Stoffer, men der haves ingen praktisk Metode til at skelne mellem disse forskellige Bestanddele i Raafedt, selv om det ved Præparation af større Mængder Raafedt vel nok er muligt at komme Spørgsmaalet noget nærmere. Et Par enkelte Smaaforsøg kan dog nævnes som værende vistnok de første af denne Art overfor disse Stoffer. Ved Titring med vinaandig $\frac{0}{10}$ Natron

¹⁾ Heri er den tvivlsomme D_2 udeladt, medtages den, bliver Middeltallet 1,96 $\frac{0}{0}$.

søgte bestemt Mængden af fri fede Syrer i Raafedt i fire af de mindst farvede Fedtprøver. Stor Nøjagtighed kan ikke opnaas, da det ikke er let at se Overgangen fra sur til neutral Reaktion paa Grund af Fedtopløsningens Farve, men med en Nøjagtighed af omtrent 0,5 % blev ved gentagen Titration fundet følgende Tal:

Tabel 16.

Fri Fedtsyre beregnet som Oliesyre i Procent af Raafedt.

D ₁ Hundegræs Hø	23,2 %	E ₁ Engsvingel Hø	27,6 %
D ₂ Hundegræs Halm	19,9 %	E ₂ Engsvingel Halm	25,5 %

Saavidt der overhovedet af disse faa Prøver kan dømmes noget, indeholder Høet mere „fri Fedtsyre“ end Halmen, men fremtidige Undersøgelser maa vise, hvorvidt dette er et konstant Træk hos „Raafedt“ fra saadanne Plantedele.

V. Bestemmelse af de kvælstofholdige Stoffer.

Disse Bestemmelser er udført af cand. polyt. Frk. JOHANNE WILLE og cand. polyt. Frk. AUGUSTA LUND, Assisterter ved Landbohøjskolens agrikulturkemiske Laboratorium, som jeg skylder megen Tak for den Omhyggelighed, hvormed dette Arbejde blev udført. Bestemmelserne er foretaget som angivet i „Fælles Arbejds-metoder“ S. 9—10 dels som Bestemmelse af »Total Kvælstof« dels som „Æggehvidekvælstof“ (STUTZER's Metode, se Anf. Std.), hvoraf atter ved Subtraktion lader sig beregne Mængden af „Amidkvælstof“. De angivne Tal er Resultatet af gentagne veloverensstemmende Forsøg. Af Æggehvidekvælstof er beregnet Ren-æggehvide ved Multiplikation med Faktoren 6¹⁾.

Tabel 17.

Græsarterne.		Procent i Tørstoffet:			
Stoffets Art		Total Kvælstof	Æggehvide Kvælstof	Amid Kvælstof	Beregnet Renæggehvide
A ₁	Italiensk Rajgræs Hø	0,99	0,85	0,14	5,10
A ₂	— Halm	0,42	0,33	0,09	1,98
B ₁	Draophavre Hø	1,03	0,90	0,13	5,40
B ₂	— Halm	0,54	0,40	0,14	2,40
C ₁	Agerhejre Hø	1,20	0,83	0,37	4,98
C ₂	— Halm	0,38	0,35	0,03	2,10
D ₁	Hundegræs Hø	1,06	0,91	0,15	5,46
D ₂	— Halm	0,61	0,56	0,05	3,36
E ₁	Engsvingel Hø	1,17	0,97	0,20	5,82
E ₂	— Halm	0,44	0,36	0,08	2,16

¹⁾ Smlg. 58de Beretning fra d. kgl. Veterinær- og Landbohøjskoles Laborat. for landøkonom. Forsøg. Kbhvn. 1905, S. 73

For Høets Vedkommende ligger altsaa Indholdet af Renæggehvide mellem 4,98 $\frac{0}{10}$ og 5,82 $\frac{0}{10}$ i Middeltal 5,35 $\frac{0}{10}$ af Tørstof. For Halmens Vedkommende er Grænserne for Æggehvideindholdet 1,98 $\frac{0}{10}$ —3,36 $\frac{0}{10}$ med et Middeltal af 2,38 $\frac{0}{10}$. Der er altsaa gennemsnitlig mere end dobbelt saa meget Renæggehvide i Hø som i Halm.

Der foreligger, saavidt vides, ingen Undersøgelser, der af de fundne Mængder Amidkvælstof tillader at beregne de tilsvarende Mængder af Amidstoffer, der sikkert nok ikke er noget enkelt Stof, men en eller flere Stofgrupper¹⁾. Beregnes af de fundne Tal i Tab. 17 Forholdet mellem Amidkvælstof og Æggehvidekvælstof, bliver disse:

Amidkvælstof : Æggehvidekvælstof i Hø.

A ₁ Italiensk Rajgræs Hø	1 : 6,0	} Middeltal 1 : 6
B ₁ Draphavre	- 1 : 6,9	
D ₁ Hundegræs	- 1 : 6,1	
E ₁ Engsvingel	- 1 : 4,5	

Altsaa Forholdstal, der er hinanden ret nærliggende, medens Proven C₁ Agerhejre Hø af ubekendte Grunde giver et helt afvigende Forholdstal nemlig 1 : 2,3.

For Halmprøvernes Vedkommende er der næppe nogen Regelmæssighed at spore. Ganske vist er for

A ₂ Italiensk Rajgræs Halm	1 : 3,7	} Middeltal 1 : 3,7
B ₂ Draphavre	1 : 2,9	
E ₂ Engsvingel	— 1 : 4,5	

men for

C ₂ Agerhejre	— 1 : 12
D ₂ Hundegræs	— 1 : 11

Nærmere kan man næppe komme Sagen, førend Spørgsmaalet om Amidstoffernes Art tages op til indgaaende Undersøgelse ved Anvendelse af meget store Mængder rent og ensartet Materiale²⁾. Man tør dog efter de her foreliggende Undersøgelser udtale, at Amidstoffernes Mængde baade i Hø og Halm af de undersøgte Græsarter er saa ringe, at den praktisk talt ikke kan spille nogen som helst Rolle for Husdyrenes Ernæring.

Da Udførelsen af den kvantitative Bestemmelse af Æggehvidekvælstof efter STUTZER's Metode er ret omstændelig og besværlig, hvorimod en simpel Bestemmelse af Totalkvælstofmængden efter KJELDAHL's Metode let lader sig udføre i Løbet af nogle Timer, vilde det være af stor Betydning, hvis man kunde for Praksis Vedkommende erstatte Bestemmelsen af Æggehvidekvælstof med Bestemmelsen af Totalkvælstof. Dette synes at være Tilfældet for Oliekagers Vedkommende³⁾, og som det her skal vises, kan det ogsaa lade sig gøre for Græsarternes Hø og Halm. Forholdstallet, som maa benyttes, naar man af den fundne Mængde Æggehvide-

¹⁾ Jfr. anførte 58de Beretning fra Forsøgslaboratoriet S. 58—59.

²⁾ Smstd. S. 58.

³⁾ Smlg. FR. CHRISTENSEN & G. JØRGENSEN: „Benyttelse af Loven af 26de Marts 1898. Tidsskrift for Landøkonomi 1912 S. 608 o. flg.

kvælstof vil beregne Renæggevidemængden i Hø og Halm, er, som omtalt, 6. Vil man benytte Totalkvælstofprocenten som Middel til at finde Mængden af Renæggehvide, maa der naturligvis benyttes et andet Forholdstal. Ved at dividere den i Tab. 17 fundne Procentmængde Totalkvælstof ind i Procenttallet i samme Tabel for Renæggehvide findes følgende Forholdstal:

Tabel 18.

Renæggevideprocent: Totalkvælstofprocent.

Hø		Halm	
A ₁ ...	5,15	A ₂ ...	4,71
B ₁ ...	5,23	B ₂ ...	4,44
C ₁ ...	4,15	C ₂ ...	5,53
D ₁ ...	5,15	D ₂ ...	5,51
E ₁ ...	5,00	E ₂ ...	4,91
Middeltal	4,91		5,02

Udelader man Prøven C₁, der som før omtalt helt igennem, hvad Kvælstoffet angaar, viser anomale Forhold, bliver Middelfaktoren for Hø 5,10. I Middeltal baade for Hø og Halm tilsammen bliver Faktoren 5,06, som kan bruges. Men det er rigtigere at benytte Faktoren 5,1 for Hø og 5,0 for Halm. Benyttes disse Faktorer til Beregning af Renæggevideprocenten, bliver de saaledes fundne Tal i Sammenligning med Tallene fundne ved Bestemmelse af Æggehvidekvælstof følgende:

Tabel 19.

Renæggevide.

	A ₁	B ₁	D ₁	E ₁	A ₂	B ₂	C ₂	D ₂	E ₂
Af Totalkvælstof	5,05 ⁰ / ₁₀	5,25 ⁰ / ₁₀	5,11 ⁰ / ₁₀	5,27 ⁰ / ₁₀	2,10 ⁰ / ₁₀	2,70 ⁰ / ₁₀	1,90 ⁰ / ₁₀	3,05 ⁰ / ₁₀	2,22 ⁰ / ₁₀
Af Æggehvidekvælstof.	5,10 ⁰ / ₁₀	5,40 ⁰ / ₁₀	5,16 ⁰ / ₁₀	5,82 ⁰ / ₁₀	1,98 ⁰ / ₁₀	2,10 ⁰ / ₁₀	2,10 ⁰ / ₁₀	3,36 ⁰ / ₁₀	2,16 ⁰ / ₁₀
Differents	÷ 0,05 ⁰ / ₁₀ ÷ 0,15 ⁰ / ₁₀ ÷ 0,05 ⁰ / ₁₀ ÷ 0,15 ⁰ / ₁₀ ÷ 0,12 ⁰ / ₁₀ ÷ 0,30 ⁰ / ₁₀ ÷ 0,20 ⁰ / ₁₀ ÷ 0,31 ⁰ / ₁₀ ÷ 0,06 ⁰ / ₁₀								

De fundne Differentser mellem Æggehvide beregnet af Totalkvælstof og Æggehvide beregnet af Æggehvidekvælstof ligger altsaa mellem + 0,30 ⁰/₁₀ og ÷ 0,31 ⁰/₁₀. Hvis Æggehvideprocenten var Resultatet af en almindelig kvantitativ analytisk Bestemmelse maatte man blive betænkelig ved disse Grænser for Afvigelsen, men da som anført Æggehvideprocenten er fremkommet ved at multiplicere det analytiske fundne Tal for Renæggehvide med 6, maa mulige analytiske Forsøgsfejl ogsaa i Renæggehvideprocenten være multipliceret med 6, saa at den analytiske Usikkerhed altsaa har været ± 0,05 ⁰/₁₀. I Betragtning af Materialets Art kan Fordringer til Nøjagtighed næppe stilles strengere ved saadanne Kvælstofbestemmelser.

Det synes altsaa, at man ved Beregning af Renæggehvide i Hø og Halm af Græsser af nævnte Art i Stedet for den vidtløftige Bestemmelse af Æggehvidekvælstof kan til praktisk Brug benytte den langt simplere Bestemmelse af Totalkvælstof. Faktoren, hvormed

der skal multipliceres for at finde Renæggehvideprocenten er for Hø 5,1, for Halm 5,0¹⁾.

Fremtidige Undersøgelser af Hø og Halm fra andre Aargange vil vise om dette holder Stik, men følgende skal dog anføres: I tidligere anførte 58de Beretning fra Forsøgslaboratoriet findes S. 58 anført Middeltal af 17 Analyse af Totalkvælstof og Æggehvidekvælstof i „Hø“ og „Halm“.

Hø: Totalkv. 1,448 $\frac{0}{10}$ Æggehvidekv. 1,266 $\frac{0}{10}$ } beregnes heraf Ægge-
Halm: — 0,702 $\frac{0}{10}$ — 0,632 $\frac{0}{10}$ }

hvidestof med de omtalte Faktorer henholdsvis 5,1 og 6,0 for Hø og 5,0 og 6,0 for Halm findes følgende:

	Æggehvidestof af Totalkv.	Æggehvidestof af Æggehvidekv.	Diff.
Hø	7,38 $\frac{0}{10}$	7,60 $\frac{0}{10}$	\div 0,22 $\frac{0}{10}$
Halm	3,51 $\frac{0}{10}$	3,79 $\frac{0}{10}$	\div 0,28 $\frac{0}{10}$

Altsaa ogsaa her kan man af Totalkvælstofprocenten beregne Indholdet af Renæggehvide ved Multiplikation med de nævnte Faktorer.

Kløverarterne blev underkastet Analyse paa samme Maade som Græsarterne; Resultaterne var følgende:

Tabel 20.

Kløverarterne: Procent i Tørstoffet.

Stoffets Art	Total Kvælstof	Æggehvide Kvælstof	Amid Kvælstof	Beregnet Renæggehvide
F ₁ Sildig Rødkløver Hø	3,01	2,60	0,41	15,60
F ₂ — Halm	1,34	1,20	0,14	7,20
G ₁ Kællingetand Hø	2,54	2,08	0,46	12,45
G ₂ — Halm	1,21	1,05	0,16	6,30
H ₁ Gul Rundbælg Hø ²⁾	2,54	1,66	0,88	9,96
H ₂ — Halm	0,69	0,51	0,18	3,06

For Kløverarternes Vedkommende ligger altsaa Indholdet af Renæggehvide mellem 9,96 $\frac{0}{10}$ og 15,60 $\frac{0}{10}$ af Tørstoffet. Der er 2–3 Gange saa meget Æggehvidestof i de undersøgte Kløverhøprøver som i de undersøgte Høprøver af Græsarterne.

I Kløverarternes Halm varierer Æggehvideprocenten mellem 3,06 $\frac{0}{10}$ og 7,20 $\frac{0}{10}$ og er altsaa i Gennemsnit betydelig større end i de undersøgte Græsarters Halm.

Mængden af Amidkvælstof er i Kløverhalmen ikke væsentlig forskellig fra samme Stof i Græsarternes Halm, medens Amidkvælstoffet i Kløverhøet er til Stede i forholdsvis stor Mængde, nemlig i Rødkløverhø 0,41 $\frac{0}{10}$, i Kællingetandhø 0,46 $\frac{0}{10}$ og

¹⁾ At man til videnskabelig Brug er nødt til at bestemme baade Æggehvidekvælstof og Amidkvælstof naar det gælder særlige Undersøgelser af disse Stoffer, er en Selvfølge.

²⁾ Gul Rundbælg Hø, der viser anomal Højde i Amidkvælstofprocenten, er senere blevet underkastet fornyet Analyse. Som Middeltal af flere Bestemmelser (Examensanalyser) blev fundet: Totalkvælstof 2,52 $\frac{0}{10}$, Æggehvidekvælstof 1,69 $\frac{0}{10}$, Amidkvælstof 0,83 $\frac{0}{10}$, saa at ovenanførte Tal altsaa maa anses for rigtige.

i Gul Rundbælghø 0,88⁰ o, medens Middeltallet for Amidkvælstof i Græsarternes Hø (med Udeladelse af den anomale C₁) er 0,15⁰ o. De for Græsarterne fundne Faktorer, 5,1 for Hø og 5,0 for Halm, ved hvilke man kunde finde Renæggehvideprocenten af Totalkvælstofprocenten viser, naar de benyttes overfor Kløverarterne, følgende Forhold (H₁, Gul Rundbælghø, er udeladt):

Tabel 21.

Æggehvideprocent i Tørstof.			
	Beregnet af Totalkvælstof	Beregnet af Æggehvidekvælstof	Different-
F ₁	15,31	15,60	÷ 0,29
F ₂	6,70	7,20	÷ 0,50
G ₁	12,95	12,48	+ 0,47
G ₂	6,05	6,30	÷ 0,25
H ₂	3,45	3,06	+ 0,29

Usikkerheden er altsaa her ved Kløverarterne noget større, nemlig $\pm 0,5^{0/0}$, i Renæggehvideprocenten, end den var ved Græsarterne ($\pm 0,3^{0/0}$). En Variation i Renæggehvideprocenten af $\pm 0,5^{0/0}$ svarer omtrent til en Variation i selve de analytiske Bestemmelser af Æggehvidekvælstof paa c. $\pm 0,08^{0/0}$. Der maa dog betydelig flere Undersøgelserækker til, før man med Sikkerhed tør udtale sig om, hvorvidt en Bestemmelse af Totalkvælstof ved Kløverarterne i Almindelighed kan afløse Bestemmelsen af Æggehvidekvælstof ved Beregning af Renæggehvide.

V. Bestemmelse af Kulhydraterne.

A. Bestemmelse af Sukker.

Efter forskellige foreløbige Forsøg blev Sukkerbestemmelsen udført paa følgende Maade. Det viste sig nødvendigt at bruge Vinaand til Sukkerextraktionen, saaledes som almindeligvis anvendes ved Roer og lignende sukkerholdige Plantedele¹⁾. Benyttelsen af Vinaand ved Extraktionen forhindrer, at der i Opløsningen gaar noget af de dextrin- og stivelseholdige Plantedele, som maaske findes i Høet, som vilde delvis opløses ved Kogning med Vand. Derimod foraarsager Benyttelsen af Vinaand, at Opløsningen kommer til at indeholde dels Fedt dels Klorofyl og maaske andre Farvestoffer fra Høet, men disse Stoffer kan som det vil ses, senere let skilles fra det opløste Sukker. Extraktionen foregik paa følgende Maade: En afvejet Mængde af det fintmalede lufttørre Stof, i Reglen 5,00 grm, blev udkogt med c. 300 cm³ 92⁰ o holdig Vinaand, filtreret gennem et løst Filter og udvasket med varm Vinaand. Der viste sig her ikke at være nogen Fordel ved at benytte SOXLETH's Extraktionsapparat, da en fuldstændig Extraktion af Sukkeret paa an-

¹⁾ Smlg. O. T. CHRISTENSEN og C. F. A. TUXEN: „Øvelser i kvantitativ og agrikulturkem. Analyse“. 2den Udg. ved O. T. Christensen og K. Rørdam. Kbhvn. 1905. S. 81.

førte Maade krævede c. 2 Timer, medens Extraktionen ved Benyttelsen af Soxleth's Apparat kræver mindst 24 Timer inden den afløbne Væske er farveløs. Til den vinaandholdige Extrakt sættes ca. 50 cm.³ Vand, og Vinaanden afdestilleres. Remanensen i Kolben, der er stærkt farvet, fortyndes med lunkent Vand og filtreres fra udskilt Fedt og Klorofyl, der i Reglen sætter sig fast paa Kolbens Sider. Filtratet, der nu kun er lidt gulligt farvet, fyldes efter Afkøling op til 250 cm.³. Heraf vil altsaa 100 cm.³ svare til 2,000 grm lufttørt Stof. Bestemmelsen af Sukker i Extrakten udføres efter den vægtanalytiske Inversionsmetode. 100 cm.³ Extrakt anbringes i en ERLÉNMEYER's Kolbe og der tilsættes 10 cm.³ fort. Saltsyre. Kolben hensættes i kogende Vandbad i 30 Minutter, neutraliseres med Natron, hvorefter der tilsættes 30 cm.³ Fehlings Væske. Der opvarmes atter i 30 Minutter i kogende Vandbad. Cuprooxydet frafiltreres under Sugning paa et 10 cm askefrit Filter, udvaskes med kogende Vand, tørres, forkulles forsigtigt, glødes tilsidst i en Iltstrøm, afkøles i Exsikkator og vejes som Cuprooxyd¹⁾. Heraf beregnes Kobbermængden og der fradrages saameget Kobber som svarer til den Mængde Kobberilte et Filter af den angivne Størrelse erfaringsmæssigt tilbageholder. Af Kobbermængden beregnes Mængden af Invertsukker ved Hjælp af KJELDAHL's Tabeller²⁾.

De nedenfor i Tab. 22 og 23 angivne Duplikatbestemmelser er udført med en for hvert Tilfælde særlig tilberedt Extraktmængde, altsaa ikke ved Gentagelse af Forsøget i en ny Portion af samme Extrakt. Man faar derved, som det vil ses, ganske vist ikke altid saa nøje overensstemmende Bestemmelser, som kan erholdes ved at udføre Duplikatbestemmelser i samme Extrakt, men Sandsynligheden for at ramme det rigtige Resultat maa siges at være større.

Tabel 22.

Sukkerbestemmelse i Græsarterne.						
Stoffets Art		Kobber		Invertsukker		
2,000 grm. lufttørt Stof giver:		Gram	Gram	Procent i Tørstof	Procent i Tørstof. Middeltal	
A ₁	Italiensk Rajgræs Hø	—	—	—	11,50 ³⁾	
A ₂	—	Halm	0,1820	0,0950	5,03	5,18
	—	—	0,1914	0,1007	5,32	
B ₁	Drophavre Hø	—	0,3115	0,1614	8,26	8,28
	—	—	0,3129	0,1622	8,30	
B ₂	—	Halm	0,2410	0,1205	6,12	6,13
	—	—	0,2415	0,1208	6,13	
C ₁	Agerhejre Hø	—	0,1911	0,1005	5,15	5,08
	—	—	0,1857	0,0973	5,00	
C ₂	—	Halm	0,0581	0,0279	1,43	1,38
	—	—	0,0541	0,0260	1,33	

¹⁾ Smgl. PRAGER i „Zeitschr. für angew. Chemie“, 1894, S. 520 og E. v. LIPPMANN: „Die Chemie d. Zuckerarten“. 3te Aufl. Braunschweig 1904, S. 597.

²⁾ J. KJELDAHL: „Undersøgelser over Sukkerarternes Forhold mod alkaliske Kobberopløsninger“. Medd. fra Carlsberg Laborat. IV Bd. 1. Kbhvn. 1895.

³⁾ Angaaende Analyser af dette Stof se det efterfølgende.

Tabel 22 (forsat).

Stoffets Art		Kobber		Invertsukker	
2,000 grm. lufttørt Stof giver:		Gram	Gram	Procent i Tørstof	Procent i Tørstof, Middeltal
D ₁ Hundegræs Hø	—	0,0418	0,0199	1,07	} 1,08
—	—	0,0422	0,0201	1,08	
D ₂ — Halm	—	0,0522	0,0250	1,40	} 1,32
—	—	0,0469	0,0225	1,23	
E ₁ Engsvingel Hø	—	0,0433	0,0207	1,13	} 1,17
—	—	0,0465	0,0223	1,21	
E ₂ — Halm	—	0,0473	0,0227	1,18	} 1,17
—	—	0,0462	0,0221	1,15	

Som det fremgaar af Tab. 22 er Sukkerindholdet i Græsarterne meget vekslende. Italiensk Rajgræs Hø indeholder 11,50 % og omtrent det halve 5,18 % i Halmen. Draphavre og Agerhejre Hø har ogsaa et stort Sukkerindhold, medens Hundegræs og Engsvingel indeholder ligelidt Sukker, ca. 1 % i Hø og Halm.

En virkelig indgaaende Undersøgelse af Sukkerarterne i Græsserne vil kræve, at det friske ikke til Hø eller Halm tørrede Materiale først underkastes Undersøgelse, da det er meget sandsynligt, at Sukkerarterne forandres ved Stoffets Lagring, saaledes som det er paavist for andre Plantedeles Vedkommende, særlig for Roer¹⁾.

Naar Sukkeret i Tab. 22 er angivet som Invertsukker, ligger der ikke i denne Angivelse, at Sukkerarten i paagældende Plantedele virkelig er tilstede udelukkende som Rørsukker eller andre Polysaccharider, der har været opløst i Vinaanden og derefter ved Saltsyrebehandlingen er blevet hydrolyseret og beregnet som Invertsukker. Det er tvertimod sandsynligt, at største Delen af Sukkeret er tilstede som en direkte reducerende Sukkerart. Vinaandsextrakterne af paagældende Plantedele viser nemlig stærk Reduktion med Fehling's Væske uden at være inverterede. Men efter at Reduktionen er fuldendt ved Opvarmning i 30 Minuter paa kogende Vandbad med Fehling's Væske i Overskud og Frafiltrering af det udskilte Cuprooxyd, viser Ekstrakterne, naar de derefter inverteres med Saltsyre og neutraliseres paany, tydelig Sukkerreaktion. Forholdet er derfor ret kompliceret og maa, som nævnt, gøres til Genstand for et særskilt Studium, helst paa friske Plantedele.

For „Italiensk Rajgræs“, der er saa paafaldende stærkt sukkerholdigt, blev der anstillet en særlig Undersøgelserække, idet der blev bestemt baade, a) hvormeget Sukker der fandtes i Ekstrakterne som direkte reducerende Sukkerarter og b) hvormeget Sukker der opstod af hele Ekstrakten ved Inversion paa førangivne Maade. Tillige blev der bestemt c) hvormeget Invertsukker der fandtes i Filtratet fra det direkte reducerende Sukker, naar dette Filtrat blev inverteret og neutraliseret. Disse Undersøgelser blev ogsaa gennemført for en Prøve af Italiensk Rajgræs Hø af Høst 1912, der iøvrigt blev behandlet ganske paa samme Maade som Prøven

¹⁾ Anførte 58de Beretning fra Forsøgslaboratoriet S. 123 o. flg.

af samme Plantearts Hø fra 1911. Prøverne maa siges at vise god Overensstemmelse, naar henses til, at de er fra forskellige Aargange.

Tabel 23.

A ₁ Italiensk Rajgræs Hø, Høst 1911, med 92,61 % Tørstof i det lufttørre Stof.						
	Anvendt Stof lufttørt	Anvendt Fehling's Væske	Fundet Cu. Gram	Invertsukker Gram	Invertsukker Procent af Tørstof	Middeltal
Ekstrakten ikke inverteret	2,000 ¹⁾	50 cm ³	0,3436	0,1809	9,77	9,63
	1,000	30 —	0,1756	0,0912	9,84	
	1,000	30 —	0,1735	0,0899	9,70	
	1,000	30 —	0,1690	0,0873	9,42	
	1,000	30 —	0,1695	0,0876	9,45	
Ekstrakten inverteret	2,000	80 —	0,3934	0,2138	11,54	11,50
	2,000	80 —	0,3912	0,2123	11,46	
Italiensk Rajgræs Hø, Høst 1912, med 93,13 % Tørstof i det lufttørre Stof.						
Ekstrakten ikke inverteret	2,000	50 cm ³	0,3282	0,1716	9,21	9,30
	2,000	50 —	0,3338	0,1750	9,39	
Ekstrakten inverteret	2,000	80 —	0,4166	0,2298	12,34	12,42
	2,000	80 —	0,4207	0,2328	12,49	

I Filtratet fra Cuprooxydbundfaldet fra den ikke inverterede Extrakt fandtes efter Inversion

Hø af Høst 1911 i Tørstof 1,46 %	Invertsukker
— 1912 — 2,92 %	—

Lægges disse Bestemmelser til de respektive Bestemmelser som de ikke inverterede Extrakter gav, nemlig henholdsvis 9,77 % (Hø 1911) og 9,21 % (Hø 1912), faas for Høet 1911 — 11,23 % og for Hø 1912 — 12,13 %. Begge disse Tal er lidt mindre end de ved Inversion af Extrakten fundne Tal, der som angivet var for Hø 1911 — 11,50 % og for Hø 1912 — 12,42 %, hvilket var at vente efter de gentagne Behandlinger i skiftevis sure og alkaliske Væsker. Differensen mellem Sukkermængden fundet ved den dobbelte Behandling og Sukkermængden fundet ved Inversion af hele Extrakten udgør for Høet fra 1911 — 0,27 % og for Høet fra 1912 — 0,29 %.

At „Italiensk Rajgræs“ med Hensyn til Indholdet af Sukkerarter i forskellig Henseende frembyder Interesse fremgaar ogsaa af andre Undersøgelser. Saaledes har E. SCHULZE paavist at af *Lolium italicum*, der blev høstet grønt efter at have naaet en Højde af 30 cm uden at have blomstret, kan udtrækkes dels noget Rørsukker dels et andet Polysaccharid højst sandsynligt identisk med det af SCHULZE & FRANKFURT opdagede Secalose C₁₈H₃₂O₁₆, som nævnte Forf. tidligere havde isoleret af den vinaandige Extrakt af grøn Rug, grøn Havre m. fl.²⁾ Secalose er ogsaa senere undersøgt af JESSEN-HANSEN.³⁾ Dette Kulhydrat kan krystallisere,

¹⁾ De tre første Bestemmelser er fra Portioner af samme Extrakt, de to sidste fra en anden Extrakt.

²⁾ E. SCHULZE: Zeitschrift f. physiolog. Chemie, Bd. 27. Strassburg 1899. S. 286 o. flg.

³⁾ H. JESSEN-HANSEN: Medd. fra Carlsberg Laboratoriet IV. 1896. S. 145 o. flg. Smlg. E. v. LIPPMANN l. c. S. 1669.

smager stærkt sødt, er „venstredrejende“ og giver ved Hydrolyse med Saltsyre α -Fruktose. Men andre Sukkerarter forekommer i større Mængder i Rajgræs Ekstrakten. De foreliggende Bestemmelser, der er anført i Tab. 23, viser, at de direkte reducerende Sukkerarter i Høet baade af Høst 1911 og 1912 var tilstede i en Mængde svarende til 9,63—9,30 $\frac{0}{10}$ Invertsukker af Tørstoffet, medens de Sukkerarter, der først efter Inversion med Saltsyre reagerer med Fehling's Væske, kun svarede til henholdsvis 1,87 $\frac{0}{10}$ og 3,12 $\frac{0}{10}$ Invertsukker.

Anm. Et lille Gæringsforsøg blev udført med Italiensk Rajgræs Hø, 20 grm finmalet lufttørt Hø blev udrørt med c. 300 grm kogende destilleret Vand, omrystet og afsvalet til 37°. Ved Tilsætning af lidt ren Pressegær og Henstand i 3 Døgn ved almindelig Stuetemperatur paa en med Gærspunds lukket Kolbe var Væsken i Kolben kommet i ret kraftig Gæring. Efter Frafiltrering af det uopløste dannede Filtratet en klar gullig Væske med karakteristisk Lugt og Smag som tyndt hjemmebrygget Øl.

Fra Ekstrakterne af andre Græsarter saasom *Phleum pratense* (Rottehal), *Triticum repens* (Kvikrod) og Arter af *Trisetum* (Guldhavre) har man isoleret hinanden nærstaaende Kulhydrater ($nC_6H_{10}O_5$ med mer eller mindre Vand), som har faaet Navnene Phlein, Triticin, Graminin, der alle ved Hydrolyse ligesom Secalose kun giver Fruktose¹⁾. Det er sandsynlig at disse Kulhydrater ogsaa er tilstede i de her behandlede Ekstrakter, men Reaktionen for dem er ikke synderlig karakteristiske.

For Kløverarternes Vedkommende blev der udført Sukkerbestemmelser paa samme Maade som for Græsarterne. Resultaterne var følgende:

Tabel 24.

Sukkerbestemmelse i Kløverarterne.				
Stoffets Art	Kobber	Invertsukker		Middeltal
2,000 grm lufttørt Stof giver	Gram	Gram	Procent i Tørstof	Procent i Tørstof
F ₁ Sildig Rødkløver Hø	0,1108	0,0548	2,78	2,84 $\frac{0}{10}$
—	0,1154	0,0573	2,90	
F ₂ — Halm	0,0220	0,0110	0,60	0,58 $\frac{0}{10}$
—	0,0217	0,0103	0,56	
G ₁ Kællingetand Hø	0,0764	0,0307	1,90	1,90 $\frac{0}{10}$
—	Ved Titration	—	1,89	
G ₂ — Halm	0,0204	0,0102	0,52	0,46 $\frac{0}{10}$
—	Ved Titration	—	0,40	
H ₁ Gul Rundbælg Hø	0,0402	0,0192	1,02	0,99 $\frac{0}{10}$
—	Ved Titration	—	0,95	
H ₂ — Halm	0,0093	0,0042	0,22	0,28 $\frac{0}{10}$
—	Ved Titration	—	0,33	

Rødkløver Hø er altsaa det sukkerrigeste med 2,84 $\frac{0}{10}$ Sukker i Tørstof, men naar dog ikke nær op paa Højde med de forholdsvis sukkerrige Græsarter, som Italiensk Rajgræs, Draphavre og Agerhejre. Halmen af Kløverarterne er meget fattig paa Sukker 0,58 $\frac{0}{10}$ —0,28 $\frac{0}{10}$, hvis da overhovedet den fundne Kobbermængde skyldes Sukkerreduktion og ikke andre forekommende maaske reducerende Stoffer.

¹⁾ Smlg. E. v. LIPPMANN l. c. S. 804 o. flg.

P. SCHWEITZER har gjort Sukkerbestemmelse i forskellige Planter (Majsblade og Stængler, Thimotégræs, „Bluegrass“ (Rapgræs) og i Rødkløver)¹⁾. Han finder beregnet for det askefri Tørstof i

	Procent Invertsukker
Rødkløver i Blomst	3,86 ⁰ / ₁₀
— i fuld Modning	4,58 ⁰ / ₁₀

Men da hans Sukkerbestemmelser er udført ved at invertere det vandige Udtræk af Plantedelene, der indeholder forskellige Stoffer (Pektinstoffer, dextrinagtige Stoffer, opløselig Stivelse m. m.), som ved Invertering med Syre vil kunne give Stoffer, der kan reducere Fehling's Væske, kan man ikke sammenligne hans Bestemmelser med de her udførte, hvorved i det væsentlige kun de i Vinaand opløselige Bestanddele kommer til at reagere.

B. Bestemmelse af Pentosaner.

At disse Stoffer kan forekomme i betydelige Mængder i Hø og Halm er en bekendt Sag. E. v. LIPPMANN har givet en Oversigt over de foreliggende Undersøgelser, af hvilke det fremgaar, at der i Enggræsser findes 15,44—18,22 ⁰/₁₀ Pentosaner, i Timotégræs 15,33—21,07 ⁰/₁₀, i Hø kan derimod ved Gæring Mængden gaa ned til 0,5 ⁰/₁₀. Betydelig større Mængder, op imod 30 ⁰/₁₀, findes i Kornsorternes Straa²⁾. Pentosanernes Værdi for Husdyrenes Ernæring er noget omstridt og synes at veksle med Mængden af andre for Fordøjelsen lettere tilgængelige Stoffer³⁾. I de her omhandlede Prøver viser der sig at være Pentosanmængder af lignende Størrelser som ovenfor anført for henholdsvis Græsser og Kløverarter, altsaa et langt større Indhold af Pentosan end af Sukker, og disse Stoffers relative Mængde synes ikke at staa i noget som helst Forhold til hinanden.

Pentosanbestemmelsen blev udført paa den sædvanlige Maade ved at destillere en afvejede Stofmængde med 12 ⁰/₁₀ holdig Saltsyre under stadig Tilsætning af Syre af samme Styrke under Destillationen. Herved overdestilleres sammen med Vanddampene en til Pentosanmængden ækvivalent Mængde Furfurol, der kan bestemmes paa forskellige Maader. Furfurolen i Destillatet blev fældet med Floroglucin opløst i 12 ⁰/₁₀ holdig Saltsyre. Fældningsmidlet og Destillatet udgjorde 400 cm³, og det udfældede Floroglucid blev frafiltreret paa et tørret og vejet Filter efter 18 Timers Henstand. Efter Udvaskning tørres ved 100° og vejes i lukket Glas. Af Floroglucidets Vægt kan Vægten af Furfurol og Pentosan beregnes efter KRÖBER's Tabeller⁴⁾. Udførlig Diskussion af Metoden findes forskellige Steder let tilgængelig

¹⁾ P. SCHWEITZER: „Investigations of the bodies called fiber and carbohydrates in feeding-stuffs“. Journ. of the American chemical society. Vol. XXVI Easton 1904 S. 252 o. flg.

²⁾ E. v. LIPPMANN l. c. S. 53—54.

³⁾ — — S. 1830.

⁴⁾ S. KÖNIG: „Die Untersuchung landwirtsch. und gewerbl. wichtiger Stoffe“. 1906 S. 243. KRÖBER's Originalangivelser findes i „Journal für Landwirtschaft“ 1900 S. 357 o. flg.

og vil være overflødig her¹⁾. Kun bør maaske fremhæves, at den Opløsning af Anilinacetat, der benyttes til at paavise om Furfuroloverdestilleringen er ophørt, maa være frisk tilberedt for at give sikre Resultater. Ved Kogning med Saltsyren bliver Høet og Halmen temmelig stærkt angrebet, saa at Remanensen i Kolben efter Destillationen er brunfarvet. Frafiltreres den uopløste Rest, vil den brunfarvede Væske vise sig at være stærkt sukkerholdig. Sukkermængden udgør i Følge nogle faa Forsøg 15—20 % af det anvendte Stof og maa antages at være fremkommet ved den varme Saltsyres hydrolyserende Indvirkning paa Cellulosen og andre Kulhydrater i Plantedelene. I alt Fald giver Cellulose fremstillet af disse, som det senere skal vises, ved lignende Behandling, en lignende Sukkermængde.

Tabel 25.

Pentosanbestemmelse i Græsarterne.

Stoffets Art	Anvendt Mængde lufttørt Stof	Fundet Mængde Floroglucid	Pentosaner		I Middeltal
	Gram	Gram	Gram	i Procent af Torstof	Procent af Torstof
A ₁ Italiensk Rajgræs Hø	1,0270	0,2075	0,1882	19,78	19,89
— —	1,6025	0,3322	0,3038	20,00	
A ₂ — Halm	1,5560	0,2825	0,2539	17,27	17,54
— —	1,0438	0,1930	0,1756	17,81	
B ₁ Draphavre Hø	0,9980	0,2305	0,2085	21,44	21,39
— —	1,3705	0,3170	0,2848	21,28	
— —	1,3730	0,3192	0,2868	21,46	
B ₂ — Halm	1,2955	0,3145	0,2856	22,14	22,02
— —	1,2885	0,3085	0,2772	21,78	
— —	0,9300	0,2240	0,2028	22,13	
C ₁ Agerhejre Hø	1,1268	0,2370	0,2111	19,60	18,99
— —	0,8870	0,1735	0,1590	18,50	
— —	0,8695	0,1690	0,1546	18,87	
C ₂ — Halm	1,3923	0,3240	0,2910	21,44	20,88
— —	0,7770	0,1700	0,1541	20,38	
— —	0,8685	0,1940	0,1764	20,83	

¹⁾ Smlg. Forsøgslaboratoriets 58de Beretning, S. 134 o. flg.

Anm. Efter at Arbejdet var udført er følgende ny Modifikation i Metoden ved Furfurolfældningen fremkommet uden at der har været Lejlighed til at prøve dem:

- K. H. BÖDENER & H. TOLLENS har foreslaaet en Forandring i Metoden ved Floroglucinfældningen — varm i Stedet for kold Udfældning — som skal give endnu bedre Resultater (Jahresber. der Agrikultur-Chemie 3. XIV. 1912 S. 624).
- M. ISBECQUE har (Annales de Gembloux 21 Bruxelles 1912 S. 347) gennemprøvet det af JÄGER & UNGER foreslaaede Fældningsmiddel for Furfurol, Barbitursyre og fundet at det giver næsten ganske samme Resultater, som der faas ved Fældning med Floroglucin. Barbitursyre-Fældning skal være nok saa bekvem som Floroglucinfældningen.
- I Følge ISBECQUE (anf. Std.) skal BRAUNS (Pharmaceutisch Weekblad 46 S. 326) have vist, at man kan skælné mellem de forskellige Klasser af de Furfurol-givende Stoffer. De egentlige Pentosaner giver Furfurol allerede ved Destillation med 2 % Saltsyre, hvorimod Furfuroiderne kræver den stærkere Saltsyre (12 %) for at afgive Furfurol. Holder dette Stik ved nærmere Prøvelse, maa det siges at være et meget væsentlig Fremskridt i analytisk Henseende.

Tabel 25 (fortsat).

Stoffets Art	Anvendt Mængde lufttørt Stof	Fundet Mængde Floroglucid	Pentosaner		I Middeltal
	Gram	Gram	Gram	i Procent af Tørstof	Procent af Tørstof
D ₁ Hundegræs Hø	0,8845	0,1925	0,1752	21,35	20,37
— —	0,9188	0,1877	0,1710	20,06	
— —	0,7975	0,2010	0,1826	19,72	
D ₂ — Halm	1,0000	0,1585	0,1152	15,91	15,81
— —	1,0000	0,1573	0,1442	15,81	
— —	1,0000	0,1562	0,1432	15,73	
E ₁ Engsvingel Hø	1,4295	0,3195	0,2870	21,91	20,77
— —	1,0200	0,2030	0,1834	19,73	
— —	1,3190	0,2780	0,2499	20,68	
E ₂ — Halm	1,0970	0,2280	0,2063	20,53	20,55
— —	1,3320	0,2917	0,2621	20,56	

Pentosanmængden ligger altsaa mellem 15,84 ⁰/₁₀₀ og 22,02 ⁰/₁₀₀. Nogen Regel for Mængden af Pentosaner i de til hinanden svarende Hø- og Halmprøver synes der ikke at være, og Forskellen er gennemgaaende kun ringe. Ser man bort fra Prøven af Hundegræs Halm, hvor Pentosanmængden sandsynligvis af en eller anden Grund er anomal lav, bliver Middeltallene af samtlige øvrige Bestemmelser for:

Hø i Tørstof 20,35 ⁰/₁₀₀ Pentosan.
Halm — 20,25 ⁰/₁₀₀ —

Det er af ikke ringe Interesse at DE CHALMOT har paavist et ganske lignende Forhold hos en lang Række af forskellige Træarter. Der er hos dem saagodt som ingen Forskel i Mængden af Furfurolgivende Stof i unge og gamle Træer af samme Art, snarest var der her ligesom i de undersøgte Græsarter en lille Formindskelse af det Furfurolgivende Stof med Alderen. Saaledes fandtes for Træ af Eg og Platan¹⁾:

<i>Quercus nigra</i>	Furfurol- procent	<i>Platanus occidentalis</i>	Furfurol- procent
2 — 12 Aar gammel	10,6	1—10 Aar gammel	9,8
69 —	10,5	71—79 —	9,7
109—110 —	10,3		

Men andre Træarter viste lidt varierende Forhold.

For Kløverarternes Vedkommende blev Resultaterne af Pentosanbestemmelserne følgende:

¹⁾ DE CHALMOT: Amer. Chem. Journ. 16 S. 225.

Smgl. CROSS & BEVAN: „Cellulose“ 2nd Edit. London 1903 S. 183—184, hvor der gaas nærmere ind paa Spørgsmaalet Pentosaner ctr. Furfurolgivende Stof.

Tabel 26.

Pentosanbestemmelsen i Kløverarterne.

Stoffets Art	Anvendt Mængde	Fundet Mængde	Pentosaner		Middeltal
	lufttørt Stof	Floroglucid	Gram	Procent af Tørstof	Procent af Tørstof
F ₁ Sildig Rødkløver Hø	1,2142	0,1050	0,0979	8,81	8,61
— —	0,8580	0,0685	0,0661	8,41	
F ₂ — Halm	0,8101	0,1065	0,0984	12,73	12,51
— —	0,9340	0,1165	0,1080	12,29	
G ₁ Kællingetand Hø	1,6375	0,1900	0,1723	10,85	
G ₂ — Halm	1,3710	0,2475	0,2233	16,49	
H ₁ Gul Rundbælg Hø	1,5040	0,1750	0,1598	11,28	
H ₂ — Halm	1,4005	0,2410	0,2176	16,24	

Kløverhø er altsaa fattigere paa Pentosaner end Kløverhalmen. Kællingetand og Gul Rundbælg har næsten fuldkommen samme Pentosanindhold i henholdsvis Hø- og Halmprøverne af disse Plantearter, og det blev derfor anset for uforholdsmæssigt at gøre Duplikatbestemmelser her, medens Rødkløver baade i Hø og Halm indeholder noget mindre end de to andre. Middeltallet af alle disse Prøver bliver for

Høet	10,37 %	Pentosan i Tørstof
Halmen	15,08 %	— — —

Skønt der er for faa Bestemmelser til at man kan tillægge disse Middeltal altfor udstrakt almindelig Betydning, er det dog øjensynligt, at Kløverarterne er betydelig mindre pentosanhaltende end Græsarterne.

C. Bestemmelse af Cellulose.

Paa Grund af den overordentlig store tekniske Betydning, som Cellulose i mer eller mindre ren Form har til Anvendelse ved Papirfabrikationen, ved Fremstilling af Textilstoffer, Sprængmidler og andet teknisk Brug, er de cellulosegivende Stoffer og Cellulosens tekniske og kemiske Forhold Æmner, der er behandlet i en uhyre omfattende Litteratur. Bortset fra de mangfoldige Tidsskrifter, Specialværker og lignende, der behandler de mere tekniske Detailler, har man foruden i de forskellige store Haandbøger i organisk Kemi god Vejledning i den bekendte CROSS & BEVAN: „Cellulose, an outline of the structural elements of plants“. 2nd. edit. London 1903 (med forskellige Tillæg). Hertil er i Løbet af afvigte Aar kommet den særdeles udførlige og omfattende CARL G. SCHWALBE: „Die Chemie der Cellulose“. (Berlin 1911 665 S.).

Skønt der saaledes skulde synes at være Celluloseundersøgelser nok til Vejledning, tør det dog siges, at de forskellige ved agrikulturkemisk Undersøgelse af Foderstoffer gangbare Metoder, hvorved man søger at faa et Indhold af „Raacellulose“ (Rohfaser), „Træstof“ eller lignende ubestemt betegnede Stoffer, er temmelig utilfredsstillende af flere Grunde. Man kan til en vis Grad sige, at Aarsagen hertil er, at man fra den første Begyndelse af disse Undersøgelser gik ud fra en Teori,

for hvilken man ikke havde noget egentligt Bevis. Man antog, at paagældende Plantestoffer indeholdt Cellulose i en saadan Form, at den mer eller mindre fuldkomment lod sig rense ved de benyttede Opløsningsmidler for „Urenhederne“¹⁾. Men hvad enten man anvendte HENNEBERG's Metode (Weende-Metoden), hvor man benytter skiftevis Kogning med Svovlsyre og Kaliopløsning, eller G. LANGE's Metode ved Kogning eller Smeltning med højst koncentreret Kaliopløsning, eller SCHULTZE's Metode med Behandling med Vand, Vinaand, Æter, Salpetersyre, Kaliumklorat o. s. v., faar man et Produkt af et meget uanseligt oftest meget brunfarvet Udseende — maaske lidt bedre ved Schultze's Metode end ved de andre Fremgangsmaader — men i hvert Fald langt fra den Hvidhed, som man i alle Fald ved er ejendommelig for den noget hypotetiske Cellulose, som man har søgt at fremstille af de paagældende Plantedele. De almindelige agrikulturkemiske Fremgangsmaader fører altsaa ikke til Maalet: Bestemmelse af Cellulose, da det, man faar fremstillet, øjensynligt ikke er ren Cellulose, og det næppe lader sig afgøre eller i hvert Fald ikke er afgjort i hvilket Forhold Produktet staar til Cellulose. C. G. SCHWALBE omtaler dette Spørgsmaal paa følgende Maade²⁾: „Üeber die Chemie der zusammengesetzten Cellulosen der Gräser ist noch wenig bekannt, so gewaltig auch das Tatsachenmaterial ist, welches von Agrikulturchemikern über diese landwirtschaftlich so wichtige Familie von Gewächsen zusammengetragen worden ist. Schon seit vielen Jahrzehnten werden Heu und Stroharten auf ihren Gehalt an Rohfaser, an Fett, an stickstoffhaltigen und stickstofffreien Extraktivstoffen hin untersucht. Viel weiter ist man aber in Erkenntnis der genannten Stoffe noch nicht vorgeschritten, auch bleibt es unentschieden, ob man sich diese verschiedenartigen Substanzen als Gemenge, als Adsorptionsverbindung oder als chemische Verbindung zu denken hat“.

V. STORCH udtalte sig i 1905 efter en lang Række Forsøg paa agrikulturkemiske Cellulosebestemmelser væsentlig paa samme Maade³⁾: „Det vil af det foregaaende være fremgaaet, at man ved den kemiske Analyse af Foderstoffer har stødt paa saare store Vanskeligheder ved at finde det virkelige Indhold af Æggehvide-stoffer, samt at disse Vanskeligheder endnu kun til dels kan siges at være overvundne. Vi skal nu søge at vise, at Bestemmelsen af en anden af Foderstoffernes og for deres Vurdering vigtig Bestanddel, nemlig Cellestoffet, frembyder endnu større Vanskeligheder ja endog saa store, at det maa erkendes, at der for nærværende Tid ikke gives nogen Metode, ved hvilken Foderstoffernes Indhold af rent Cellestof kan bestemmes selv blot tilnærmelses-

¹⁾ Metoderne til Bestemmelse af Cellulose findes udførligt beskrevne i Forsøgslaboratoriets 58de Beretning, i de før nævnte Hovedværker af CROSS & BEVAN og C. G. SCHWALBE, i de almindelige Haandbøger i agrikulturkemisk Analyse m. fl. Std. Som en særlig klar og omhyggelig gennearbejdet og tillige let tilgængelig Fremstilling, der ganske vist tager noget mere Sigte paa Teknikens end paa Agrikulturkemiens Krav, kan fremhæves MAX RENKER: „Über Bestimmungsmethoden der Cellulose“ 2te Aufl. Berlin 1910.

²⁾ Anførte Haandbog: „Die Chemie der Cellulose“. S. 385 o. flg.

³⁾ Anførte 58de Beretning fra Forsøgslaboratoriet S. 79. Fremhævelsen foretaget af K. R.

vis nøjagtigt". Det maa altsaa anses for udelukket, at man ved de almindelig kendte agrikulturkemiske Fremgangsmaader til Cellulosebestemmelse kunde vente et tilfredsstillende Resultat og Forf.'s egne ganske vist ikke nær saa omfattende Erfaringer gik i samme Retning. Det kunde derfor ligge nær at prøve en eller anden af de tekniske Fremgangsmaader, hvorved man af et givet Raastof fremstiller Cellulose som rent Bomuld (Sygevat), Cretonne, bleget Hørfiber, Filtrepapirmasse og lignende, der i alt Fald i Retning af Hvidhed, ofte ogsaa ved det ringe Askeindhold, staar langt over de Produkter, der fremstilles ved de agrikulturkemiske Metoder.

Nu vides det, at den blegede Cellulose, der findes i saadanne teknisk fremstillet hvide Produkter er i Besiddelse af forskellige Egenskaber, der i kemisk Henseende tyder paa, at det fremstillede Stof ikke er den egentlige overfor næsten alle Reagenser reaktionsfri Cellulose, der sikkert nærmest maa være en polyvalent Alkohol. Ganske vist kan Cellulose af Bomuld opløses i konc. Svovlsyre under Dannelse af Æthersvovlsyrer ligesom andre Alkoholer, og ved Fortynding og Kogning hydrolyseres disse kvantitativt til Dextrose¹⁾, hvad der vel kan tyde paa en forholdsvis simpel Sammensætning, men VIGNON har ved Destillation med Saltsyre faaet betydelige Mængder Furfurol af bleget Bomuld²⁾, hvad der maa hidrøre fra Pentosaner eller Furfuroider, og SCHWALBE har i Overensstemmelse hermed faaet betydelig Reaktion ved at koge samme Stof (Sygevat) med Fehling's Væske³⁾, saa at selv Bomuldscellulosen, som maa anses for en af de reneste Former, er af kompleks Sammensætning, eller indeholder snarere endnu nogle Urenheder. I endnu højere Grad gælder dette Cellulose fremstillet af Træ eller af Straa, der ikke kan opløses i konc. Svovlsyre uden dyberegaaende Dekompositioner og delvis Forkulning, og ved Fortynding og Kogning kun giver en meget ringere Mængde Dextrose end den i Arbejde tagne Cellulose⁴⁾.

Blandt de forskellige nærmest til teknisk Brug afpassede Metoder kan overfor Hø og Halm være Tale om at anvende enten CROSS & BEVAN's Klormetode eller H. MÜLLER's Brommetode. Den sidste viste sig at være den bekvemteste at anvende og gav, som det vil ses af det følgende, forholdsvis vel overensstemmende Resultater ved Duplikatbestemmelser. Den udvundne Cellulose er i Reglen ganske hvid, oftest helt snehvid, af et meget tiltalende ensartet Udseende. At det ogsaa i kemisk Henseende er temmelig ensartede Produkter, der udvindes af de forskellige undersøgte Græsarter, lader sig vel bevise, men tillige fremgaar det af Undersøgelserne, at det ikke er den egentlige Cellulose $nC_6H_{10}O_5$, der bliver isoleret paa denne Vis. Det ligger ganske vist uden for dette Arbejdes Plan nøjere at fordybe sig i de forskellige Cellulosearters vanskelige Kemi⁵⁾, men det

¹⁾ CROSS & BEVAN: „Cellulose“ S. 49.

²⁾ VIGNON: Compt. rend. 126 (1898) S. 1355.

³⁾ SCHWALBE; Zeitschrift f. angew. Chemie 1907, S. 2172.

⁴⁾ CROSS & BEVAN: S. 49. Smgl. S. 85.

Smstd. S. 313 o. flg.

er dog lykkedes med nogenlunde Sikkerhed at faa fastslaaet, hvad det er for et Stof, der bliver fremstillet ved Brommetoden.

Der kan først henvises til, at Cross & Bevan allerede tidligere har vist, at: „The monocotyledonous fibre-agregates (Esparto, Bamboostems, Sugarcane) are largely made up of pectocelluloses with a greater or less proportion of lignocelluloses“¹⁾.

P. SCHWEITZER er kommet Spørgsmaalet nærmere ved sine sammenlignende Undersøgelser af Cellulose i Majsblade og -Stængler, Thimotégræs, Rajgræs og Kløver og viser, at Brommetoden giver et betydeligt højere Udbytte af Cellulose end den ellers i U. S. almindeligt benyttede O. A. C. Metode²⁾. Han oplyser, at Grunden hertil er, at „Pektosen forbliver i Forbindelse med Cellulosen“ ved denne Metode, saa at man altsaa herved ifølge Schweitzer isolerer et af de i paa-gældende Planter forekommende celluloseholdige Molekulkomplekser, nemlig Pektocellulose³⁾.

Naar man nu nærmere vil gaa ind paa Spørgsmaalet om, hvilken Celluloseart de her undersøgte Planter giver ved Brombehandlingen, er der visse Forhold, der maa fremdrages. Foreløbigt kan vi for Kortheds Skyld kalde den fremstillede Celluloseart for Bromcellulose i Lighed med Sulfitcellulose o. l.

Kemisk ren Cellulose $n C_6 H_{10} O_5$ bestaar af 44,45 % C, 6,17 % H, 49,38 % O, og giver ikke Furfurol ved Destillation med Saltsyre. Den kan opløses i konc. Svovlsyre uden Sønderdeling og giver, som nævnt, ved Fortynding og Kogning Dextrose i den til Kulstofindholdet svarende Mængde. Bromcellulose har, som det skal vises, et noget ringere Kulstofindhold, giver ret meget Furfurol ved Destillation med Saltsyre og kan ganske vist opløses i konc. Svovlsyre omtrent uden at farves før efter længere Tids Henstand, men giver efter Hydrolyse kun omtrent $\frac{1}{3}$ af den til Kulstofindholdet svarende Dextrose, (eller andre Sukkerarter beregnet som Dextrose efter Reduktionsevnen). Bromcellulose er altsaa ikke ren Cellulose.

Man maa derefter formode, at Bromcellulose enten er Lignocellulose, Pektocellulose, Oxycellulose (et kunstigt frembragt Iltningsprodukt) eller maaske en Blanding af to eller tre af disse Stoffer.

For Lignocellulose menes i Almindelighed, at man i dens Forhold overfor Reagenserne Floroglucinsaltsyre og en svag svovlsur Opløsning af Anilinsulfat har sikre Kendetegn. Bromcellulose viser ingen Reaktion umiddelbart ved at behandles med disse Reagenser. Først efter længere Tids Henstand bliver Floroglucinsaltsyre noget gulfarvet, og Opløsningen af Anilinsulfat farves saagodt som ikke selv efter tolv Timers Henstand med Bromcellulose.

¹⁾ CROSS & BEVAN: S. 220.

²⁾ „Official and provisional methods of analysis“, „Association of official agricultural chemistry“, U. S. Dep. of agricultur. Bureau of chemistry, Bull. 107. Ed. by H. W. WILEY Wash. 1908 S. 56. Metoden bestaar i Extraktion med Alkohol, Æter og skiftevis Kogning med Svovlsyre og Natronlud.

³⁾ SCHWEITZER: Journ. of the American chem. soc. Vol. XXVI S. 251—262. Angaaende nærmere Undersøgelse af Pektocellulose henviser S. til en tidligere Afh. i Ann. Rep. of the Missouri Agr. Expt. Stat. 1898. Denne Afh. har ikke været tilgængelig.

Bromcellulose kan derfor i hvert Fald kun indeholde meget lidt Lignocellulose.

Oxycellulose af noget forskellig Beskaffenhed kan, som bekendt, fremstilles af Cellulose eller celluloseholdigt Stof ved Behandling med helst varme kraftige Iltningsmidler saasom Salpetersyre og Kaliumklorat, stærke Opløsninger af Kaliumpermanganat, intensiv Klorering eller Bromering m. m.¹⁾. Bromcellulose bliver ogsaa fremstillet ved Hjælp af Iltningsmidler om end paa langt lemfældigere Maade, men det var dog paa Forhaand ikke udelukket, ja vel endog sandsynlig, at Bromcellulosen maatte indeholde Oxycellulose. I flere Henseender har Bromcellulose Reaktionen tilfælles med Oxycellulose, men adskiller sig dog fra dette Stof i saa væsentlige Forhold, at det maa anses for udelukket, at Bromcellulose og Oxycellulose er samme Stof. Oxycellulose giver ved Kogning med Fehling's Væske Reduktion og ved Destillation med Saltsyre Furfurol. Den farver ved Kogning med fortyndet Natronopløsning Væsken karakteristisk skinnende guldgul og kan i Kulden uden at farves helt opløses i konc. Natronopløsning. Opløsningen udskiller ved Kogning en Del af den opløste Cellulose. Bromcellulose forholder sig paa samme Maade overfor Fehling's Væske, men de kvantitative Forhold er anderledes med Hensyn til de udvundne Furfurolmængder, og ligeledes er Kulstofprocenten i Bromcellulose højere end i Oxycellulose, som det fremgaar af det følgende. Yderligere er Bromcellulosen saagodt som uopløselig i Natron og farves ikke ved Kogning med fortyndet Natron. Ved langvarig Kogning med konc. Natron (25 $\frac{0}{10}$) dekomponeres Stoffet noget under Opløsning af brunfarvede Humusstoffer.

Der foreligger ikke faa Analyser af Præparater af Oxycellulose, men kun forholdsvis faa af disse kan gøre Fordring paa at have været rene ensartede Produkter. Et af de reneste var utvivlsomt den af FABER & TOLLENS af Bomuld ved kraftig Iltning fremstillede Oxycellulose²⁾. De angiver at beregnet for det tørre askefri Stof vindes der ved Destillation med Saltsyre 1,5–1,8 $\frac{0}{10}$ Furfurol. Ved Elementæranalyse finder de S sammensætningen i to Forsøg at være

$$\text{C} = \begin{array}{c} 42,29 \frac{0}{10} \\ 42,01 \frac{0}{10} \end{array} \quad \text{H} = \begin{array}{c} 6,15 \frac{0}{10} \\ 6,10 \frac{0}{10} \end{array} \quad \text{O} = \begin{array}{c} 51,56 \frac{0}{10} \\ 51,89 \frac{0}{10} \end{array}$$

og beregner deraf en Formel $\text{C}_6 \text{H}_{10} \text{O}_5 + \text{C}_6 \text{H}_{10} \text{O}_6$ (eller et Multiplum) med 42,35 $\frac{0}{10}$ C, 5,88 $\frac{0}{10}$ H, 51,78 $\frac{0}{10}$ O.

NASTUKOFF har fremstillet Oxycellulose af Filtrepapir ved grundig Iltning med Kaliumpermanganat og fandt i to Forsøg³⁾

$$\text{C} = \begin{array}{c} 42,12 \frac{0}{10} \\ 42,26 \frac{0}{10} \end{array} \quad \text{H} = \begin{array}{c} 6,20 \frac{0}{10} \\ 6,11 \frac{0}{10} \end{array}$$

¹⁾ C. G. SCHWALBE: *Chemie d. Cellulose*. S. 291 o. flg.

²⁾ FABER & TOLLENS: *Ber. d. D. chem. Ges.* **32** (1899) S. 2592.

³⁾ NASTUKOFF *Ber. d. D. chem. Ges.* **33** (1900) S. 2237–2243. Ved svagere Iltning faar N. derimod Produkter med 43,13–43,78 $\frac{0}{10}$ C, som er Blandinger af upaavirket Cellulose og Oxycellulose.

Pektocellulose har efter SCHWEITZER¹⁾ en Sammensætning, der kan udtrykkes ved Formlen $C_{43}H_{74}O_{37}$ med 43,64 % C, 6,34 % H, 50,02 % O, altsaa ret afvigende fra Oxycellulose, som den iøvrigt ogsaa skiller sig fra ved med Saltsyre at give langt mere Furfurol. Af Italiensk Rajgræs Hø fremstilledes paa nedenangivne Maade med Brommetoden en Bromcellulose, som indeholdt 2,40 % Aske, men iøvrigt i alle sine Forhold var et ensartet rent Produkt.

Beregnet paa det tørre askefri Stof fandt cand. polyt. Frk. JOH. WILLE følgende Sammensætning:

a) 0,1007 grm askefri Bromcellulose gav:	b) 0,1046 grm askefri Bromcellulose
0,1637 — $C O_2 = 43,96\%$ C	0,1679 — $C O_2 = 43,78\%$ C
0,0545 — $H_2 O = 6,04\%$ H	0,0582 — $H_2 O = 6,18\%$ H.

I Middeltal altsaa 43,87 % C, 6,11 % H, 50,02 % O.

Denne Bromcellulose giver ved Destillation med Saltsyre 7,16 % Furfurol og kan saa godt som fuldstændigt opløses i Kobberilteammoniak (efterlader kun 0,51 % uopløst). Reaktionerne overfor Floroglucinsaltsyre, Anilinsulfat og Natron, samt den kemiske Sammensætning og Furfurolmængden, der vindes ved Destillation med Saltsyre, viser, at Bromcellulose ikke er identisk med Oxycellulose, men at den temmelig nøje stemmer overens med Schweitzer's Pektocellulose, der er fremstillet paa lignende Maade af nærstaaende Planter.

En anden Bromcellulose fremstillet af Hundegræs Halm blev ogsaa undersøgt. Den indeholder kun 0,5 % Aske, men to Opløsningsforsøg i Kobberilteammoniak efterlod henholdsvis 4,16 % og 4,03 % i Middeltal 4,10 % uopløst. Cellulosen indeholder tillige 0,11 % Kvælstof utvivlsomt henhørende til det uopløste Stof, som S. KÖNIG & F. MURDFIELD har givet Navnet Kutin²⁾, der er kulstofrigere end Cellulose. Den kvantitative Analyse af Bromcellulosen, som ogsaa blev udført af Frk. JOH. WILLE, synes ogsaa at tyde herpaa. Som Middeltal af to Analyser blev fundet

44,21 % C, 6,01 % H, 49,67 % O (0,11 % N).

Denne Bromcellulose gav i to Forsøg 8,62 % og 8,44 % Furfurol. Hvad Sammensætning den rene Bromcellulose her vil have, naar Kutinet fradrages, lader sig ikke beregne med Nøjagtighed, men den vil næppe afvige meget fra den foregaaende.

Anm. Kutin angives at indeholde c. 68 % C, 10 % H, 22 % O. Benyttes disse Tal og antages et Indhold af 4,0 % Kutin (uden Hensyn til Kvælstoffet) faas en Sammensætning af den kutinfri Cellulose som nedenfor anført:

C = 43,3 %
H = 5,9 %
O = 50,8 %

altsaa meget nær ved den rene Pektocelluloses Sammensætning. Men Kutinets Sammensætning er ikke sikkert fastslaaet.

¹⁾ SCHWEITZER *anf. Std.*

²⁾ J. KÖNIG: „Die Untersuchung landwirtschaftlich und gewerblich wichtiger Stoffe“. 3te Aufl. Berlin 1906. S. 1057.

Bromcellulose saaledes som den her foreligger er derfor utvivlsomt Pektocellulose med et lille Indhold af Kutin, et Indhold, som lader sig bestemme ved at behandle Pektocellulosen med Kobberilteammoniak, som det senere skal vises. I denne Henseende stemmer det udvundne Stof ogsaa med SCHWEITZER's Pektocellulose, som ved Opløsning i samme Reagens efterlod i Middeltal 3,28 $\frac{0}{10}$ ¹⁾.

Hvorvidt den her fremstillede Pektocellulose er ganske identisk med Pektocellulose af Hør, som CROSS & BEVAN anser for den egentlige Type paa dette Stof²⁾, har der ikke været Lejlighed til at afgøre. Udover den kvantitative Sammensætning og den Omstændighed, at den ikke opløses i Natron, og at der fremkommer temmelig meget Furfurol ved Saltsyredestillation, er der ikke mange karakteristiske særlige Kendetegn for Pektocellulose. Hvad Sammensætningen angaar, maa det endda bemærkes, at BUNCKE & WOLFFENSTEIN³⁾ ved svagere Iltning af Filtrepapir med Brintoverilte har fremstillet et Produkt, som de kalder Hydracellulose (for at betone Aldehyd Karakteren) som angives nærmest at have Formlen $6C_6H_{10}O_5 + H_2O$, hvad der svarer til 43,64 $\frac{0}{10}$ C, 6,26 $\frac{0}{10}$ H, 50,10 $\frac{0}{10}$ O, altsaa praktisk talt ganske Sammensætning som Pektocellulose. 1 grm Hydracellulose angives at kunne reducere 15 cm³ Fehling's Væske, hvad der svarer til 0,132 grm Cu, saa at Kobbertallet (se det efterfølgende) er omtrent det samme for Hydracellulose som for Pektocellulose. Men Hydracellulosen giver, saavidt det fremgaar af den foreliggende ufuldstændige Litteratur, ikke Furfurol med Saltsyre, men hydrolyseres.

Der er endnu et Forhold, som bør fremhæves til Gunst for Brommetodens Anvendelse ved Cellulosebestemmelser i agrikulturmæssig Øjemed. Det er lykkedes Forf. ved samme Fremgangsmaade, som blev benyttet ved de her omtalte Foderplanter, baade af Ko- og Hestegødning i store Mængder at isolere et Stof, der af Kogødning er ganske snehvidt, af Hestegødning hvidt med gullig Tone, et Stof, der saavidt Undersøgelserne rækker, er fuldkommen identisk med Pektocellulosen fremstillet af Foderplanterne. Pektocellulosen fremstillet af Kogødning er et ganske ensartet, paa naturlig Vis ved Dyrenes Tygning m. m. finmalet Stof, medens det af Hestegødning ikke er nær saa vel bearbejdet og ensartet. Man kunde i dette Stof endnu skelne Ledknuderne, ganske vist i bleget og sønderknust Tilstand.

Man tør maaske heraf slutte, at Pektocellulosen er ufordøjelig for de anførte Husdyr, men en nærmere Undersøgelse af disse Forhold har ligget uden for dette Arbejdes Plan. Det vil være af ikke ringe Betydning, hvis det viser sig, at man kan benytte Brommetoden til at bestemme Pektocellulosen baade i Foderplanter og i Dyrenes Gødning.

Brommetoden ved Bestemmelse af Cellulose er beskrevet paa ikke væsentlig forskellig Maade dels i H. MÜLLER's originale Angivelse⁴⁾ dels hos

¹⁾ SCHWEITZER anf. Std.

²⁾ CROSS & BEVAN anf. Std. S. 218. Hos Hør er „the fibre proper a pectocellulose“.

³⁾ BUNCKE & WOLFFENSTEIN: „Ueber Cellulose“. Ber. d. D. chem. Ges. 32 II (1899) S. 2493 o. flg.

⁴⁾ Gengivet efter M. RENKER's anførte Afhandling S. 50.

SCHWALBE¹⁾ m. fl. Std. Den blev efter forskellige foreløbige Forsøg udført paa følgende Maade:

En afvejte Mængde af det malede lufttørre Stof overhældes i et rummeligt Pulverglas med indleben Prop med Bromvand. Der indtræder straks en Reaktion, hvorved Bromet absorberes og Stoffet i kendelig Grad affarves. Under jævnlig Tilning af lidt stærkt Bromvand omrystes Massen og henstaar til næste Dag. Om fornødent tilsættes mere Bromvand, og naar Reaktionen i Reglen efter et eller halvandet Døgn Forløb er tilendebragt, bringes Massen, der indeholder Brom i Overskud, paa Filter og udvaskes først med koldt, derefter meget grundigt med varmt Vand²⁾. Dernæst overhældes med fortyndet Ammoniakvand (5 % holdigt), og den nu sortebrune Masse udvaskes med varmt Vand. Herved er den blevet næsten helt affarvet. Har Brombehandlingen første Gang været tilstrækkelig, vil en fornyet Behandling af en udtaget Prøve og bagefter stedfunden Ammoniakbehandling ikke i væsentlig Grad frembringe nogen mørk Farve. Den efter Ammoniakbehandlingen og paafølgende Udvaskning paa Filtret værende Masse overhældes med 2 % holdig Opløsning af Kaliumpermanganat og udvaskes grundigt. Derefter opløses de udskilte Manganilter i fortyndet Saltsyre (2 %) under Tilsætning af Svovlsyrlingvand. Massen vaskes Svovlsyrefri og sprøjtes med Vinaand over paa et tørret og vejte Filter, udvaskes grundigt tilsidst med absolut Alkohol og tørres ved 100° og vejes. Tørringen tager paafaldende lang Tid (2 Døgn), saa at det kunde synes, at der var dannet en Forbindelse af Vinaand og Cellulose, der først spaltes ved langvarig Opvarmning³⁾. Af den tørrede og vejede Cellulose udtages en aliquot Del til Askebestemmelse Cellulosen brænder let bort ved først at ophedes meget langsomt over Flammespreder. Til Cellulosebestemmelserne blev benyttet 20,00 grm lufttørt finmalet Stof. I nedenstaaende Tabel 27 er angivet Indholdet af askefri Pektocellulose beregnet for Tørstoffet.

¹⁾ C. G. SCHWALBE: anf. Std. S. 617.

²⁾ Det kan sikkert være bekvemt at anvende det fra Fedtextraktionen i Soxleth's Apparat tiloversblevne Stof, hvis det kan faas ud af Filtrepatronerne uden Tab. Efter et enkelt Forsøg ser det ud som man i saa Fald helt kan undvære Brombehandlingen. 20,00 grm lufttørt, malet Rajgræs Hø blev ekstraheret og udvasket med kogende Vinaand, saa at alt Fedt, Voks, Klorofyl og Sukker gik i Opløsning. Bundfaldet paa Filtret blev behandlet med 2 % Ammoniakvand og gav et meget mørkt farvet Filtrat medens Bundfaldet blev meget lysere. Efter fuldstændig Udvaskning med kogende Vand, blev det bleget med KMnO_4 og SO_2 ligesom Bromcellulosen og vasket med Vinaand, suget tørt paa et vejte Filter med Æter og tørret ved 100°. 20,00 grm lufttørt Hø med 92,61 % Tørstof gav 5,8915 grm Cellulose med 1,60 % Aske altsaa 5,797 grm askefri Cellulose = 31,29 % af Tørstof. Ved Brommetoden er der, som det fremgaar af Tab. 27, fundet i to Forsøg 31,46 % — 31,88 % askefri Bromcellulose, altsaa praktisk talt det samme Tal, der fandtes ved ovenomtalte Fremgangsmaade. Dette Forsøg blev først udført, efter at hele den øvrige Undersøgelse var sluttet; fremtidige Undersøgelser maa derfor vise, om denne Overensstemmelse i Resultaterne er tilfældig, eller grunder sig paa, at det er samme Stof i samme Mængde, der isoleres ved de to Behandlingsmaader.

³⁾ Flere Steder i Celluloselitteraturen vil man finde noteret lagttagelser, der væsentlig gaar ud paa samme Forhold, men nærmere Undersøgelser heraf foreligger ikke saa vidt vides.

Tabel 27.

Indholdet af Pektocellulose i Græsarterne¹.

			askefri Pektocellulose % af Tørstof	Middeltal
A ₁	Italiensk Rajgræs	Hø	31,46	} 31,65
	—	—	31,83	
A ₂	—	Halm	31,60	} 31,57
	—	—	31,54	
B ₁	Draphavre	Hø	38,39	} 38,72
	—	—	39,04	
B ₂	—	Halm	42,88	} 42,67
	—	—	42,45	
C ₁	Agerhejre	Hø	35,23	} 35,45
	—	—	35,66	
C ₂	—	Halm	47,37	} 47,38
	—	—	47,38	
D ₁	Hundegræs	Hø	37,67	} 37,08
	—	—	36,50	
D ₂	—	Halm	37,28	} 37,37 ²⁾
	—	—	37,46	
E ₁	Engsvingel	Hø	36,44	} 36,68
	—	—	36,92	
E ₂	—	Halm	39,95	} 40,13
	—	—	40,31	

For Draphavre, Agerhejre og Engsvingel er der, som man vil se, ret betydelig Forskel i Pektocelluloseindholdet af Hø og Halm medens Italiensk Rajgræs og Hundegræs indeholder de samme Mængder i Hø og Halm af samme Plante.

Bestemmelse af Kobbertal i Pektocellulose. Til nærmere Karakterisering af de forskellige Cellulosearter har C. G. SCHWALBE indført Bestemmelsen af det saakaldte Kobbertal³⁾. En afvejet Mængde Stof koges i en vis Tid med en afmaalt Mængde af Fehling's Væske under Tilsætning af en afpasset Mængde Vand. Der foregaar herved efter Cellulosens Beskaffenhed en svagere eller stærkere Reduktion, og Cellulosen farves højrød af udskilt Cuprooxyd. Den kobberilteholdige Cellulose frafiltreres og udvaskes, og i Bundfaldet kan Kobbermængden bestemmes efter en af de gængse Metoder. Ved af det vundne Resultat at beregne hvormegit Kobber, der svarer til 100 grm askefri Cellulose, faas Kobbertallet, i hvilket der dog maa udføres forskellige Korrektioner.

Ved at behandles paa ovennævnte Maade viste alle de fremstillede Præparater af Pektocellulose ret stærk Reduktionsevne. Men Mængden af reduceret Kobberilte er i høj Grad afhængig af Tiden, som følgende Forsøg kan vise.

¹⁾ Egentlig Pektocellulose + Kutin, som det senere skal vises.

²⁾ Ved Benyttelse af 100,0 grm Stof blev fundet i to Forsøg 36,94 % og 38,45 % askefri Pektocellulose, men saa store Portioner lader sig ikke let behandle med kvantitativ Nøjagtighed. Middeltallet bliver 37,70 %.

³⁾ C. G. SCHWALBE: „Chemie d. Cellulose“ S. 625.

Ren hvid Pektocellulose af Hundegræs Halm (D_2) blev afvejet i Portioner fra $\frac{1}{4}$ — $\frac{1}{2}$ Gram og kogt i en snæverhalset Kolbe med 50 cm³ Fehling's Væske og 100 cm³ Vand.

Beregnet for 1,000 grm askefri Pektocellulose blev udfældet:

10 Min. Kogning	=	0,0453	grm Cu
20 — — —	=	0,0971	—
30 — — —	=	0,1162	—
60 — — —	=	0,1232	—

En anden Forsøgsrække med Pektocellulose af Italiensk Rajgræs Hø (A_1) gav ved lignende Fremgangsmaade følgende Resultater beregnet for 1,000 gram askefri Pektocellulose.

1 Times Opvarmning paa Vandbad. Henstand afproppet til næste Dag:	=	0,0998	grm Cu
Opvarmning paa Vandbad i en løselig tilproppet, snæverhalset Kolbe i 3 Døgn:	=	0,3252	—
Samme Behandling i 4 Døgn:	=	0,3574	—
2 Timers stærk Kogning over aabent Blus i en Kolbe med Tilbagesvaling:	=	0,4081	—

Skønt det er givet, at den af J. KJELDAHL paaviste Egenreduktion af Fehling's Væske ved Opvarmning¹⁾ har haft sin Part i den udskilte Kobberiltemængde, fremgaar det dog med Sikkerhed, at Kobberiltemængden, som reduceres af Pektocellulosen, er afhængig af Reduktionstiden, og af Maaden hvorpaa Opvarmningen foretages. En kortere Tids Opvarmning over aabent Blus virker kraftigere end en længere Tids Opvarmning paa Vandbad²⁾.

Bestemmelsen af Kobberiltet blev udført ved nøjagtig at afveje nedenforangivne Mængde af Pektocellulose, der anbragtes i en snæverhalset Kolbe paa 200 cm³ sammen med 50 cm³ Fehling's Væske og 100 cm³ Vand. Kolben blev anbragt 1 Time i kogende Vandbad, blev tilproppet og henstod til næste Dag, hvorefter Bundfaldet under Sugning frafiltreredes paa et askefrit Filter. Denne Behandlingsmaade blev foretrukket, da Kobberiltet herved blev krystallinsk og let lod sig filtrere, saa at man undgik Tilsætningen af Diatomékisel, som C. G. SCHWALBE anbefaler for at lette Filtringen³⁾. Bundfaldet blev behandlet som det kobberilteholdige Bundfald fra Sukkerbestemmelserne⁴⁾. I den fundne Vægtmængde maa fradrages: 1) Cellulosens Askeindhold, 2) Kobberiltemængden som Filtret har tilbageholdt ved Adsorption, 3) Kobberiltemængden som den anvendte Cellulose har tilbageholdt ved Adsorption. Den sidste Korrektion blev bestemt i to Prøver med følgende Resultater:

¹⁾ J. KJELDAHL: Medd. f. Carlsberg Laborat. 4de Bd. Kbhvn. 1895. S. 6 o. flg.

²⁾ Smlg. anf. Std. S. 25.

³⁾ C. G. SCHWALBE anf. Std.

⁴⁾ Denne Afh. S. 22.

E₁ Engsvingel Hø.

Pektocellulose	= 0,3095	gram heri 1,01 % Aske.
÷ Aske	= 0,0034	
Askefri Pektocellulose	= 0,3061	
Fundet Cu O	= 0,0169	
÷ Celluloseaske	= 0,0034	
korr. Cu O	= 0,0135	
Cu	= 0,0108	1,000 gram askefri Cellulose
÷ Cu inds. af Filtret	= 0,0010	tilbageholder 0,0320 gram Cu
Indsuget af Cellulosen Cu ...	= 0,0098	

G₂ Agerhejre Halm.

Pektocellulose	= 0,2805	heri 0,66 % Aske.
÷ Aske	= 0,0019	
Askefri Pektocellulose	= 0,2786	
Fundet Cu O	= 0,0154	
÷ Celluloseaske	= 0,0019	
korr. Cu O	= 0,0135	
Cu	= 0,0108	1,000 gram askefri Cellulose
÷ Cu inds. af Filtret	= 0,0010	tilbageholder 0,0352 gram Cu
Indsuget af Cellulosen Cu ...	= 0,0098	

Man tør herefter antage, at i Gennemsnit tilbageholder 1,000 gram Pektocellulose 0,0336 gram. Cu.

Følgende Bestemmelser blev udført. I den angivne Kobbermængde er indregnet Korrektioner for Celluloseasken og for den af Filtret optagne Kobbermængde.

Tabel 28.

Stoffet	Anvendt Mængde Pektocellulose beregnet askefri	Fundet Cu	Beregnet Cu for 1,000 gram askefri Pektocellulose
A ₁ Italiensk Rajgræs Hø	0,7091 gram	0,0708 gram	0,0998 gram
A ₂ Halm	0,8193	0,0811	0,0990
B ₁ Draphavre Hø	0,9448	0,0696	0,0882
B ₂ — Halm	0,5692	0,0458	0,0847
C ₁ Agerhejre Hø	0,4064	0,0511	0,1258
C ₂ — Halm	0,5233	0,0631	0,1181
D ₁ Hundegræs Hø	0,3795	0,0331	0,0846
D ₂ — Halm	0,2560	0,0243	0,0971
E ₁ Engsvingel Hø	0,8023	0,0995	0,1240
E ₂ — Halm	0,4263	0,0517	0,1212

Beregner man Kobbertallet for 100 gram askefri Pektocellulose med Fradrag af det adsorptivt bundne Kobber, faas følgende Tal:

Tabel 29.

Kobbertal.			
A_1	\backslash 6,62	D_1	\backslash 5,10
A_2	\backslash 6,54	D_2	\backslash 6,35
B_1	\backslash 5,16	E_1	\backslash 9,04
B_2	\backslash 5,11	E_2	\backslash 8,76
C_1	\backslash 9,20		
C_2	\backslash 8,45		

De forskellige Hø- og Halmprøver af samme Planteart følges godt sammen med Hensyn til Kobbertallet, som man jo ikke kan vente bestemt med nogen særlig stor Nøjagtighed, da dets Størrelse, som vist, er afhængig af Tiden, hvori Reduktionen foretages og mulige Forsøgsfejl multipliceres med 100. Denne Overensstemmelse i Kobbertal mellem Pektocelluloserne fra Hø- og Halmprøverne af den samme Planteart viser utvivlsomt, at disse Stoffer er identiske. Derimod synes de forskellige Kobbertal for Pektocelluloserne af de forskellige Plantearter at antyde, at disse Stoffer vel er hinanden meget nærstaaende, men dog maaske ikke fuldkommen identiske. Middeltallet af alle 10 Bestemmelser er 7,03. Man mangler endnu nærmere Sammenligningspunkter med Pektocellulosens Kobbertal. C. G. SCHWALBE har ganske vist udført en lang Række Bestemmelser af Kobbertal¹⁾ i forskellige Stoffer, men de er alle afledt af Bomuld eller Filtrepapir. Blegel, udkogt og malet reneste Bomuldsatin har et Kobbertal af kun 0,014, medens Oxy-cellulose fremstillet af samme Stof ved kraftig Iltning med Natriumhypochlorit har et Kobbertal = 34,9. Nærmest ved Pektocellulosens Kobbertal staar Kobbertallene af „Hydrocellulose“, som er fremstillet af Filtrepapir ved Behandling med 3 % holdig Svovlsyre, eller Hydrocelluloser fremstillet af merceriseret Bomuld ved lignende Behandling.

Som allerede omtalt giver Pektocelluloserne fremstillet af Græsarterne ved Destillation med 12 % holdig Saltsyre ikke ubetydelige Mængder Furfurol, som i Destillatet kan fældes med Floroglucin. Deraf kan ved Krøber's Tabeller beregnes den tilsvarende Mængde Furfurol og Pentosan, men som det vil vides, haves der dog ingen Sikkerhed for, at den fundne Furfurolmængde i Realiteten stammer fra en Pentosangruppe, som er knyttet til Pektocellulosens Molekul, og som frigøres ved Saltsyrebehandlingen, eller maaske hidrører fra andre Atomgrupper i Pektocellulosen, der kan give Furfurol. Ikke desto mindre maa Furfurolmængden, der faas af Pektocellulosen, anses for et ret vigtigt Karakteristikum for Stoffet. Destillationen med Saltsyre og Fældningen af Floroglucidet i Destillatet blev udført som ved Pentosanbestemmelsen i selve Stoffet.

Der blev først udført følgende foreløbige Forsøg for at se, med hvor stor indbyrdes Overensstemmelse man kunde udføre saadanne Bestemmelser. Hertil blev

¹⁾ C. G. SCHWALBE: Z. f. angew. Chemie XX 1907 S. 2177. M. RENKER's Kobbertal i „Ueber Bestimmungsmethoden d. Cellulose“, 2te Aufl. Berlin 1910 S. 94 drejer sig væsentlig kun om Sulfitcellulose behandlet paa forskellig Maade og kan heller ikke direkte paralleliseres med Pektocellulosen.

udvalgt Pektocelluloserne fra D₂, Hundegræs Halm. Som Noten S. 38 viser, er der udført to Præparationer af Pektocellulose fra Hundegræs Hø ved Benyttelse af 100 grm Stof. Præparation I gav 38,45 % Pektocellulose (beregnet askefri), II gav 36,94 %. Disse Præparationer gav følgende Mængde Floroglucid og Furfurol.

	Pektocellulose askefri	Floroglucid	Furfurol	Furfurol Procent	Middeltal
I	0,9006 grm 0,4713 —	0,1560 grm 0,0785 —	0,0837 grm 0,0435 —	9,26 9,18	} 9,22 %
II	0,5002 grm 0,3716 —	0,0785 grm 0,0555 —	0,0433 grm 0,0314 —	8,62 8,44	

Middeltal af I—II er 8,88 % Furfurol = 15,20 % Pentosan af askefri Pektocellulose.

Der er altsaa ret god Overensstemmelse mellem Duplikatbestemmelser af Furfurol i samme Præparat, medens Præparater med saa store Variationer som I og II i Pektocellulosen ogsaa varierer noget i Furfurolmængde eller Pentosanmængden. Den laveste Pektocellulose indeholder mindst Pentosan. Yderligere blev der foretaget en femte Bestemmelse med Benyttelse af den Præparation af 20 grm Hundegræs Halm, der, som Tab. 27 viser, gav 37,46 % Pektocellulose (Præparat III).

III 1,0591 grm askefri Pektocellulose gav 0,1065 grm Floroglucid = 8,90 % Furfurol = 15,25 % Pentosan i god Overensstemmelse med Middeltallet af Præparaterne I—II. Det blev derfor anset for mindre nødvendigt helt at gennemføre Duplikatbestemmelser ved alle Destillationerne, da Arbejdet derved forøges i saa høj Grad. Resultaterne af de enkelte Bestemmelser er anført i Tab. 30.

Tabel 30.

Furfurol og Pentosan i Pektocellulose.

Stoffets Art	Anvendt Pektocellulose beregnet askefri	Fundet Floroglucid Gram	Furfurol Gram	Procent af Pektocellulose	
				Furfurol	Pentosan
A ₁ Italiensk Rajgræs Hø	1,9700	0,2677	0,1411	7,16	12,39
A ₂ — Halm	1,6484	0,2363	0,1248	7,57	13,10
B ₁ Draphavre Hø	1,0447	0,2235	0,1186	11,35	19,37
B ₂ — Halm	1,0249	0,2380	0,1261	12,30	20,97
C ₁ Agerhejre Hø	1,0209	0,1465	0,0788	7,71 ¹⁾	13,19
C ₂ — Halm	1,2754	0,2503	0,1324	10,38 ²⁾	17,70
D ₁ Hundegræs Hø	0,8719	0,1330	0,0717	8,20 ³⁾	14,02
D ₂ — Halm	1,0579	0,1765	0,0913	8,90	15,25
E ₁ Engsvingel Hø	1,0577	0,1350	0,0728	6,88 ⁴⁾	11,75
E ₂ — Halm	1,4271	0,2730	0,1442	10,10	17,21

¹⁾ Et andet stærkere bleget Præparat gav i to Bestemmelser 6,69 % og 6,60 % Furfurol.

²⁾ Et andet mindre rent Præparat gav 11,61 % Furfurol.

³⁾ Det samme Præparat gav ved fornyet Analyse 8,10 % Furfurol.

⁴⁾ Et andet Præparat gav 7,10 % Furfurol.

I Agerhejre og Engsvingel er Furfurolmængden 2—3 $\frac{0}{10}$ større i Pektocelluloserne, der stammer fra Halmen, end i de der stammer fra Høet, og Differentsten mellem de af Furfurolmængderne beregnede Pentosanmængder bliver derfor 4—6 $\frac{0}{10}$. De andre 8 Prøver har derimod praktisk talt givet samme Furfurolmængde for de fra samme Planteart stammende Pektocelluloser.

I nedenstaaende Tab. 31 er sammenstillet Mængden af Pektocellulose i Tørstoffet i Overensstemmelse med Tab. 27 og Procentindholdet af Furfurol, der er udvundet ved Saltsyredestillationen af disse Pektocelluloser.

Tabel 31.

	Pektocellulose askefri Procent af Tørstof	Furfurol Procent af Pektocellulose
A ₁	31,65	7,16
A ₂	31,57	7,57
B ₁	38,72	11,35
B ₂	42,67	12,30
C ₁	35,45	7,71
C ₂	47,38	10,38
D ₁	37,08	8,20
D ₂	37,37	8,90
E ₁	36,08	6,88
E ₂	40,13	10,10

Skønt det endnu næppe er muligt at danne sig noget nærmere Billede af Pektocellulosens Sammensætning i de enkelte Tilfælde, og skønt man ikke ved noget om de furfurolgivende Atomgruppers Natur i Pektocellulosen, synes det dog med Sikkerhed at fremgaa af Tabel 31, at de Prøver, der indeholder mest Pektocellulose i Tørstoffet, ogsaa i disse Pektocelluloser indeholder mest furfurolgivende Stof.

Yderligere fremgaar det af Tab. 31, at for de Plantearters Vedkommende (Draphavre og Engsvingel), hvor Mængden af Pektocellulose i Tørstoffet tiltager med Planternes Alder, beror denne Tiltagen for en væsentlig Del paa, at Mængden af furfurolgivende Atomgrupper tiltager, medens for de Planter Vedkommende (Italiensk Rajgræs, Agerhejre og Hundegræs), hvor der ikke sker nogen væsentlig Tilvækst i Pektocellulosemængden med Alderen, forandres Pektocelluloserens Sammensætning, hvad de furfurolgivende Atomgrupper angaar, heller ikke med Alderen.

At der i ingen af de omhandlede fem Græsarter med Alderen sker nogen væsentlig Forandring i Pektocelluloserne med Hensyn til de aldehydagtige Atomgrupper, der forårsager Reduktion ved Behandling med Fehling's Væske, fremgaar allerede af Tab. 29.

For at prøve paa, om man dog ikke til en vis Grad kunde erfare noget om, hvilke Omsætninger der foregaar i Pektocelluloserne ved Destillation med Saltsyre under Furfuroidannelsen, blev Remanenserne i Kolberne efter Destillationen for at

kunne filtreres fortyndet noget med Vand og filtreret gennem et løst Filter. Herved faas et Bundfald paa Filtret og et Filtrat. Bundfaldet bestaar af uopløst Cellulose, der dog er temmelig stærkt brunfarvet og noget slimet, antageligt af Pektinstoffer, og Filtratet, der er sukkerholdigt, er ligeledes noget brunfarvet og slimet. Cellulosen paa Filtret blev rensed ved Behandling med 2 % Ammoniakvand, bleget med en tynd Opløsning af Kaliumpermanganat og derefter udvasket med Vand, Svovlsyrlingvand, Vand, Vinaand og suget tørt paa et vejet Filter og tørret ved 100°. Efter Vejning forbrændes det hele eller en aliquot Del til Askebestemmelse. Denne Cellulose er snehvid med et ganske ringe Askeindhold og synes efter de faa derover foretagne Forsøg at mangle Reduktionsevne overfor Fehling's Væske. Filtratet fra Kolbeindholdet blev neutraliseret med tilsat fast Natriumkarbonat og kogt 30 Minutter over aabent Blus med Fehling's Væske i Overskud. Cuprooxydet blev frafiltreret paa et dobbelt Filter, glødet og vejet som Cu O, som tidligere omtalt under Sukker. Sukkeret blev beregnet som Dextrose af den fundne Kobbermængde med Fradrag af den Mængde, Filtrene erfaringsmæssigt tilbageholder fra Fehling's Væske. I nogle Tilfælde blev der fundet en vis Regelmæssighed herved, som det vil fremgaa af nedenstaaende Resultater.

Tabel 32.

Procent af anvendt Pektocellulose.

	Restcellulose	Dextrose	Restcellulose + Dextrose
B ₁	40,54	25,52	66,06
C ₁	42,23	23,86	66,09
E ₁	43,73	22,65	66,38
E ₂	52,73	12,57	65,30

Undersøgelserne viser yderligere, at Mængden af det opstaaede Sukker og den tiloversblevne Cellulose til en vis Grad er omvendt proportionale og afhængige af Kogningstiden, som det var at vente, men at der dog lides et Tab i Stofmængde (Cellulose + Dextrose), jo længere Kogningstiden er.

Tabel 33.

Procenter af anvendt Pektocellulose.

	Restcellulose	Dextrose	Restcellulose + Dextrose
B ₂ kogt til Furfurolet er afdestilleret	47,61	17,76	65,40
kogt yderligere c. 1 Time	37,36	23,32	60,68
E ₁ kogt til Furfurolet er afdestilleret	43,73	22,65	66,38
kogt yderligere c. 1 Time	38,14	26,66	64,80
C ₁ kogt til Furfurolet er afdestilleret	42,23	23,86	66,09
kogt c. 3 Timer ¹⁾	21,58	15,58	37,16

¹⁾ Ved denne langvarige Behandling forkulles Cellulosen stærkt og der overdestillerer Myresyre og Eddikesyre ved den stærke Saltsyres destruerende Virkning.

Da det ikke er givet, at Tiden for Furfurolets Overdestillering staar i noget Forhold til Tiden for Pektocellulosens Sønderdeling ved Saltsyrebehandlingen, lader der sig næppe beregne nogen nærmere Omsætningsproces af ovenstaaende.

Overfor konc. Svovlsyre forholder Pektocellulosen sig noget forskelligt efter Maaden hvorpaa Stofferne bringes til at reagere paa hinanden. Ved yderst forsigtigt i ganske smaa Portioner at indbringe 0,1114 grm (beregnet askefri) skarpttørret Pektocellulose af A₁ Italiensk Rajgræs Hø i c. 25 cm³ stærkt afkølet konc. Svovlsyre opløses Cellulosen næsten uden at Væsken farves. Efter Henstand til næste Dag fortyndedes Opløsningen med 250 cm³ Vand og kogtes $\frac{1}{2}$ Time over aabent Blus, neutraliseredes med fast Natriumkarbonat og blev kogt med Fehling's Væske. Herved udfældedes Kobberilte = 0,1225 grm Cu (efter Korrektion for Filterets Adsorption) svarende til 0,0553 grm Dextrose = 48,70 % Dextrose. Ved et nyt Forsøg blev lufttør Pektocellulose af samme Art 0,2595 grm (beregnet tør og askefri) overhældt paa en Gang i et lille Bægerglas med c. 30 cm³ konc. Svovlsyre af almindelig Stuetemperatur. Stoffet opløstes delvis, men blev stærkt brunsort farvet og var helt sort næste Dag. Ved Fortynding udskiltes en Mængde forkullet Stof, saa at Undersøgelsen ikke blev ført videre. Ganske snehvid lufttør Pektocellulose af D₂ Hundegræs Halm blev i Smaaportioner bragt ned i en Flaske med konc. Svovlsyre og omrystet for hver Tilsætning; 0,6229 grm (beregnet askefri og tør) opløstes herved i 30 cm³ konc. Svovlsyre, men Opløsningen var ved Henstand til næste Dag blevet brunfarvet. Efter Fortynding, Kogning og Neutralisering blev med Fehling's Væske udfældet Kobberilte svarende til 0,1895 grm Dextrose = 30,4 %. I denne Henseende forholder Pektocellulose sig altsaa anderledes end ren Cellulose og Lignocellulose. Ren Cellulose (af Bomuld), der opløses i konc. Svovlsyre, giver den til Kulstofindholdet svarende Mængde Dextrose. Lignocellulose forkulles derimod omtrent straks af konc. Svovlsyre, medens som anført Pektocellulose synes ved yderst forsigtig Behandling at kunne opløses i Svovlsyre uden Forkulning, men giver kun en Brøkdel af den til Kulstofmængden svarende Dextrose.

Bestemmelsen af Pektocellulose i Kløverarterne blev udført paa samme Maade som i Græsarterne. For Kløverhalmen er der ingen Vanskelighed; der faas her en snehvid let udvaskelig Cellulose, medens Kløverhøet er tilbøjeligt til at give en lidt slimet undertiden noget graalig Cellulose. Medens Askeindholdet i Cellulosen fra Kløverhalmen ikke var større end i Cellulosen fra Græsarternes Halm 0,5—1,0 %, var Askeindholdet i Cellulosen af Sildig Rødkløver Hø 4,37 %—4,58 %, i Cellulosen af Kællingetand Hø 2,49—2,29 % og i Gul Rundbælg Hø 2,96 %—2,85 %.

Resultaterne af Analyserne, der blev udført ved at benytte 20,00 lufttørt Stof, var følgende:

Tabel 34.

Indholdet af Pektocellulose i Kløverarterne.

		Askefri Pektocellulose % af Tørstof	Middeltal
F ₁	Sildig Rødkløver Hø	23,57	23,72
		23,87	
F ₂	— Halm	43,53	43,53
	— —	43,52	
G ₁	Kællingetand Hø	25,90	25,73
	— —	25,56	
G ₂	— Halm	43,70	44,04
	— —	44,38	
H ₁	Gul Rundbælg Hø	27,30	27,07
	— —	26,84	
H ₂	— Halm	48,46	48,86
	— —	49,26	

Der er i Kløverarterne en stærkt udpræget Forskel mellem Mængderne af Pektocellulose i Hø og Halm af de samme Planter. Ved Modningen stiger Mængden af Pektocellulose til nær henimod det dobbelte af hvad der findes i den grønne Plante.

P. SCHWEITZER har ved Benyttelsen af samme Metode, som her er anvendt, for Rødkløver af amerikansk Avl fundet¹⁾

	Pektocellulose % af det lufttørre Stof	Pektocellulose % beregnet af Tørstof
Redclover in bloom ²⁾	28,17	30,64
— seed ripe ³⁾	39,10	42,88

Schweitzer's Rødkløver i Blomst har sandsynligvis været paa et lidt mere fremskredet Voksestadium end F₁ i Tab. 34, medens hans modne Rødkløver og F₂ viser god Overensstemmelse.

I Pektocellulose fra Kløverarterne blev der kun foretaget nogle faa Bestemmelser af Kobbertallet som imidlertid var af samme Størrelser som i Pektocellulosen fra Græsserne (6 9). Derimod blev Bestemmelsen af Furfurol gennemført for alle Prøver med følgende Resultater.

Tabel 35.

	Anvendt Pektocellulose	Fundet Floroglucid	Furfurol	Furfurol	Pentosan
	Gram	Gram	Gram	%	%
F ₁ Sildig Rødkløver Hø	1,0687	0,1330	0,0717	6,71	11,48
F ₂ — Halm	1,0820	0,1174	0,0636	5,88	10,07
	0,9847	0,1000	0,0546	5,55	9,50
G ₁ Kællingetand Hø	1,5017	0,2410	0,1292	8,61	14,67
G ₂ — Halm	1,0165	0,1750	0,0935	8,93	15,27
H ₁ Gul Rundbælg Hø	2,5502	0,1170	0,2342	9,18	15,65
H ₂ — Halm	1,3587	0,2270	0,1204	8,86	15,11

¹⁾ P. SCHWEITZER anf. Std.

²⁾ Stoffet indeholder 8,03 % Vand.

³⁾ — — — — — 8,95 %

Resultaterne af Bestemmelserne af Furfurol i Pektocelluloserne fra Kællingetand og Gul Rundbælg er praktisk talt fuldkommen overensstemmende, medens Furfurolbestemmelserne i Kløverhø og Kløverhalm er lidt lavere. Pektocellulosen i disse Præparater synes at være blevet bleget noget stærkere end i de andre og indeholder kun 0,07 % Kvælstof, medens Kvælstofindholdet i Pektocelluloserne fra Kællingetand og Gul Rundbælg er 0,10—0,15 %, og lader sig ikke fjerne ved den foretagne Behandling.

Sammenlignende Undersøgelser mellem Pentosanmængden i Pektocellulosen og Pentosanmængden i selve Høet og Halmen.

Pentosanmængderne, der er angivet i Tab. 30 og Tab. 35, indgaar som Parter i Pentosanbestemmelserne i selve Høet og Halmen, der er angivet i Tab. 25 og Tab. 26. Det vil derfor være ønskeligt at anstille en Sammenligning mellem de fundne Værdier for at udrede, hvormeget af den samlede Pentosanmængde der hører til Pektocellulosen, og hvormeget der er selvstændigt Stof. Det er sandsynligt, at dette er til Stede som virkelige Pentosaner, Araban eller maaske nærmere Xylan, medens det er sandsynligt, at den Mængde, der er angivet som Pentosan knyttet til Pektocellulosen, ikke er egentlig Pentosan, men en furfurolgivende Atomgruppe indbygget i Pektocellulosens Molekul. Der synes at aabne sig Veje til at skelne mellem de egentlige Pentosaner og de furfurolgivende Atomgrupper af anden Art gennem den af BRAUNS foreslaaede Fremgangsmaade¹⁾, men der er hidtil ikke udført nogen saadan Undersøgelse, saa at man er nødt til at beregne de furfurolgivende Atomgrupper under et som Furfurol eller Pentosan. I nedenstaaende Tab. 36 er anført Procenterne af Pektocellulose i Tørstoffet, Procenterne af beregnet Pentosan i Pektocellulose og af disse to Kolonner beregnet den 3die, der angiver Pentosan i Pektocellulose beregnet som Procent af Tørstoffet. I 4de Kolonne er angivet Totalmængden af Pentosan som Procent af Tørstof i Overensstemmelse med Tab. 25—26, og endelig er i 5te Kolonne angivet Mængden af fri Pentosaner i Tørstoffet, hvilken Mængde faas ved at subtrahere 3die Kolonne fra 4de Kolonne.

Tabel 36.

Stoffets Art	Pektocellulose % af Tørstof	Pentosan % af Pektocellulose	Pentosan fra Pektocellulose beregnet som % af Tørstof	Totalmængden af Pentosan % af Tørstof	Frit Pentosan i % af Tørstof
A ₁ Italiensk Rajgræs Hø	31,65	12,39	3,92	19,89	15,97
A ₂ — Halm	31,67	13,10	4,14	17,54	13,40
B ₁ Draphavre Hø	38,72	19,37	7,50	21,39	13,89
B ₂ — Halm	42,67	20,97	8,91	22,02	13,08
C ₁ Agerhejre Hø	35,45	13,19	4,67	18,99	14,32
C ₂ — Halm	47,38	17,70	8,38	20,88	12,50
D ₁ Hundegræs Hø	37,98	13,85	5,13	20,37	15,24
D ₂ — Halm	37,37	15,25	5,69	15,81	10,15
E ₁ Engsvingel Hø	36,68	11,75	4,30	20,77	16,47
E ₂ — Halm	40,13	17,21	6,90	20,55	13,65

¹⁾ Smlg. Citatet S. 28.

Mængden af frit Pentosan er i et enkelt Tilfælde, Draphavre, lige stort i Hø og Halm, men i alle de andre Tilfælde 2—3 $\frac{0}{10}$ større i Høet end i Halmen af samme Plante.

For Kløverarterne stiller Forholdet sig væsentlig anderledes end for Græsserne. Pentosanmængderne er i det hele taget betydelig mindre end i Græsserne, saaledes som det vil fremgaa ved at sammenligne Tab. 36 med nedenstaaende Tabel. Der er i alle Tilfælde mere frit Pentosan i Halmen end i Høet af samme Plante.

Tabel 37.

Stoffets Art	Pektocellulose $\frac{0}{10}$ af Torstof	Pentosan $\frac{0}{10}$ af Pektocellulose	Pentosan fra Pektocellulose beregnet som $\frac{0}{10}$ af Torstof	Totalmængden af Pentosan $\frac{0}{10}$ af Torstof	Frit Pentosan i $\frac{0}{10}$ af Torstof
F ₁ Sildig Rødkløver Hø	23,72	11,48	2,72	8,61	5,89
F ₂ — Halm	43,43	10,07	4,64	12,51	7,87
G ₁ Kællingetand Hø	25,73	14,67	3,78	10,85	7,07
G ₂ — Halm	44,01	15,27	6,73	16,49	9,76
H ₁ Gul Rundbælg Hø	27,07	15,65	4,24	11,28	7,04
H ₂ — Halm	48,86	15,11	7,38	16,21	8,86

Kutinbestemmelse i Pektocellulose. Ved Anvendelsen af SCHWEIZER's Reagens, Kobberilteammoniak i konc. Opløsning, opløses selve Pektocellulosen let og fuldstændigt, men saaledes som den er fremstillet paa tidligere beskreven Maade af Hø og Halm efterlades altid en i Reglen lille Rest af et Stof af en fra Pektocellulosen meget afvigende Beskaffenhed. Stoffet fik Navne som Cytine, Cutine m. fl., og er paavist og tildels undersøgt af forskellige Celluloseforskere som FREMY, FLÜCKIGER, WESSENBURGH m. fl., men det er egentlig først J. KÖNIG og F. MURDFIELD, der har givet nærmere Anvisning til kvantitativ Bestemmelse af Stoffet²⁾ og givet det Navnet Kutin, som det nu almindeligvis gaar under. Skønt man ser kvantitative Bestemmelser af Kutinmængden i forskellige Plantearter og Plantedele anført ikke saa faa Steder i agrikulturkemiske Undersøgelser, da Stoffet har frembudt sig som Opløsningsrest under de ofte foretagne Forsøg paa at rense Cellulose af den ene eller anden Herkomst og Fremstillingsmaade ved Opløsning i Schweizer's Reagens, kendes der dog ikke synderlig meget til Kutinets nærmere Natur, og man har egentlig intet Bevis for, at der er nogen Forbindelse mellem Planternes Cuticula og Kutinet, saaledes som det bliver tilbage ved Behandlingen af Cellulosen med Kobberilteammoniak. En botanisk-anatomisk Undersøgelse af Kutin er saavidt vides aldrig foretaget. Stoffet maa vistnok i kemisk Henseende siges at staa temmelig nær ved Kork og indeholder ligesom dette voksagtigt Stof og fede Syrer, saa at det tildels kan forsæbes ved Behandling med Alkali. Den eneste nærmere kemiske

¹⁾ CROSS & BEVAN; „Cellulose“ S. 228.

²⁾ J. KÖNIG: „Die Untersuchung landwirtschaftlich und gewerblich wichtiger Stoffe“. 3te Aufl. Berlin 1906 S. 1057.

Undersøgelse af Kutin, som kendes, er udført i 1906 af J. KÖNIG¹⁾. Den er ikke større, end den kan citeres *in extenso*, og den lyder saaledes:

»Den anderen Bestandteil der Rohfaser, der weder durch Wasserstoffsuperoxyd und Ammoniak oxydierbar ist noch durch Kupferoxydammoniak gelöst wird, haben wir für sich in etwas grösseren Mengen zu gewinnen gesucht, um seine Eigenschaften und seine Elementarzusammensetzung direkt ermitteln zu können; die Elementarzusammensetzung schwankte bei Gras- und Kleeheu, Erbsenstroh, Roggen- und Weizenkleie zwischen 68—70 % Kohlenstoff und 10—12 % Wasserstoff. Die Schwankungen sind auch bei dem isolierten Körper durch seinen hohen Aschengehalt bedingt, der durchweg bis 30 % beträgt. Die Asche besteht aus fast reiner Kieselsäure, die sich nur durch Zerstören mit Flusssäure von dem eigentlichen Kutin trennen lässt. Das eigentliche Kutin lässt sich durch 20 %-ige Kalilauge bis auf einen kleinen Rest verseifen, aus der Seife durch Äther in geringen Mengen ein durch seinen Geruch an Koniferin erinnernder Körper gewinnen, während nach dem Ansäuern der Seifenlösung durch Äther eine Säure ausgezogen werden kann, die von wachsähnlicher Beschaffenheit ist, bei 10° fest wird und stark nach Vanillin riecht. Das Kutin der Rohfaser ist daher gleich dem „cytine“ Fremy's als ein ester-(wachs)-ähnlicher Körper aufzufassen, der aber von dem Suberin verschieden ist«.

Det er derfor ikke sandsynligt, at Kutin, saaledes som det tilbagebliver ved ovennævnte Behandling, i kemisk Henseende er noget enkelt Stof, men derimod en Blanding af flere. Kutinbestemmelser har derfor næppe nogen selvstændig Værdi, da man knap nok hverken botanisk eller kemisk ved, hvad det er, man fraskiller ved Kutinbestemmelsen, men den maa dog siges at have Betydning som Middel til at skille Cellulosen af med en Urenhed.

Med denne Betragtning for Øje, kunde det maaske synes fra et kemisk Synspunkt at ligge nærmere at bestemme den rene Cellulose, der er opløst i Schweizer's Reagens, end at bestemme Urenhederne, der bliver tilbage. Der er ogsaa udført mange Forsøg paa at gaa denne Vej ved Cellulosebestemmelsen, men de maa alle uden Undtagelse siges at være mislykkede. J. KÖNIG angiver ganske vist, at man af Kobberilteammoniakopløsningen kvantitativt kan udfælde med 80 %-holdig Alkohol den opløste Cellulose, men meddeler samtidig i en Anmærkning, at det ikke er ren Cellulose, der udfældes, da den indeholder mere Kulstof end denne paa Grund af et Indhold af „Methyl-Methoxyl- eller Acetylgrupper“²⁾. BUNCKE & WOLFFENSTEIN har allerede 1899 vist, at der af Cellulose ved Behandling med Schweizer's Reagens og Udfældning af det opløste med Syre opstaar Acidcellulose, der ved Udvaskning og Tørring under Afgivelse af Vand gaar over til en Lakton ($C_{36}H_{60}O_{31}$) med andre kemiske og helt andre fysiske Egenskaber end Cellulose³⁾.

¹⁾ J. KÖNIG: „Bestimmung der Cellulose des Lignins und Kutins in der Rohfaser“. Zeitschrift f. Untersuchung d. Nahrungs- und Genussmittel. 12. Bd. 1906. S. 391.

²⁾ J. KÖNIG: «Die Untersuchung landw. & gewerbl. Stff.» S. 1057.

³⁾ G. BUNCKE & R. WOLFFENSTEIN: B. d. D. ch. G. 32 II (1899) S. 2493 o. flg.

I Overensstemmelse hermed viste Pektocellulose følgende Forhold: 0,7647 grm Pektocellulose (beregnet askefri) af Hundegræs Halm rystes i 2 Timer i Rystemaskine med 200 cm³ konc. Kobberilteammoniak, filtreres gennem Asbest i Goochdigel og udvaskes med Ammoniak og kogende Vand. Der tilbagebliver efter Tørring 0,0318 grm uopløst = 4,16 % af Pektocellulose. Opløsningen gøres svagt sur med fortyndet Svovlsyre og udskiller et hvidt, fnugget amorft Bundfald, der filtreres fra paa et vejlet Filter, udvaskes med Vand, Vinaand, Æter, suges tørt og tørres ved 100°. Det danner nu amorfe hornagtige Klumper, som beskrevet af BUNCKE & WOLFFENSTEIN. Vægten udgjorde 0,6260 grm med kun 0,0004 grm Aske (Kiselsyre). Den askefri Lakton er altsaa 0,6256 grm = 81,8 % af den anvendte Pektocellulose.

Ved Tilberedning af Schweizer's Reagens har MAX RENKER angivet en meget praktisk Metode¹⁾, som bestaar i at fordele Kobberdrejespaaner i to større Pulverglas med indsleben Prop og overhælde Kobberet i det ene Glas med højst koncentreret Ammoniakvand. Ved at hælde Opløsningen skiftevis fra det ene Glas til det andet, kan man, medens der ved Luftens Indvirkning foregaar en kraftig Iltning af Kobberet i det tømte Glas, i Løbet af nogle Timer tilberede en meget stærk Opløsning af Kobberilteammoniak, der kun behøver at filtreres gennem Asbest eller Glasuld for at være færdig til Brug. Herved undgaar man næsten helt det Tab af Ammoniak, som i betydelig Grad finder Sted ved den af CROSS & BEVAN anbefalede af C. R. A. WRIGHT benyttede Metode, som bestaar i at suge Luft gennem Ammoniakvand, der dækker Ruller af tynde Kobberblade.²⁾ Iøvrigt beror hverken Renker's eller Wright's „Metoder“ paa noget nyt af dem iagttaget Forhold. Allerede PÉLIGOT anbefalede 1858 at fremstille Kobberilteammoniak ved at ryste Kobber med Ammoniakvand og Luft.³⁾ At Kobber angribes af Ammoniak, har iøvrigt været kendt fra ældgammel Tid. Det vil ogsaa her være Stedet at gøre opmærksom paa en anden kuriøs, men temmelig udbredt Fejltagelse. Flere Steder i den nyere Celluloselitteratur ser man anført som originale Iagttagelser af paagældende Forskere det ret interessante Forhold, at Kobberilteammoniak, der indeholder Cellulose i Opløsning, ved at henstaa i Luften kommer til at indeholde Nitrit. Dette blev ganske bekræftet ved Forf.'s Undersøgelser. En filtreret Opløsning af Cellulose i Kobberilteammoniak, der havde henstaaet 4 Døgn i et Bærgglas dækket med en Glasplade, gav ved at blive gjort svagt sur med Svovlsyre en meget stærk Udvikling af „røde Damp“ af Kvælstofoverilte. Adskillige Steder ser man anført, at denne Nitritdannelse skyldes Cellulosens Tilstedeværelse i Opløsningen, hvad der jo ogsaa for saa vidt maa siges at være en nærliggende Slutning. CROSS & BEVAN anfører saaledes, at PRUDHOMME¹⁾ i 1891 er den første, der har gjort denne Iagttagelse, og

¹⁾ MAX RENKER: *anf. Std.* S. 23.

²⁾ CROSS & BEVAN: *anf. Std.* S. 10.

³⁾ PÉLIGOT: *Compt. rendu.* 47 1858, S. 344.

¹⁾ PRUDHOMME: „*Journal of the society of dyers and colourists*“, 1891 S. 148. Anført efter CROSS & BEVAN S. 11.

nyere Forfattere anfører den samme Iagttagelse, endog som noget de selv har Prioriteten paa. At Cellulosen skulde have nogen Indflydelse paa Iltningen af Kobberilteammoniak er dog paa Forhaand aldeles usandsynligt, da den, som det vil vides, indvirker stærkt reducerende paa Fehlings's Væske. Iltningen maa nødvendigvis være en Proces som foregaar uden Cellulosens Medhjælp, vel endog til Trods for Cellulosens reducerende Indvirkning. Ved nærmere Undersøgelse fremgik det ogsaa tydeligt nok, at det er Kobberilteammoniaken, der med Kobberilte som Katalysator ilter sig, uden at Cellulosen spiller nogen Rolle. Nitritdannelsen i Kobberilteammoniak uden Cellulose er ikke nogen ny Opdagelse, men en gammelkendt Sag. MALAGUTTI & SARZEAU har allerede 1818 paavist, at der opstaar Nitrit ved Indvirkning af Ammoniak paa Kobberilte under Luftens Adgang, og SCHÖNBEIN har i 1856 paavist det samme Forhold, naar Ammoniak og Luft indvirker paa Kobber, og senere er Forholdet ogsaa studeret af andre¹⁾.

Ved at udføre Opløsningsforsøg med Pektocellulose og Kobberilteammoniak blev der fundet følgende Forhold, som angivet i Tab. 38. Her er tillige i 4de Kolonne angivet Mængden af Kutin i Hø- og Halmtørstoffet fundet ved at benytte 3die Kolonne i Tab. 38, Tab. 27 og Tab. 34.

Tabel 38.

Kutinbestemmelse i Pektocellulose fra Græs- og Kløverarterne.

Stoffets Art	Anvendt Pektocellulose Askefri	Fundet Mængde Kutin	Kutin	
			Procent af Pektocellulose	Procent af Hø- eller Halmtørstoffet
A ₁ Italiensk Rajgræs Hø	1,1846	0,0060	0,51	0,16
A ₂ — Halm	0,6403	0,0055	0,86	0,27
B ₁ Draphavre Hø	0,3282	0,0055	1,68	0,65
B ₂ — Halm	0,3705	0,0080	2,16	0,92
C ₁ Agerhejre Hø	0,3572	0,0042	1,18	0,42
C ₂ — Halm	0,4013	0,0085	2,18	1,03
D ₁ Hundegræs Hø	0,2896	0,0105	3,66	1,35
D ₂ — Halm	{ 0,7647	{ 0,0318	4,16	} 1,53
	{ 0,6085	{ 0,0245	4,03	
E ₁ Engsvingel Hø	0,3625	0,0085	2,34	0,86
E ₂ — Halm	0,4585	0,0108	2,36	0,95
F ₁ Sildig Rødkløver Hø	0,3448	0,0120	3,48	0,91
F ₂ — Halm	0,2198	0,0080	3,64	1,51
G ₁ Kællingetand Hø	0,2303	0,0110	6,08	1,56
G ₂ — Halm	0,2103	0,0115	5,47	2,40
H ₁ Gul Rundbælg Hø	0,3877	0,0145	3,74	1,01
H ₂ — Halm	0,3026	0,0100	3,90	1,46

4de Kolonne i Tab. 38 viser, at i alle Planterne, hvad enten det er Græsser

¹⁾ Smgl. GMELIN-KRAUT: Handbuch der Anorg. Chemie 6te Aufl. III 1875 S. 652.

7de Aufl. V (1) 1909 S. 792.

eller Kloverarter, stiger Kutinmængden med Planternes Alder, hvad der, hvis Kutin er en Dannelse svarende til Kork, var at vente.

Endelig kan der ved at benytte 4de Kolonne i Tab. 38, der angiver Mængden af Kutin i Tørstoffet, og de i Tab. 27 og Tab. 34 angivne Procentmængder af Pektocellulose i Tørstoffet naas til følgende Tal i Tab. 39, der angiver Procentmængderne af kutinfrit og askefrit Pektocellulose i Tørstof.

Tabel 39.

Kutinfri Pektocellulose. Procent af Tørstof							
Græsarter			Kløverarter				
A ₁	Italiensk Rajgræs	Hø	31,49 ⁰ / ₁₀	F ₁ Sildig Rødkløver	Hø	22,81 ⁰ / ₁₀	
A ₂	—	Halm	31,30	F ₂	Halm	41,99	
B ₁	Drophavre	Hø	38,07	G ₁	Kællingetand	Hø	24,17
B ₂	—	Halm	41,75	G ₂	—	Halm	41,64
C ₁	Agerhejre	Hø	35,03	H ₁	Gul Rundbælg	Hø	26,06
C ₂	—	Halm	46,35	H ₂	—	Halm	47,40
D ₁	Hundegræs	Hø	35,73				
D ₂	—	Halm	35,84				
E ₁	Engsvingel	Hø	35,82				
E ₂	—	Halm	39,18				

De ved Tabellerne 27 og 34 anførte Bemærkninger om Pektocelluloseprocenterne i Tørstoffet gælder i forstærket Grad om den kutinfri Pektocellulose i Tab. 39, saa at det vil være unødvendigt at gentage dem her.

Forsøg paa Bestemmelse af Cellulose ved Glycerin-Svovlsyre Metoden.

En Metode, der nærmest gaar ud paa ved Hydrolyse at befri Cellulosen for alle andre Kulhydrater og tillige ved Ophedning med et Opløsningsmiddel at faa andre indblandede Stoffer (Æggeghidestoffer og Fedt m. m.) bortskaffet, er den af J. KÖNIG angivne Glycerin-Svovlsyre Metode¹⁾. Metoden vinder stedse større Tilslutning ved agrikulturkemiske Undersøgelser, men giver i den oprindelige Form ganske vist ikke noget helt pentosanfrit Produkt, selv om Mængden er bragt stærkt ned. E. KRÖBER har saaledes i König-Cellulose af Enghø kun fundet 0,45–0,59 ⁰/₁₀ Pentosan, medens de samme Stoffer, naar Cellulosen blev fremstillet efter Weende Metoden, gav over 4 ⁰/₁₀ Pentosan²⁾. Som Max Renker, der har prøvet Metoden ved Cellulose af forskellig Oprindelse, ogsaa angiver, er König-Cellulose, saaledes som den umiddelbart fremkommer efter Ophedning med Glycerin-Svovlsyre, selv naar Sulfitcellulose danner Udgangspunktet, brunlig farvet³⁾. König har derfor ogsaa fuldstændiggjort Metoden ved at benytte en Blegning med Ammoniakvand og Brintoverilte, hvorved tillige de ligninagtige Stoffer, som König-Cellulosen maatte

¹⁾ J. KÖNIG: Ztschr. f. Unters. d. Nahrungs und Genussmittel 1. 8 (1898).

— Die Unters. landw. und gewerbl. wicht. Stff. 3te Aufl. 1906. S. 249.

²⁾ Jahresb. f. Agriculturchemie 1901 S. 146.

³⁾ MAX RENKER: anf. Std. S. 36.

indeholde, fjernes¹⁾. Ved at ophede i en Kolbe med tilbagegaaende Svaleapparat en afvejet Mængde af Stoffet med Glycerin-Svovlsyre efter Königs Angivelse²⁾ i en Time til 133° (Temperaturen stiger ikke højere i Luftbad), og efter Fortynding med varmt Vand og Udvaskning tilsidst med Vinaand og Æter, faas af de her omhandlede Hø- og Halmprøver et mørkt farvet Produkt. Ved Behandling med Ammoniakvand, der bliver stærkt farvet, og paafølgende Blegning med Kaliumpermanganat og Svovlsyring, som beskrevet ved Blegning af Pektocellulosen, faas derimod et ret smukt hvidt Produkt, som blev udvasket tilsidst med Vinaand og Æter og tørret ved 100°. I en aliquot Del blev der derefter foretaget Askebestemmelse. For Græsarterne er der 1,5—2,0 % Forskel i Vægten af den brune Raacellulose (beregnet askefri) og den blegede rene König-Cellulose (beregnet askefri), men for Kløverarterne er Forskellen meget større, op til 6 %. Den blegede König-Cellulose synes at være fuldstændig kvælstoffri og giver ikke paaviselige Mængder af Furfurol, naar 0,5—1,0 grm destilleres med Saltsyre. Derimod er den ikke askefri, men indeholder Askemængder af ganske samme Størrelsesforhold som Pektocelluloserne, saaledes som omtalt S. 45.

Til Analyserne blev benyttet 5,00 grm. lufttørt Stof, der blev ophedet med c. 350 cm³ Glycerin-Svovlsyre i 1 Time og behandlet videre som ovenfor beskrevet. Inden Temperaturen er steget til 133°, ved hvilken den holder sig konstant, er Høprøverne, i Særdeleshed Kløverhøet, meget tilbøjeligt til at skumme, saa at Ophedningen maa foretages forsigtigt under jævnlig Omrystning uden Svaleapparat, til Temperaturen er steget saaledes, at Vandet i Prøverne tildels er bortdestilleret. I flere Tilfælde blev der til Kontrol foretaget Duplikatbestemmelser (med mindre Stofmængder); de afveg kun i et enkelt nedenfor angivet Tilfælde over 0,1 % fra de anførte Tal.

Ved Benyttelsen af 5,00 grm lufttørt Stof³⁾ blev fundet følgende Forhold:

Tabel 40.

Cellulose ved Glycerin-Svovlsyremetoden.

Græsarterne.		Askefri Cellulose	Procent af Tørstof
		Gram	
A ₁	Italiensk Rajgræs Hø	1,1642	25,14
A ₂	— Halm	1,1859	25,10
B ₁	Draphayre Hø	1,2165	25,52
B ₂	— Halm	1,1006	28,43
C ₁	Agerhejre Hø	1,2737	26,26
C ₂	— Halm	1,6624	34,08
D ₁	Hundegræs Hø	1,3182	28,54
D ₂	— Halm	1,3144	28,80 ⁴⁾

¹⁾ J. KÖNIG: sidst anf. Std. S. 1057.

²⁾ Glycerin (Vægtfld. 1,23) med 20 grm konc. Svovlsyre pr. Liter.

³⁾ Tørstofprocenten er angivet i Tabel 4—5.

⁴⁾ Ved at benytte Pektocellulosen til Glycerin-Svovlsyre Behandlingen blev fundet i to Forsøg 27,88 %—28,26 %, i Middeltal 28,05 %.

Tabel 40. (fortsat).

	Askefri Cellulose Gram	Procent af Tørstof
E ₁ Engsvingel Hø	1,2645	27,60
E ₂ — Halm	1,4565	30,40
Kløverarterne.		
F ₁ Sildig Rødkløver Hø	0,8002	17,46
F ₂ — Halm	1,4205	30,88
G ₁ Kællingetand Hø	0,9108	18,74
G ₂ — Halm	1,6901	34,24
H ₁ Gul Rundbælg Hø	0,9667	20,44
H ₂ — Halm		36,65 ¹⁾

Den ved Glycerin-Svovlsyre Metoden fundne Cellulosemængde er, som man vil se ved Sammenligning med Tab. 27 og Tab. 34, og som det var at vente, betydelig mindre end Mængden af Pektocellulose i de samme Stoffer. I de 10 Prøver af Græsarterne udgør König-Cellulose i Middeltal 74,24 % af Pektocellulose²⁾, og i de 6 Prøver af Kløverarterne er Middeltallet næsten det samme, 74,30 %. Middeltallet af furfurolgivende Stoffer i Pektocellulosen beregnet som Pentosan udgør for Græsserne 15,5 % og for Kløverarterne 13,7 % af Pektocellulosen (Smlg. Tab. 36—37), saa at Summen af Cellulose + Pentosan i Procent af Pektocellulose for Græsserne er 89,7 % og for Kløverarterne 88,0 %. De manglende 10—12 % udgøres af 2 % Urenheder (Kutin), og Resten, de 8—10 %, er de pektinagtige Stoffer, som findes i Pektocellulosen, og som paa forskellig Maade giver sig til Kende, naar man søger at skille Stoffet ad i dets nærmere Bestanddele, men som man ingen Midler har til direkte at bestemme Mængden af. Da man ikke har Sikkerhed for, at König-Cellulose, saaledes som den foreligger, er ren Cellulose og tillige indeholder al den rene Cellulose, der findes i Pektocellulosemolekulet, lader der sig for Tiden næppe drage yderligere Slutninger af de fundne Reaktionsforhold. En nærmere Udredning af disse Forhold vil være af den største Interesse baade i rent videnskabelig og i agrikulturkemisk Henseende. Vejen til at løse disse Spørgsmaal maa dog være en lidl forskellig fra den her fulgte, hvor praktiske Hensyn har været de ledende. De angivne Plantearter er ganske vist undersøgt paa to bestemte Tidspunkter af Vækstperioden, men er dog først omdannet til „Hø“ og „Halm“, saaledes som de anvendes i Landbrugets Praksis. Vil man forsøge paa at klare Cellulose-

¹⁾ Middeltal af to Bestemmelser, hver paa 5,000 grm: 38,51 % 34,80 %, hvoraf den første endnu var noget brunlig, den anden vistnok for stærkt bleget.

²⁾ De fleste Prøver afviger kun faa Procent fra disse Middeltal (Max. 79,50 %, Min. 71,10 %), men Draphavre har i denne Henseende en Særstilling. König-Cellulosen er i B₁ = 65,9 % af Pektocellulose og i B₂ = 66,6 %. Til Gengæld findes en større Mængde furfurolgivende Stoffer, der beregnet som Pentosan i B₁ er = 19,4 % af Pektocellulose og i B₂ er = 20,9 %, saa at Cellulose + Pentosan bliver omtrent det samme (85,3 % for B₁ og 87,7 % for B₂ som ovenfor angivet.

spørgsmaalet videnskabeligt, maa man utvivlsomt benytte som Undersøgelsesmateriale en enkelt eller nogle faa som typiske udvalgte Plantearter og gennem Undersøgelser af det friske grønne Materiale paa forskellige Vækststadier faa Klarhed paa Stofferne Omskiften.

Sammenligning med andre Analyser af de samme Plantearter.

I det foregaaende er der, hvor Lejlighed tilbød sig, udført Sammenligninger mellem enkelte af de her foreliggende analytiske Resultater og Analyser af de samme Plantearter foretaget af andre. Men der er ogsaa fremhævet Vanskeligheden ved at finde virkelig anvendeligt Sammenligningsmateriale, da det trods de mangfoldige „Hø“- og „Halm“-Analyser, der foreligger, er sjældent at finde brugbare Analyser af de enkelte Plantearter. Fra 1912 foreligger der dog fra M. B. ISBECQUE i Belgien en Række Analyser af Græsser, hvoraf tre er de samme Arter¹⁾. De otte Græsarter Isbecque har analyseret er:

<i>Agrostis vulgaris.</i>	<i>Dactylis glomerata</i> ²⁾ .
<i>Alopecurus pratensis.</i>	<i>Festuca elatior.</i>
<i>Avena elatior.</i>	<i>Holcus lanatus.</i>
<i>Bromus erectus.</i>	<i>Lolium italicum.</i>

Græsarterne blev dyrket i Renkultur i den botaniske Have i Gembloux „dans les conditions culturales les plus favorables“, hvad ogsaa giver sig til Kende i deres Sammensætning. Analysemetoderne m. H. t. Bestemmelse af Vand, Aske, Fedt og Pentosan³⁾ er ganske de samme som anvendt her. Æggeghvidestof er bestemt ved SCHJERNING's Metode med Uranacetat, Cellulose ved KÖNIG's Metode med Glycerin-Svovlsyre først som Raacellulose og bagefter Iltning med Ammoniakvand og Brintoverilte. Det saaledes udvundne Stof ÷ Kutinindholdet er betegnet som Rencellulose. Da hverken „Raacellulose“ eller „Rencellulose“ efter Isbecque's Fremgangsmaader kan svare ganske nøjagtigt til de ved de her benyttede Metoder fremstillede Præparater af König-Cellulose, men man paa Forhaand maa være tilbøjelig til at antage, at Procentmængden af denne sidste maa ligge imellem Procentmængderne af Isbecque's Raa- og Rencellulose, er Procenttallene for begge disse Stoffer angivet. For at kunne sammenligne Isbecque's Tal med de her foretagne Analyser maa der udføres en Omregning, da Isbecque's Procenttal refererer sig til Stofferne med et Indhold af 10–14⁰ „Vand, medens de i denne Afhandling

¹⁾ M. B. ISBECQUE: „Analyse des graminées constituant habituellement nos prairies“. Ann. de Gembloux **21**. 1911 (Bruxelles 1912). S. 334 o. flg.

²⁾ I alle Listerne benævnes den *Dactylus glomeratus*.

³⁾ Pentosaner er bestemt baade ved Fældning med Barbitursyre og med Floroglucin, som allerede omtalt S. 28.

angivne Tal er Procenter af Tørstoffet. Man kommer ved en saadan Omregning til nedenstaaende Tal, hvortil som Sammenligningspunkter er anført Procentindholdet i de samme Plantearter af dansk Avl efter de her foreliggende Undersøgelser.

Tabel 41.

Procenter af Tørstof.

Sammenligning mellem Græs-arter af belgisk og dansk Avl.	Italiensk Rajgræs Hø		Draphavre Hø		Hundegræs Hø	
	Belgisk Avl	Dansk Avl	Belgisk Avl	Dansk Avl	Belgisk Avl	Dansk Avl
Aske	7,35	7,01	6,40	5,71	9,59 ¹⁾	7,38
Fedt	4,19	2,97	2,55	2,24	4,91	2,84
Æggehvidestof	5,09	5,10	7,62	5,10	9,20	5,46
Total Pentosan	19,30	19,89	24,65	21,39	21,34	20,34
Raacellulose	27,35		35,27		32,92	
König-Cellulose		25,14		25,52		28,54
Rencellulose	19,07		29,51		27,43	

Italiensk Rajgræs af belgisk Avl har noget større Fedtindhold end den danske Prøve, Draphavre har noget mere Æggehvidestof og Hundegræs af belgisk Avl baade mere Fedt og Æggehvidestof end den danske Prøve, hvad der maaske kan tilskrives de noget bedre Livsbetingelser, de belgiske Planter har haft, end de tilsvarende danske, men iøvrigt varierer Indholdet af de samme Stoffer ikke synderlig meget fra hinanden indenfor samme Planteart.

Fra Danmark har man gennem Statens Forsøgsvirksomhed i Plantekultur ved Landbrugscand. R. K. KRISTENSEN Beretning om et betydeligt Arbejde, der er udført ved Forsøgsstationerne i Askov over partielle Bestemmelser af Græsmarks Planter, der er dyrket i Aarene 1905—1909²⁾. Planterne voksede dels paa „Lermarken“, dels paa „Sandmarken“ og dels paa „Mosen“. Planterne af sidstnævnte Oprindelse er udeladt i denne Sammenhæng. Angaaende Tilvejebringelsen af Stoffet bemærkes følgende:

„Ved en Sammenligning mellem de enkelte Arters Indhold af de Stoffer, hvorom der her er Tale, maa det erindres, at ved Forsøgene, som Analysematerialet stammer fra, er disse Arter voksede i Blanding og høstede samtidigt. . . . De enkelte Arters Indhold af Kvælstof, Kali og Fosforsyre er for Lermarkens og Sandmarkens Vedkommende opført i Tabel 9. Da Planterne fra disse Afdelinger stillede sig temmelig ens, og der ikke her kunde spores nogen Indflydelse af Vejrliget i de forskellige Aar, er Materialet behandlet under eet paa den Maade, at der simpelthen er taget Gennemsnit af alle de foreliggende Analyser uden Hensyn til, om dette er repræsenteret med nøjagtig lige mange Analyser hvert Aar eller paa hver Afdeling. . . . Hundegræs er behandlet som to Arter, idet Analyserne fra 1. Aars Markerne er holdt for sig og Analyserne fra de ældre Græsmarker for sig“.

¹⁾ Har næppe været „Renaske“, men har indeholdt lidt Sand eller Ler.

²⁾ Tidsskrift for Landbrugets Planteavl 4. 1911 S. 543 o. flg.

Der er bestemt Mængden af Totalkvælstof (N), Kali (K_2O), Fosforsyre (P_2O_5), og Mængden er beregnet som Procenter af Tørstoffet. Analysemetoderne er saavidt vides væsentlig de samme, som de i denne Afhandling anvendte.

Resultaterne af Undersøgelserne er anført i nedenstaaende Tab. 42. Øverste Talrække i hver vandret Kolonne hidrører fra Askov Forsøgsstations Undersøgelser; nederste Talrække hidrører derimod fra Forf.s Undersøgelser og er mærket med *.

Tabel 42.

Indholdet af Kvælstof, Kali og Fosforsyre i Hø.
Procent af Tørstof.

		Kvælstof N 0/0	Kali K_2O 0/0	Fosforsyre P_2O_5 0/0
Italiensk Rajgræs.	Askov.	0,80	1,40	0,44
—	*	0,99	1,81	0,51
Draphavre.	Askov.	0,82	1,57	0,49
—	*	1,03	1,63	0,42
Agerhejre.	Askov.	0,99	1,71	0,58
—	*	1,20	2,19	0,72
Hundegræs I.	Askov.	1,18		
— II.	Askov.	0,79	1,64	0,58
—	*	1,06	1,47	0,42
Engsvingel.	Askov.	0,81	1,57	0,53
—	*	1,17 ¹⁾	2,21	0,58
Sildig Rødkløver.	Askov.	2,55	1,20	0,61
—	*	3,01	1,01	0,49
Kællingetand.	Askov.	2,23	0,97	0,53
—	*	2,54	0,87	0,38
Gul Rundbælg.	Askov.	2,06	1,17	0,58
—	*	2,54	1,39	0,49

Askov Analyserne, der hver repræsenterer fra 7—20 enkelte Analyser af forskellige Prøver af samme Planteart, maa siges gennemgaaende at vise god Overensstemmelse med de her udførte Analyser, der, som det vil vides, kun repræsenterer en enkelt Prøve af samme Planteart²⁾. Med Hensyn til selve de analytiske Bestemmelseres Udførelse, er der for Kvælstof og Kali intet at bemærke udover det tidligere anførte under Omtalen af Askebestemmelserne. Derimod maa der dvæles lidt ved Fosforsyrebestemmelserne. De er, som tidligere omtalt, udført ved at forkulle Stoffet ved meget svag Varme og derefter brænde Kullet bort ved stærkere Varme. I den derved opstaaede Aske er Fosforsyren bestemt som Molybdat, efter at Kiselsyren er fraskilt ved Inddampning med Saltsyre. I den nyeste Tid er der dog fremkommet Undersøgelser, som viser, at det er sandsynligt, at der ved

¹⁾ Engsvingel Halm indeholder 0,44 0/0 N.

²⁾ Det fremgaar dog af Analyserne, at Askov-Prøverne utvivlsomt er høstet paa et gennemgaaende lidt mere fremskredne Modningsstadium end de i denne Afh. omhandlede Prøver.

saadan direkte Indaskning kan opstaa et Tab af Fosforsyre. E. FLEURENT og L. LEVI søger saaledes at vise, at det navnlig er fedtrige Stoffer, der giver Anledning til, at Fosforbindelser forflygtiges under Indaskningen¹⁾. De blander derfor 10 grm af Stoffet med Kalkmælk, der indeholder 0,150 grm Ca O, og indtørrer. Massen forkulles derefter først i en Kulsyrestrøm og brændes derefter i Ilt efter TH. SCHLOESSING's Metode²⁾. De finder derefter beregnet for 1 Kilogram:

Tabel 43.

Fosforsyremængden.

Efter E. FLEURENT & L. LEVI.	Ved direkte Forbrænding	Ved Forbrænding med Kalk.	Differents beregnet som Procent af Aske
Blé de la région de Paris	15,444 grm	16,916 grm	8,88 %
Blé russe Ghirka	17,688 —	19,193 —	7,96
Blé dur Taganrog	18,840 —	20,339 —	7,37

Der er dog i Fleurent's Afhandling slet ingen Oplysninger om, hvor store Variationerne kan være ved Analyser af samme Stof foretaget paa samme Maade.

A. PONTE har ved en italiensk Landbrugsstation gjort lignende Forsøg med Fosforsyrebestemmelser: 1) efter direkte Forbrænding, 2) ved elektrisk Forbrænding og 3) ved Forbrænding efter Tilsætning af Kalk. Fosforsyrebestemmelser efter Forbrænding paa de to sidstnævnte Maader stemmer godt sammen og giver gennemgaaende lidt højere Værdier end Fosforsyre efter direkte Forbrænding. Forskellen er fra 2 % til 11 % af Fosforsyremængden³⁾. Det lader sig herefter ikke benægte, at skal man have udført meget nøjagtige Bestemmelser af Fosforsyre i organisk Stof, vil en Forbrænding af Stoffet efter Tilsætning af Kalkmælk være at foretrække for en direkte Forbrænding, men ved de her omhandlede Stoffer, hvor Fosforsyremængden i intet Tilfælde gik op over 0,58 % af Tørstoffet, er det dog næppe det betydeligt forøgede Arbejde værd at foretage Bestemmelser efter Fleurent's Forslag. Forf. har foretaget en Række sammenlignende Fosforsyrebestemmelser i Hø ved direkte Forbrænding og ved Forbrænding efter Tilsætning af Kalkmælk. De sidste er ganske vist alle højere end de første, men Forskellen er kun nogle faa Enheder i 2den Decimal af Procenterne. Næppe nogen kan drage synderlig vidtgaaende agrikulturkemiske Slutninger af om, man for at vælge af de mest yderliggaaende Eksempler i Tørstoffet af Italiensk Rajgræs Hø ved direkte Indaskning finder 0,51 P₂ O₅ og ved Forbrænding med Kalk 0,57 P₂ O₅.

Hvor det derimod gælder om at konstatere en mulig Forskel i Fosforsyreindholdet i forskellige Organer af samme Plante eller ved rent videnskabelige plantefysiologiske Forsøg, vil FLEURENT's besværligere, men nøjagtigere Metode være at benytte.

¹⁾ E. FLEURENT & LUCIEN LEVI: „Sur une méthode de détermination exacte des cendres dans l'analyse des matières végétales et animales“. Compt. rend. 152. Paris 1911. S. 715 o. flg.

²⁾ Encyclopédie chimique „Chimie agricole“ p. 224 o. flg. (citeret efter Fleurent).

³⁾ A. PONTE: Le Staz. sperim. agrar. ital. 44 1911 S. 459—460.

VIII. Oversigt.

For at kunne vinde en Oversigt over Indholdet af de forskellige Stofgrupper i Hø- og Halmprøverne er Resultaterne sammenstillet i Tab. 44—48. I Tab. 44 er gengivet Procentindholdet dels af Tørstof og Vand i de foreliggende Hø- og Halmprøver, dels af de forskellige Stofgrupper i Tørstoffet. Da nogle af de angivne Stoffer tildels indgaar som Bestanddele i andre af de angivne Stoffer (f. Eks. Æggehvidekvælstof og Amidkvælstof i Totalkvælstof, Bestanddele af „Total Pentosan“ i „Fri Pentosan“ og i Pektocellulose) kan man ikke ved at summere Tallene i Kolonnerne i Tab. 44 faa et Tal, der angiver Procentindholdet af alle bestemte Bestanddele. Dette fremgaar derimod af Tab. 45—48, hvori tillige findes angivet Middelsammensætningen af Høet og Halmen dels af Græsserne dels af Kløverarterne.

Ved Benyttelsen af disse Tabeller maa erindres følgende:

Æggehvidestofprocenten er, som tidligere angivet, paa sædvanlig Vis beregnet af det ved Stutzer's Metode bestemte Æggehvidekvælstof ved Multiplikation med Faktoren 6. For at faa et til Regning brugeligt Tal for Amidstofferne, er Amidkvælstoffet multipliceret med Faktoren 5, hvad der svarer til, om Amidstofferne bestod af omtrent lige Dele Asparagin (med 21,2 % N) og Glutamin (med 19,2 % N). For Græsserne, hvor Procentindholdet af Amidkvælstof er saa ringe, 0,14 % i Middeltal af 10 Bestemmelser, kan en saadan Beregning næppe volde større Betænkeligheder, ikke heller for Kløverhalmens Vedkommende, hvor der ikke er mere Amidkvælstof end i Græsserne, medens det for Kløverhøet, hvor Middeltallet af Amidkvælstof er 0,58 % og i Gul Rundbælg, hvor det gaar op til 0,88 %, maa erkendes at være meget usikkert. Er „Amidstofferne“ i Kløverhø f. Eks. for en væsentlig Del Xantin (med 36,8 % N) eller Allantoin (med 35,4 % N), der begge er paavist i unge Planter, kan Faktoren ikke være over 3, saa at den i Tabellen angivne Mængde af Amidstoffer i saa Fald er betydeligt for høj (maaske kun $\frac{3}{5}$ af det anførte Tal).

Invertsukker repræsenterer som tidligere nævnt forskellige Sukkerarter, saa at Betegnelsen egentlig burde være Sukkerarter ækvivalente med angivne Mængde Invertsukker m. H. t. Forholdet overfor Fehling's Væske.

Fri Pentosan er, som det fremgaar af det foregaaende, Differentsten mellem Total Pentosan, saaledes som den lader sig bestemme ved at destillere selve Stoffet med Saltsyre, og den Mængde Pentosan, der lader sig beregne af Furfurolmængden, som Pektocellulosen afgiver ved Saltsyredestillation.

Kutinfri Pektocellulose er fundet ved at bestemme Pektocellulose paa tidligere angivne Maade og fra det fundne Tal dels fradrage Askemængden i Pektocellulosen, dels Kutinmængden bestemt ved Opløsning i Schweizer's Reagens.

Som man vil se af Tab. 45, er den samlede Mængde af alle de bestemte Stoffer i Høet af Græsarterne 72,68 %, hvorefter Mængden af ubestemt Stof udgør 27,32 %.

Disse Stofmængder blev i ældre Analyser opført som kvælstoffri Extraktstoffer, men, som det forlængst fra anden Side er vist, bør denne Betegnelse ikke benyttes¹⁾, navnlig da den bringer Forvirring i Praksis ved Forstaaelsen af Resultaterne af den kemiske Analyse af Foderstoffer, da man ved en saadan Betegnelse er tilbøjelig til at tro, at der under Rubriken kvælstoffri Extraktstoffer foreligger en virkelig Bestemmelse af et tilstedeværende Stof, medens Rubriken dels indbefatter en Summation af de uundgaelige Forsøgsfejl, som alle de virkelig bestemte Stoffer er behæftet med, dels Mængden af Stoffer, som netop ikke har kunnet bestemmes kvantitativt. Jo flere Stofgrupper det lykkes at fastslaa Procentmængden af, desto mindre vil „de kvælstoffri Extraktstoffer's“ Mængde blive.

Mængden af ubestemt Stof udgør i Middeltal:

For Græsarterne	{	Høet	27,32 ⁰ / ₁₀	af Tørstof
		Halmen	34,15 ⁰ / ₁₀	—
For Kløverarterne	{	Høet	37,85 ⁰ / ₁₀	—
		Halmen	33,21 ⁰ / ₁₀	—

Hvis man vil betragte Mængdeforholdene af de egentlige Næringsstoffer Fedt + Æggehvidestof + Sukker i Hø- og Halmprøverne, bliver Middeltallene for Procentindholdet i Tørstoffet:

For Græsarterne	{	Høet	15,38 ⁰ / ₁₀	Fedt + Æggehvidestof + Sukker.
		Halmen	7,10 ⁰ / ₁₀	— — —
For Kløverarterne	{	Høet	18,39 ⁰ / ₁₀	— — —
		Halmen	7,76 ⁰ / ₁₀	— — —

Kløverarternes Hø staar altsaa højest i Næringsstofindhold og har som Tab. 47 viser ogsaa langt det største Indhold af Æggehvidestof, 12,68 ⁰/₁₀ imod Græssernes Hø med 5,35 ⁰/₁₀ Æggehvidestof.

Regner man derimod Pentosanerne med til Næringsstoffer, bliver Forholdet:

For Græsarterne	{	Høet:	28,56 ⁰ / ₁₀	Fedt + Æggehvidestof + Sukker + Pentosan,
		Halmen:	19,96 ⁰ / ₁₀	— — —
For Kløverarterne	{	Høet:	25,16 ⁰ / ₁₀	— — —
		Halmen:	16,59 ⁰ / ₁₀	— — —

Det kan foreløbigt med Henblik paa Praksis næppe lønne sig at gaa stort videre ad kemisk Vej med det her foreliggende Stof, selv om der øjensynligt endnu er mange Spørgsmaal af videnskabelig Natur at løse, saasom nærmere Undersøgelse af „Raafedt“, af „Amidstofferne“, mulige Bestemmelser af Stivelse, hvoraf der dog kun synes at være meget lidt i de foreliggende Prøver, nærmere Undersøgelse af Sukkerarterne m. m. Det kan dog kun gennem fysiologiske Fodringsforsøg fastslaaes, hvormeget der fordøjes under forskellige Omstændigheder af de allerede kemisk bestemte Næringsstoffer, og tillige hvor stor Mængde Pektocellulose der fordøjes, om overhovedet noget. At en betydelig Mængde af dette Stof genfindes

¹⁾ Anf. 58de Beretning fra Forsøgslaboratoriet S. 103 o. flg.

og let kan udvindes baade af Køers og Hestes Exkrementer, er allerede tidligere antydet i denne Afhandling, men Spørgsmaalet herom ligger lidt udenfor den her behandlede Opgaves Ramme og kan kun løses ved fysiologisk Hjælp. Det bør dog maaske i denne Sammenhæng fremhæves, at Bestemmelse af den Art Cellulose, der i uforandret Tilstand passerer Planteædernes Fordøjelseskanal, og Bestemmelsen af det samme Stof i Foderplanterne og Foderstofferne i Øjeblikket maa siges at være upaatvivlelig den vigtigste Opgave indenfor den agrikulturkemiske Side af Fodringsproblemet.

Som ogsaa fremhævet før i denne Afhandling, er der fra Agrikulturkemiens Side benyttet mange Aar og et uhyre Arbejde paa at søge efter et uopnaeligt Ideal: den rene Cellulose. Metoderne til at bestemme dette ganske hypotetiske Stof i Fodermidlerne erkendes nu fra alle kompetente Sider ikke at føre til Maalet. Grunden er øjensynligt, at Cellulose ikke forekommer som saadan, hverken i det, der fodres med, eller i det Fordøjelsen levner af Foderstofferne. Derimod forekommer baade i den ene og den anden Art Produkter komplekse Forbindelser af forskellige Kulhydrater, der maaske nok i deres Grundkærne indeholder noget svarende til Cellulose, men tillige for en væsentlig Del Atomgrupper af anden Art. Man har altid tidligere forsøgt paa enhver tænkelig Maade at fremstille ren Cellulose af disse Komplexer, men forgæves. Enten blev det fremstillede Produkt ikke rent, eller hvis man naaede til et forholdsvis rent Produkt, var der gaaet saa meget tabt paa Vejen, at Fremgangsmaaden var uanvendelig som kvantitativ Bestemmelse. „Ren Cellulose“ var Undersøgelsens Maal; selve de komplekse Stofgrupper har man kun sjældent søgt at faa fat paa. Det er derfor endnu lidet, man med Sikkerhed kan fremføre om disse Stofgrupper, som vel nok tør betegnes som „Cellulosearter“, og meget Arbejde af ren videnskabelig kemisk Natur maa udføres, før der naas Klarhed. Forf. mener dog i det foregaaende bl. a. at have vist, at man ved at benytte den her tilrettelagte Brommetode kan naa til at bestemme en bestemt Stofgruppe, der maa antages at være en Pektocellulose. Det er et Atomkomplex, som forekommer i de undersøgte Græsser og Kløverarter under naturlige Forhold og udgør en meget betydelig Procentdel af Tørstoffet og med forholdsvis stor Nøjagtighed lader sig kvantitativt bestemme, naar man følger den angivne Metode. Samme Stofgruppe forekommer ligeledes i Fordøjelsesprodukterne og lader sig isolere af disse ved at benytte samme Metode, som anvendes overfor Fodermidlerne. Det er at haabe, at man herved kan naa et lille Skridt henad Vejen, der fører til fuldstændig kemisk Værdsættelse af de forskellige til Foder benyttede Plantearter under forskellige Omstændigheder samtidigt med at indvinde Oplysninger om Stofgruppernes Omsætninger i Planterne.

Tabel 44.

	I Torstøffet.															
	Italiensk Rajgræs		Draophavre		Agerhejre		Hundegræs		Engsvingel		Sildig Rødkløver		Kællinge- tand		Gul Rundbælg	
	A ₁	A ₂	B ₁	B ₂	C ₁	C ₂	D ₁	D ₂	E ₁	E ₂	F ₁	F ₂	G ₁	G ₂	H ₁	H ₂
	Ho	Halm	Ho	Halm	Ho	Halm	Ho	Halm	Ho	Halm	Ho	Halm	Ho	Halm	Ho	Halm
Af det lufttørre Stof Procent	Torstorf		Torstorf		Torstorf		Torstorf		Torstorf		Torstorf		Torstorf		Torstorf	
Vand	92,61	94,48	97,66	98,55	96,94	97,51	92,83	91,05	91,63	95,71	91,62	91,99	97,24	98,75	94,19	97,57
	7,39	5,52	2,34	1,45	3,06	2,49	7,17	8,95	8,37	4,29	8,32	8,01	2,76	1,05	5,81	2,13
Aske	7,01	5,20	5,71	5,68	7,28	4,51	7,38	7,15	8,95	6,36	8,90	5,20	6,15	3,98	10,91	3,19
Fedt	2,97	1,90	2,24	1,52	2,79	1,70	2,84	2,97	2,22	1,72	4,10	2,30	4,46	1,97	2,55	1,11
Total Kvælstof	0,39	0,42	1,63	0,54	1,20	0,38	1,06	0,61	1,17	0,44	3,01	1,54	2,54	1,21	2,54	0,69
Amid Kvælstof	0,11	0,09	0,13	0,11	0,37	0,03	0,15	0,05	0,20	0,08	0,41	0,11	0,16	0,16	0,88	0,18
Æggehvide Kvælstof	0,85	0,33	0,90	0,40	0,83	0,35	0,91	0,56	0,97	0,36	2,60	1,20	2,08	1,05	1,66	0,51
Æggehvitestof	5,10	1,98	5,10	2,40	4,98	2,10	5,16	3,36	5,82	2,16	15,60	7,20	12,18	6,30	9,96	3,06
Invertsukker	11,50	5,18	8,28	6,13	5,08	1,38	1,08	1,32	1,17	1,17	2,84	0,58	1,30	0,46	0,99	0,28
Total Pentosan	19,80	17,54	21,39	22,02	18,99	20,88	20,37	15,21	20,77	20,55	8,61	12,61	10,85	16,19	11,28	16,21
Fri Pentosan	15,97	13,40	13,89	13,08	14,32	12,50	15,21	10,15	16,17	13,65	5,89	7,67	7,07	9,76	7,04	8,86
Pektocellulose	31,65	31,57	38,52	42,67	35,15	47,38	37,08	37,37	36,68	40,11	23,72	43,33	25,73	44,04	27,07	48,86
Kutin	0,16	0,27	0,65	0,92	0,42	1,03	1,35	1,53	0,86	0,55	0,91	1,31	1,56	2,10	1,65	1,46
Kutinfri Pektocellulose	31,49	31,30	38,67	41,75	35,66	46,35	35,73	35,81	35,82	39,18	22,81	41,99	24,17	41,61	26,66	47,40
König-Cellulose	25,11	25,10	25,52	28,13	29,36	34,08	28,51	28,80	27,60	30,10	17,16	30,58	18,71	31,21	20,11	40,55

I Torstoffet.

Indholdet i Græsserne.

Tabel 45. Høprøverne.

	A ₁	B ₁	C ₁	D ₁	E ₁	Middeltal
Aske	7,01 ⁰ / ₁₀	5,71 ⁰ / ₁₀	7,28 ⁰ / ₁₀	7,38 ⁰ / ₁₀	8,65 ⁰ / ₁₀	7,21 ⁰ / ₁₀
Fedt	2,97	2,34	2,79	2,84	2,22	2,61
Æggehvidestof	5,10	5,40	4,98	5,16	5,82	5,35
Amidstoffer	0,70	0,65	1,85	0,75	1,00	0,99
Invertsukker	11,50	8,28	5,08	1,08	1,17	5,42
Fri Pentosan	15,97	13,89	14,32	15,21	16,47	15,18
Kutinfri Pektocellulose	31,49	38,07	35,03	35,73	35,82	35,23
Kutin	0,16	0,65	0,12	1,35	0,86	0,69
Ialt bestemte Stoffer	74,90	74,89	71,75	69,83	72,01	72,68

Tabel 46. Halmprøverne.

	A ₂	B ₂	C ₂	D ₂	E ₂	Middeltal
Aske	5,20 ⁰ / ₁₀	5,68 ⁰ / ₁₀	4,51 ⁰ / ₁₀	7,15 ⁰ / ₁₀	6,36 ⁰ / ₁₀	5,78 ⁰ / ₁₀
Fedt	1,90	1,52	1,70	2,97	1,72	1,96
Æggehvidestof	1,98	2,40	2,10	3,36	2,16	2,10
Amidstoffer	0,45	0,70	0,15	0,25	0,40	0,39
Invertsukker	5,18	6,13	1,38	1,32	1,17	3,04
Fri Pentosan	13,40	13,08	12,50	10,15	13,65	12,56
Kutinfri Pektocellulose	31,30	41,75	46,35	35,84	39,18	38,88
Kutin	0,27	0,92	1,03	1,53	0,95	0,94
Ialt bestemte Stoffer	59,68	72,18	69,72	62,57	65,59	65,95

Indholdet i Kløverarterne.

Tabel 47. Høprøverne.

	F ₁	G ₁	H ₁	Middeltal
Aske	8,90 ⁰ / ₁₀	6,15 ⁰ / ₁₀	10,94 ⁰ / ₁₀	8,66 ⁰ / ₁₀
Fedt	4,40	4,46	2,55	3,80
Æggehvidestof	15,60	12,48	9,96	12,68
Amidstoffer	2,05	2,30	4,40	2,92
Invertsukker	2,84	1,90	0,99	1,91
Fri Pentosan	5,89	7,07	7,04	6,67
Kutinfri Pektocellulose	22,81	24,17	26,06	24,35
Kutin	0,91	1,56	1,01	1,16
Ialt bestemte Stoffer	63,40	60,09	62,95	62,15

Tabel 48. Halmprøverne.

	F ₂	G ₂	H ₂	Middeltal
Aske	5,20 ⁰ / ₁₀	3,08 ⁰ / ₁₀	3,19 ⁰ / ₁₀	3,82 ⁰ / ₁₀
Fedt	2,30	1,97	1,14	1,80
Æggehvidestof	7,20	6,30	3,06	5,52
Amidstoffer	0,70	0,80	0,90	0,80
Invertsukker	0,58	0,46	0,28	0,44
Fri Pentosan	7,87	9,76	8,86	8,83
Kutinfri Pektocellulose	41,99	41,64	47,42	43,68
Kutin	1,54	2,40	1,46	1,80
Ialt bestemte Stoffer	67,38	66,41	66,29	66,69

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Rettelse.

S. 36 L. 12 f. o. BUNCKE læs BUMCKE.

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STUDIES
ON THE
CYCLOSTOMATA OPERCULATA

BY

G. M. R. LEVINSEN

WITH 7 PLATES AND 2 FIGURES IN THE TEXT

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURV. OG MATHEMATISK AFD. X. 1

KØBENHAVN

HØVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHADEL

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1. Warming, Eug. Familien Podostemaceae. 1ste Afhandling. Med 6 Tavler. Résumé et explic. des planches en français. 1881	3.	15.
2. Lorenz, L. Om Metallernes Ledningsevne for Varme og Elektricitet. 1881	1.	30.
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A MONOGRAPH
OF
THE GENUS DRYOPTERIS

PART I

THE TROPICAL AMERICAN PINNATIFID-BIPINNATIFID SPECIES

BY

CARL CHRISTENSEN

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURVIDENSK. OG MATH. AFD. X. 2

KØBENHAVN

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1913

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VII, med 4 Tavler. 1890—94	13.	75.
1. Gram, J. P. Studier over nogle numeriske Funktioner. Résumé en français. 1890	1.	10.
2. Prytz, K. Metoder til korte Tiders, særlig Rotationstiders, Udmaaling. En experimental Undersøgelse. Med 16 Figurer i Texten. 1890	1.	50.
3. Petersen, Emil. Om nogle Grundstoffers allotrope Tilstandsformer. 1891	1.	60.
4. Warming, Eug. Familien Podostemaceae. 4 ^{de} Afhandling. Med c. 185 mest af Forfatteren tegnede Figurer i 34 Grupper. Résumé et explication des figures en français. 1891	1.	50.
5. Christensen, Odin T. Rhodanchromammoniakforbindelser. (Bidrag til Chromammoniakforbindelsernes Kemi. III.) 1891	1.	25.
6. Lütken, Chr. Spolia Atlantica. Scopelini Musci Zoologici Universitatis Hauniensis. Bidrag til Kundskab om det aabne Havs Laxesild eller Scopeliner. Med 3 Tavler. Résumé en français. 1892	3.	50.
7. Petersen, Emil. Om den elektrolytiske Dissociationsvarme af nogle Syrer. 1892	1.	25.
8. Petersen, O. G. Bidrag til Scitamineernes Anatomi. Résumé en français. 1893	2.	75.
9. Lütken, Chr. Andet Tillæg til «Bidrag til Kundskab om Arterne af Slægten <i>Cyanus</i> Latr. eller Hvalusene». Med 1 Tavle. Résumé en français. 1893	"	85.
10. Petersen, Emil. Reaktionshastigheden ved Methylætherdannelsen. 1894	1.	50.
VIII, med 3 Tavler. 1895—98	12.	25.
1. Melnert, F. Sideorganerne hos Scarabæ-Larverne. Les organes latéraux des larves des Scarabes. Med 3 Tavler. Résumé et explication des planches en français. 1895	3.	30.
2. Petersen, Emil. Damptryksformindskelsen af Methylalkohol. 1896	1.	"
3. Buchwaldt, F. En matematisk Undersøgelse af, hvorvidt Vædske og deres Dampe kunne have en fælles Tilstandsligning, baseret paa en kortfattet Fremstilling af Varmetheoriens Hovedsætninger. Résumé en français. 1896	2.	25.
4. Warming, Eug. Halofyt-Studier. 1897	3.	"
5. Johannsen, W. Studier over Planternes periodiske Livsyttringer. I. Om antagonistiske Virksomheder i Stofskiftet, særlig under Modning og Hvile. 1897	3.	75.
6. Nielsen, N. Undersøgelser over reciproke Potenssummer og deres Anvendelse paa Rækker og Integraler. 1898	1.	60.
IX, med 17 Tavler. 1898—1901	17.	"
1. Steenstrup, Japetus, og Lütken, Chr. Spolia Atlantica. Bidrag til Kundskab om Klump- eller Maanefiskene (<i>Molidæ</i>). Med 4 Tavler og en Del Xylografer og Fotografur. 1898	4.	75.
2. Warming, Eug. Familien Podostemaceae. 5 ^{te} Afhandling. Med 42 Figurgrupper. Résumé en français. 1899	1.	60.
3. Meyer, Kirstine. Om overensstemmende Tilstande hos Stofferne. En med Videnskabernes Selskabs Guldmedaille belønnet Prisaafhandling. Med en Tavle. 1899	2.	60.
4. Jørgensen, S. M. Om Zeise's Platosemiæthylen- og Cossa's Platosemiamminsalte. Med 1 Tavle. 1900	"	75.
5. Christensen, A. Om Overbromider af Chinaalkaloider. 1900	1.	"
6. Steenstrup, Japetus. <i>Heteroteuthis Gray</i> , med Bemærkninger om <i>Rossia-Sepiola</i> -Familien i Almindelighed. Med en Tavle. 1900	"	90.
Gram, Bille. Om Proteinkornene hos oliegivende Frø. Med 4 Tavler. Résumé en français. 1901	2.	50.
8. Melnert, Fr. Vandkalvelarverne (<i>Larvæ Dytiscidarum</i>). Med 6 Tavler. Résumé en français. 1901	5.	35.
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1. Juel, C. Indledning i Læren om de grafiske Kurver. Résumé en français. 1899	2.	80.
2. Billmann, Einar. Bidrag til de organiske Kvægsølvforbindelsernes Kemi. 1901	1.	80.
3. Samsøe Lund og Røstrup, E. Marktidsele (<i>Cirsium arvense</i>). En Monografi. Med 4 Tavler. Résumé en français. 1901	6.	"
4. Christensen, A. Om Bromderivater af Chinaalkaloiderne og om de gennem disse dannede brintfattigere Forbindelser. 1902	1.	40.
XI, med 10 Tavler og 1 Kort. 1901—03	15.	05.
1. Warming, Eug. Familien Podostemaceae. 6 ^{te} Afhandling. Med 47 Figurgrupper. Résumé en français. 1901.	2.	15.
2. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejringer. I. Lamellibranchiater. Med 1 Kort og 4 Tavler. 1902.	4.	"
3. Winther, Chr. Rotationsdispersionen hos de spontant aktive Stoffer. 1902	2.	"
4. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejringer. II. Scaphopoder, Gastropoder og Cephalopoder. Med 5 Tavler. 1902	3.	40.
5. Winther, Chr. Polarimetriske Undersøgelser II: Rotationsdispersionen i Opløsninger	1.	60.
6. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejringer. III. Stratigrafiske Undersøgelser. Med 1 Tavle. Résumé en français. 1903	3.	85.
XII, med 3 Tavler og 1 Kort. 1902—04	10.	50.
1. Forch, Carl, Knudsen, Martin, og Sørensen, S. P. L. Berichte über die Konstantenbestimmungen zur Aufstellung der hydrographischen Tabellen. Gesammelt von <i>Martin Knudsen</i> . 1902	4.	75.
2. Bergh, R. Gasteropoda opisthobranchiata. With three plates and a map. (The Danish expedition to Slam 1899—1900, I.) 1902	3.	45.
3. Petersen, C. G. Joh., Jensen, Søren, Johansen, A. C., og Levinson, J. Chr. L. De danske Farvandes Plankton i Aarene 1898—1901. 1903	3.	25.
4. Christensen, A. Om Chinaalkaloidernes Dibromadditionsprodukter og om Forbindelser af Alkaloidernes Chlorhydrater med højere Metalchlorider. 1904	1.	35.

Zoologiske Skrifter

udgivne af Det Kgl. Danske Videnskabernes Selskab

(udenfor Skrifternes 6te Række, se Omslagets S. 2—3):

	Kr. Øre
Bergh, R. Bidrag til en Monographi af Marseniaderne, m. 5 Tavler. 53	4. •
— Anatomiske Bidrag til Kundskab om Æolidierne, m. 9 Tavler. 64	5. •
Eschricht, D. F. Anatomisk-physiologiske Undersøgelser over Salperne, m. 6 Tavler. 41	2. 35
— Undersøgelser over Hvaldyrene. Afhandling 1—6, m. 16 Tavler. 44—48	13. •
— Om Gangesdelphinen, m. 3 Tavler. 51	2. •
Eschricht & Reinhardt. Om Nordhvalen, m. 6 Tavler. 61	4. 65.
— — Ni Tavler til Oplysning om Hvaldyrenes Bygning m. Forklaring. 69	2. 65.
Hannover, A. Iagttagelser over indkapslede Indvoldsorme hos Frøen, m. 2 Tavler. 65	1. •
— Om Bygningen og Udviklingen af Skjæl og Pigge hos Bruskfisk, m. 4 Tavler. 67	2. •
Jungersen, Hector F. E. Ichthyotomical contributions I. The structure of the genera <i>Amphisila</i> and <i>Centriscus</i> , with 2 plates. 1908	2. 95.
Krabbe, H. Helminthologiske Undersøgelser i Danmark og paa Island, m. 7 Tavler. 65	2. 75.
— Bidrag til Kundskab om Fuglenes Bændelorme, m. 10 Tavler. Résumé en franç. 69	4. 80.
Kroyer, H. Slægten <i>Hippolytes</i> ' nordiske Arter, m. 6 Tavler. 42	3. 35.
Lütken, C. F. Additamenta ad historiam Ophiuridarum. I—III, m. 7 Tavler. Résumé en franç. 58—69	6. 85.
— Bidrag til Kundskab om Arterne af Slægten <i>Cyamus</i> Latr. eller <i>Hvallusene</i> , m. 4 Tavler. Résumé en franç. 73	2. 15.
— Velhas-Flodens Fiske, et Bidrag til Brasiliens Ichthyologi, m. 5 Tavler. Synopsis Latina. 75	6. 75.
— Til Kundskab om to arktiske Slægter af Dybhavs-Tudsefiske: <i>Himantolophus</i> og <i>Ceratias</i> , m. 2 Tavler. Résumé en franç. 78	2. •
— <i>Spolia Atlantica</i> . Bidrag til Kundskab om Formforandringer hos Fiske under deres Væxt og Udvikling, m. 5 Tavler. Résumé en franç. 80	8. 20.
Lyngé, Herm. Marine Lamellibranchiata, with 5 plates and a map. 1909	8. 60.
Meinert, Fr. Bidrag til de danske Myrers Naturhistorie, m. 3 Tavler. 60	2. 25.
Mortensen, Th. Echinoidea I., with 7 plates and a map. 1904	6. 85.
Prosch, V. Nogle nye Cephalopoder, m. 1 Tavle. 47	• 65.
Rathbun, Mary J. Brachyura, with 2 plates and a map. 1910	3. 20.
Reinhardt, J. Beskrivelse af nogle nye Slangearter, m. 3 Tavler. 43	1. 50.
— <i>Mephitis Westermanni</i> , et nyt Stinkdyr fra Brasilien, m. 1 Tavle. 57	• 65.
— Bidrag til Kundskab om Kjømpedovendyr <i>Lestodon armatus</i> , m. 3 Tavler. 75	2. 20.
— Kjømpedovendyr-Slægten <i>Coelodon</i> , m. 5 Tavler. Résumé en franç. 78	5. •
— Beskrivelse af Hovedskallen af et Kjømpedovendyr, <i>Grypotherium darwini</i> , fra La Plata-Landenes plejstocæne Dannelser, m. 2 Tavler. Résumé en franç. 79	1. 75.
Reinhardt & Prosch. Om <i>Scidaephorus Mülleri</i> , m. 5 Tavler. 46	2. 25.
Schjødte, J. C. <i>Corotoca</i> og <i>Spirachtha</i> , m. 2 Tavler. 54	1. 35.
Schmidt, Johs. Ferskvandsaalenes (<i>anguilla</i>) Udbredning i Verden I., med 1 Kort. 1909	2. 25.
Stamm, R. H. Om Musklernes Befæstelse til det ydre Skelet hos Leddyrene, med 2 Tavler. Résumé en franç. 1904	1. 95.
Steenstrup, Jap. <i>Rhizochilus antipathum</i> , m. 1 Tavle. 53	1. •
— Hectocotylidannelsen hos Octopodslægterne <i>Argonauta</i> og <i>Tremoctopus</i> , m. 2 Tavler. 56	1. 35.
— <i>Hemisepius</i> , en ny Slægt af <i>Sepia</i> -Blæksprutternes Familie, med Bemærkninger om <i>Sepia</i> -Formerne i Almindelighed, m. 2 Tavler. Résumé en franç. 75	1. 25.
— <i>Spolia atlantica</i> . Kolossale Blæksprutter fra det nordlige Atlanterhav, m. 4 Tavler. 98	2. 75.
Steenstrup & Lütken. Bidrag til Kundskab om det aabne Havs Snyltekrebs og Lernæer, m. 15 Tavler. 61	5. •
With, C. J. <i>Chelonethi</i> , with 4 plates and a map. 1906	7. 95.

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SUR LES
NOMBRES DE BERNOULLI

PAR
NIELS NIELSEN

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURVIDENSK. OG MATHEM. AFD. X. 3



KØBENHAVN

HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHANDEL

BIANCO LUNOS BOGTRYKKERI

1913

Det Kgl. Danske Videnskabernes Selskabs Skrifter,

6^{te} Række.

Naturvidenskabelig og mathematisk Afdeling.

	Kr.	Øre
I, med 42 Tavler, 1880—85	29.	50.
1. Prytz, K. Undersøgelser over Lysets Brydning i Damp og tilsvarende Vædsker. 1880	"	65.
2. Boas, J. E. V. Studier over Decapodernes Slægtskabsforhold. Med 7 Tavler. Résumé en français. 1880	8.	50.
3. Steenstrup, Jap. Sepiadarium og Idiosepius, to nye Slægter af Sepiernes Familie. Med Bemærkninger om to beslægtede Former Sepioloidea D'Orb. og Spirula Lmk. Med 1 Tavle. Résumé en français. 1881	1.	35.
4. Colding, A. Nogle Undersøgelser over Stormen over Nord- og Mellem-Europa af 12 ^{te} —14 ^{de} Novb. 1872 og over den derved fremkaldte Vandflod i Østersøen. Med 23 Planer og Kort. Résumé en français. 1881	10.	"
5. Boas, J. E. V. Om en fossil Zebra-Form fra Brasiliens Campos. Med et Tillæg om to Arter af Slægten Hippidion. Med 2 Tavler. 1881	2.	"
6. Steen, A. Integration af en lineær Differentialligning af anden Orden. 1882	"	50.
7. Krabbe, H. Nye Bidrag til Kundskab om Fuglenes Bændelorme. Med 2 Tavler. 1882	1.	35.
8. Hannover, A. Den menneskelige Hjernes kals Bygning ved Anencephalia og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 2 Tavler. Extrait et explication des planches en français. 1882	1.	60.
9. — Den menneskelige Hjernes kals Bygning ved Cyclopa og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 3 Tavler. Extrait et explic. des planches en français. 1884	4.	35.
10. — Den menneskelige Hjernes kals Bygning ved Synotia og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 1 Tavle. Extrait et explic. des planches en français. 1884	1.	30.
11. Lehmann, A. Forsøg paa en Forklaring af Synsvinklens Indflydelse paa Opfattelsen af Lys og Farve ved direkte Syn. Med 1 Tavle. Résumé en français. 1885	1.	85.
II, med 20 Tavler, 1881—86	20.	"
1. Warming, Eug. Familien Podostemaceae. 1 ^{ste} Afhandling. Med 6 Tavler. Résumé et explic. des planches en français. 1881	3.	15.
2. Lorenz, L. Om Metallernes Ledningsevne for Varme og Elektricitet. 1881	1.	30.
3. Warming, Eug. Familien Podostemaceae. 2 ^{den} Afhandling. Med 9 Tavler. Résumé et explic. des planches en français. 1882	5.	30.
4. Christensen, Odin. Bidrag til Kundskab om Manganets Iltet. 1883	1.	10.
5. Lorenz, L. Farvespredningens Theori. 1883	"	60.
6. Gram, J. P. Undersøgelser ang. Mængden af Primitiv under en given Grænse. Résumé en français. 1884	4.	"
7. Lorenz, L. Bestemmelse af Kviksølvsejlers elektriske Ledningsmodstande i absolut elektromagnetisk Maal. 1885	"	80.
8. Traustedt, M. P. A. Spolia Atlantica. Bidrag til Kundskab om Salperne. Med 2 Tavler. Explic. des planches en français. 1885	3.	"
9. Bohr, Chr. Om Iltens Afvigelse fra den Boyle-Mariotteske Lov ved lave Tryk. Med 1 Tavle. 1885	1.	"
10. — Undersøgelser over den af Blodfarvestoffet optagne Iltmængde udførte ved Hjælp af et nyt Absorptionsmeter. Med 2 Tavler. 1886	1.	70.
11. Thiele, T. N. Om Definitionerne for Tallet, Talarterne og de tallignende Bestemmelser. 1886	2.	"
III, med 6 Tavler, 1885—86	16.	"
1. Zeuthen, H. G. Keglesnitlæren i Oldtiden. 1885	10.	"
2. Levinsen, G. M. R. Spolia Atlantica. Om nogle pelagiske Annulata. Med 1 Tavle. 1885	1.	10.
3. Rung, G. Selvregistrerende meteorologiske Instrumenter. Med 1 Tavle. 1885	1.	10.
4. Mehnert, Fr. De eucephale Myggelarver. Med 4 dobb. Tavler. Résumé et explic. des planches en français. 1886	6.	75.
IV, med 25 Tavler. 1886—88	21.	50.
1. Boas, J. E. V. Spolia Atlantica. Bidrag til Pteropodernes Morfologi og Systematik samt til Kundskaben om deres geografiske Udbredelse. Med 8 Tavler. Résumé en français. 1886	10.	50.
2. Lehmann, A. Om Anvendelsen af Middelgradationernes Metode paa Lyssansen. Med 1 Tavle. 1886	1.	50.
3. Hannover, A. Primordialbrusken og dens Forbening i Truncus og Extremiteter hos Mennesket før Fødselen. Extrait en français. 1887	1.	60.
4. Lütken, Chr. Tillæg til Bidrag til Kundskab om Arterne af Slægten <i>Cyamus</i> Latr. eller <i>Hvallusene</i> . Med 1 Tavle. Résumé en français. 1887	"	60.
5. — Fortsatte Bidrag til Kundskab om de arktiske Dybhavs-Tudsefiske, særligt Slægten <i>Himantolophus</i> . Med 1 Tavle. Résumé en français. 1887	"	75.
6. — Kritiske Studier over nogle Tandhvaler af Slægterne <i>Tursiops</i> , <i>Orca</i> og <i>Lagenorhynchus</i> . Med 2 Tavler. Résumé en français. 1887	4.	75.
7. Koefoed, E. Studier i Platosforbindelser. 1888	1.	30.
8. Warming, Eug. Familien Podostemaceae. 3 ^{die} Afhandling. Med 12 Tavler. Résumé et explic. des planches en français. 1888	6.	45.
V, med 11 Tavler og 1 Kort. 1889—91	15.	50.
1. Lütken, Chr. Spolia Atlantica. Bidrag til Kundskab om de tre pelagiske Tandhval-Slægter <i>Steno</i> , <i>Delphinus</i> og <i>Prodelphinus</i> . Med 1 Tavle og 1 Kort. Résumé en français. 1889	2.	75.
2. Valentiner, H. De endelige Transformations-Grupper Theori. Résumé en français. 1889	5.	50.
3. Hansen, H. J. Cirolanidæ et familiæ nonnullæ propinquæ Musei Hauniensis. Et Bidrag til Kundskaben om nogle Familier af isopode Krebsdyr. Med 10 Kobbertavler. Résumé en français. 1890	9.	50.
4. Lorenz, L. Analytiske Undersøgelser over Primitivmængderne. 1891	"	75.

	Kr.	øre
VI, med 4 Tavler. 1890—92	13.	75.
1. Lorenz, L. Lysbevægelsen i og uden for en af plane Lysbølger belyst Kugle. 1890	2.	"
2. Sørensen, William. Om Forbeninger i Svømmeblæren, Pleura og Aortas Væg og Sammensmeltningen deraf med Hvirvelsøjlen særlig hos Siluroiderne, samt de saakaldte Weberske Knoglers Morfologi. Med 3 Tavler. Résumé en français. 1890	3.	80.
3. Warning, Eug. Lagoa Santa. Et Bidrag til den biologiske Plantegeografi. Med en Fortegnelse over Lagoa Santas Hvirveldyr. Med 43 Illustrationer i Texten og 1 Tavle. Résumé en français. 1892	10.	85.
VII, med 4 Tavler. 1890—94	13.	75.
1. Gram, J. P. Studier over nogle numeriske Funktioner. Résumé en français. 1890	1.	10.
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5. Christensen, Odin T. Rhodanchromammoniakforbindelser. (Bidrag til Chromammoniakforbindelsernes Kemi. III.) 1891	1.	25.
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8. Petersen, O. G. Bidrag til Scitamineernes Anatomi. Résumé en français. 1893	2.	75.
9. Lütken, Chr. Andet Tillæg til »Bidrag til Kundskab om Arterne af Slægten <i>Cyamus</i> Latr. eller Hval-lusene». Med 1 Tavle. Résumé en français. 1893	"	85.
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3. Buchwaldt, F. En matematisk Undersøgelse af, hvorvidt Vædske og deres Damp kunne have en fælles Tilstandsligning, baseret paa en kortfattet Fremstilling af Varmeteorien Hovedsætninger. Résumé en français. 1896	2.	25.
4. Warning, Eug. Halofyt-Studier. 1897	3.	"
5. Johannsen, W. Studier over Planternes periodiske Livsyttringer. I. Om antagonistiske Virksomheder i Stofskiftet, særlig under Modning og Hvile. 1897	3.	75.
6. Nielsen, N. Undersøgelser over reciproke Potenssummer og deres Anvendelse paa Rækker og Integraler. 1898	1.	60.
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2. Warning, Eug. Familien Podostemaceae. 5 ^{te} Afhandling. Med 42 Figurgrupper. Résumé en français. 1899	1.	60.
3. Meyer, Kirstine. Om overensstemmende Tilstande hos Stofferne. En med Videnskabernes Selskabs Guldmedaille belønnet Prisaafhandling. Med en Tavle. 1899	2.	60.
4. Jørgensen, S. M. Om Zeise's Platosemiæthyl- og Cossa's Platosemiamminsalte. Med 1 Tavle. 1900	"	75.
5. Christensen, A. Om Overbromider af Chinaalkaloïder. 1900	1.	"
6. Steenstrup, Japetus. Heteroteuthis Gray, med Bemærkninger om Rossia-Septiola-Familien i Almindelighed. Med en Tavle. 1900	"	90.
Gram, Bille. Om Proteinkornene hos oliegivende Frø. Med 4 Tavler. Résumé en français. 1901	2.	50.
8. Meinert, Fr. Vandkalvelarverne (<i>Larvæ Dytiscidarum</i>). Med 6 Tavler. Résumé en français. 1901	5.	35.
X, med 4 Tavler. 1899—1902	10.	50.
1. Juel, C. Indledning i Læren om de grafiske Kurver. Résumé en français. 1899	2.	80.
2. Billmann, Einar. Bidrag til de organiske Kvægsølvforbindelsernes Kemi. 1901	1.	80.
3. Samsoe Lund og Rostrop, E. Marktidsele (<i>Cirsium arvense</i>). En Monografi. Med 4 Tavler. Résumé en français. 1901	6.	"
4. Christensen, A. Om Bromderivater af Chinaalkaloïderne og om de gennem disse dannede brintfattigere Forbindelser. 1902	1.	40.
XI, med 10 Tavler og 1 Kort. 1901—03	15.	05.
1. Warning, Eug. Familien Podostemaceae. 6 ^{te} Afhandling. Med 47 Figurgrupper. Résumé en français. 1901	2.	15.
2. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejninger. I. Lamelibranchiater. Med 1 Kort og 4 Tavler. 1902	4.	"
3. Winther, Chr. Rotationsdispersionen hos de spontant aktive Stoffer. 1902	2.	"
4. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejninger. II. Scaphopoder, Gastropoder og Cephalopoder. Med 5 Tavler. 1902	3.	40.
5. Winther, Chr. Polarimetriske Undersøgelser II: Rotationsdispersionen i Opløsninger	1.	60.
6. Ravn, J. P. J. Molluskerne i Danmarks Kridtaflejninger. III. Stratigrafiske Undersøgelser. Med 1 Tavle. Résumé en français. 1903	3.	85.
XII, med 3 Tavler og 1 Kort. 1902—04	10.	50.
1. Forch, Carl, Knudsen, Martin, og Sørensen, S. P. L. Berichte über die Konstantenbestimmungen zur Aufstellung der hydrographischen Tabellen. Gesammelt von Martin Knudsen. 1902	4.	75.
2. Bergh, R. Gasteropoda opisthobranchiata. With three plates and a map. (The Danish expedition to Siam 1899—1900, I.) 1902	3.	45.
3. Petersen, C. G. Joh., Jensen, Søren, Johansen, A. C., og Levinsen, J. Chr. L. De danske Farvandes Plankton i Aarene 1898—1901. 1903	3.	25.
4. Christensen, A. Om Chinaalkaloïdernes Dibromadditionsprodukter og om Forbindelser af Alkaloïdernes Chlorhydrater med højere Metalchlorider. 1904	1.	35.

Mathematiske og astronomiske Skrifter

udgivne af Det Kgl. Danske Videnskabernes Selskab

(udenfor Skrifternes 6. Række, se Omslagets S. 2—3):

	Kr. Øre
Hansen, C. Recherches sur les singularités de certaines séries spéciales sur leur cercle de convergence. 1908	1. 20.
Hansen, P. C. V. En Sætning om den Eulerske Faktor svarende til Differentialligningen $M + N \frac{dy}{dx} = 0$. 73.	• 65.
Hansteen, C. Den magnetiske Inclinations Forandring i den nordlige tempererte Zone. I, med et Kort. 55.	2. •
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Hertzprung, S. Reduktion af Maskelynes Iagttagelser af smaa Stjerner. 65	1. 15.
Hjelmslev, J. Om Regning med lineære Transformationer	• 90.
Juel, C. Om ikke-analytiske Kurver. 1906.	1. 95.
— Om simple cykliske Kurver. 1911	• 65.
Nielsen, N. Recherches sur une classe de fonctions méromorphes 1904.	1. 45.
— Recherches sur les fonctions sphériques. 1906.	1. 75.
— Recherches sur quelques généralisations d'une identité intégrale d'Abel. 1907.	1. 20.
Nørlund, N. E. Ueber lineare Differenzgleichungen. 1911.	• 65.
— Untersuchungen über die Eigenbewegungen für 140 Sternen des IV. Secchischen Typus mittels älterer und eigener Beobachtungen. 1912	1. 40.
Ramus, C. Undersøgelse af Resten i Lagranges Række, samt: Om en Egenskab ved de lineære Differential-Ligninger med 2 Variable. 42	• 65.
— Om nogle Curvers Rectification ved elliptiske Functioner. 45	• 50.
— Om Ellipsoiders Tiltrækning og om de ellipsoidiske Ligevægtsfigurer af flydende Masser. 45	1. 65.
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Steen, A. Hovedsætninger om de overelliptiske Funktioner og: Om dobbelte bestemte Integraler. 49	• 65.
— Om Integrationen af Differentialligninger. Résumé en français. 68.	• 35.
— Om Ændringen af Integraler af irrationale Differentialer. 69.	• 40.
— Læren om homogene tunge Vædskers Tryk paa plane Arealer, m. 1 Tavle. Résumé en français 72	• 75.
— Om Muligheden af et Par lineære Differentialligningers Integration ved endelige explicite Funktioner. 75.	• 75.
Thiele, T. N. Om Anvendelse af mindste Kvadraters Methode i nogle Tilfælde, hvor en Komplikation af visse Slags uensartede tilfældige Fejlkilder giver Fejlene en «systematisk» Karakter. 80	• 85.
Zeuthen, H. G. Almindelige Egenskaber ved Systemer af plane Kurver, m. 5 Tavler. Résumé en français. 73	3. 60.
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d'Arrest, H. L. Siderum nebulosorum observationes Havnienses. 67	12. •
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— — — Supplément aux tables du soleil. 57.	• 35.
Jürgensen, Chr. Sur le mouvement du pendule. 53	• 65.
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AF NOGLE

GRÆSSERS OG KLØVERARTERS

KEMISKE SAMMENSÆTNING

PAA FORSKELLIGE MODNINGSSTADIER

AF

K. RØRDAM

D. KGL. DANSKE VIDENSK. SELSK. SKRIFTER, 7. RÆKKE, NATURVIDENSK. OG MATHEM. AFD. X. 4



KØBENHAVN

HOVEDKOMMISSIONÆR: ANDR. FRED. HØST & SØN, KGL. HOF-BOGHADEL

BIANCO LUNOS BOGTRYKKERI

1913

Det Kgl. Danske Videnskabernes Selskabs Skrifter, 6te Række.

Naturvidenskabelig og matematisk Afdeling.

	Kr. Øre
I, med 42 Tavler, 1880—85	29. 50.
1. Prytz, K. Undersøgelser over Lysets Brydning i Dampe og tilsvarende Vædsker. 1880	65.
2. Boas, J. E. V. Studier over Decapodernes Slægtskabsforhold. Med 7 Tavler. Résumé en français. 1880	8. 50.
3. Steenstrup, Jap. Sepidiarium og Idiosepius, to nye Slægter af Sepiernes Familie. Med Bemærkninger om to beslægtede Former Sepioloidea D'Orb. og Spirula Lmk. Med 1 Tavle. Résumé en français. 1881	1. 35.
4. Colding, A. Nogle Undersøgelser over Stormen over Nord- og Mellem-Europa af 12te—14de Novb. 1872 og over den derved fremkaldte Vandflod i Østersøen. Med 23 Planer og Kort. Résumé en français. 1881	10. "
5. Boas, J. E. V. Om en fossil Zebra-Form fra Brasiliens Campos. Med et Tillæg om to Arter af Slægten Hippidion. Med 2 Tavler. 1881	2. "
6. Steen, A. Integration af en lineær Differentialligning af anden Orden. 1882	50.
7. Krabbe, H. Nye Bidrag til Kundskab om Fuglenes Bændelorme. Med 2 Tavler. 1882	1. 35.
8. Hannover, A. Den menneskelige Hjernes kals Bygning ved Anencephalia og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 2 Tavler. Extrait et explication des planches en français. 1882	1. 60.
9. — Den menneskelige Hjernes kals Bygning ved Cyclopia og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 3 Tavler. Extrait et explic. des planches en français. 1884	4. 35.
10. — Den menneskelige Hjernes kals Bygning ved Synotia og Misdannelsens Forhold til Hjernes kalls Primordialbrusk. Med 1 Tavle. Extrait et explic. des planches en français. 1884	1. 30.
11. Lehmann, A. Forsøg paa en Forklaring af Synsvinklens Indflydelse paa Opfattelsen af Lys og Farve ved direkte Syn. Med 1 Tavle. Résumé en français. 1885	1. 85.
II, med 20 Tavler, 1881—86	20. "
1. Warming, Eug. Familien Podostemaceae. 1ste Afhandling. Med 6 Tavler. Résumé et explic. des planches en français. 1881	3. 15.
2. Lorenz, L. Om Metallernes Ledningsevne for Varme og Elektricitet. 1881	1. 30.
3. Warming, Eug. Familien Podostemaceae. 2den Afhandling. Med 9 Tavler. Résumé et explic. des planches en français. 1882	5. 30.
4. Christensen, Odln. Bidrag til Kundskab om Manganets Ilt. 1883	1. 10.
5. Lorenz, L. Farvespredningens Theori. 1883	60.
6. Gram, J. P. Undersøgelser ang. Mængden af Primitiv under en given Grænse. Résumé en français. 1884	4. "
7. Lorenz, L. Bestemmelse af Kviksølvsejlers elektriske Ledningsmodstande i absolut elektromagnetisk Maal. 1885	80.
8. Traustedt, M. P. A. Spolia Atlantica. Bidrag til Kundskab om Salperne. Med 2 Tavler. Explic. des planches en français. 1885	3. "
9. Bohr, Chr. Om Iltens Afvigelser fra den Boyle-Mariotteske Lov ved lave Tryk. Med 1 Tavle. 1885	1. "
10. — Undersøgelser over den af Blodfarvestoffet optagne Iltmængde udførte ved Hjælp af et nyt Absorption-meter. Med 2 Tavler. 1886	1. 70.
11. Thiele, T. N. Om Definitionerne for Tallet, Talarterne og de tallignende Bestemmelser. 1886	2. "
III, med 6 Tavler, 1885—86	16. "
1. Zeuthen, H. G. Keglesnitlæren i Oldtiden. 1885	10. "
2. Levisen, G. M. R. Spolia Atlantica. Om nogle pelagiske Annulata. Med 1 Tavle. 1885	1. 10.
3. Rung, G. Selvregistrerende meteorologiske Instrumenter. Med 1 Tavle. 1885	1. 10.
4. Meinert, Fr. De eucephale Myggelarver. Med 4 dobb. Tavler. Résumé et explic. des planches en français. 1886	6. 75.
IV, med 25 Tavler. 1886—88	21. 50.
1. Boas, J. E. V. Spolia Atlantica. Bidrag til Pteropodernes Morfologi og Systematik samt til Kundskaben om deres geografiske Udbredelse. Med 8 Tavler. Résumé en français. 1886	10. 50.
2. Lehmann, A. Om Anvendelsen af Middelgradationernes Metode paa Lyssansen. Med 1 Tavle. 1886	1. 50.
3. Hannover, A. Primordialbrusken og dens Forbening i Truncus og Extremiteter hos Mennesket før Fødselen. Extrait en français. 1887	1. 60.
4. Lütken, Chr. Tillæg til Bidrag til Kundskab om Arterne af Slægten <i>Cyamus</i> Latr. eller Hvallusene. Med 1 Tavle. Résumé en français. 1887	60.
5. — Fortsatte Bidrag til Kundskab om de arktiske Dybhavs-Tudseliske, særligt Slægten <i>Himantolophus</i> . Med 1 Tavle. Résumé en français. 1887	75.
6. — Kritiske Studier over nogle Tandhvaler af Slægterne <i>Tursiops</i> , <i>Orca</i> og <i>Lagenorhynchus</i> . Med 2 Tavler. Résumé en français. 1887	4. 75.
7. Koefoed, E. Studier i Platosforbindelser. 1888	1. 30.
8. Warming, Eug. Familien Podostemaceae. 3die Afhandling. Med 12 Tavler. Résumé et explic. des planches en français. 1888	6. 45.
V, med 11 Tavler og 1 Kort. 1889—91	15. 50.
1. Lütken, Chr. Spolia Atlantica. Bidrag til Kundskab om de tre pelagiske Tandhval-Slægter <i>Steno</i> , <i>Delphinus</i> og <i>Prodelphinus</i> . Med 1 Tavle og 1 Kort. Résumé en français. 1889	2. 75.
2. Valentiner, H. De endelige Transformations-Grupper Theori. Résumé en français. 1889	5. 50.
3. Hansen, H. J. Cirolanidæ et familæ nonnullæ propinquæ Musei Hauniensis. Et Bidrag til Kundskaben om nogle Familier af isopode Krebsdyr. Med 10 Kobbertavler. Résumé en français. 1890	9. 50.
4. Lorenz, L. Analytiske Undersøgelser over Primitivmængderne. 1891	75.

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1. Lorenz, L. Lysbevægelsen i og uden for en af plane Lysbølger belyst Kugle. 1890	2.	"
2. Sørensen, William. Om Forbeninger i Svømmeblæren, Pleura og Aortas Væg og Sammensmeltningen deraf med Hvirvelsøjlen særlig hos Siluroiderne, samt de saakaldte Weberske Knoglers Morfologi. Med 3 Tavler. Résumé en français. 1890	3.	80.
3. Warming, Eug. Lagoa Santa. Et Bidrag til den biologiske Plantegeografi. Med en Fortegnelse over Lagoa Santas Hvirveldyr. Med 43 Illustrationer i Texten og 1 Tavle. Résumé en français. 1892	10.	85.
VII, med 4 Tavler. 1890—94	13.	75.
1. Gram, J. P. Studier over nogle numeriske Funktioner. Résumé en français. 1890	1.	10.
2. Prytz, K. Metoder til korte Tiders, særlig Rotationstiders, Udmaalning. En experimental Undersøgelse. Med 16 Figurer i Texten. 1890	1.	50.
3. Petersen, Emil. Om nogle Grundstoffers allotrope Tilstandsformer. 1891	1.	60.
4. Warming, Eug. Familien Podostemaceae. 4 ^{de} Afhandling. Med c. 185 mest af Forfatteren tegnede Figurer i 34 Grupper. Résumé et explication des figures en français. 1891	1.	50.
5. Christensen, Odin T. Rhodanchromammoniakforbindelser. (Bidrag til Chromammoniakforbindelsernes Kemi. III.) 1891	1.	25.
6. Lütken, Chr. Spolia Atlantica. Scopelini Musel Zoologici Universitatis Hauniensis. Bidrag til Kundskab om det aabne Havs Laxesild eller Scopeliner. Med 3 Tavler. Résumé en français. 1892	3.	50.
7. Petersen, Emil. Om den elektrolytiske Dissociationsvarme af nogle Syrer. 1892	1.	25.
8. Petersen, O. G. Bidrag til Scitamineernes Anatomi. Résumé en français. 1893	2.	75.
9. Lütken, Chr. Andet Tillæg til »Bidrag til Kundskab om Arterne af Slægten <i>Cyamus</i> Latr. eller Hval-lusene». Med 1 Tavle. Résumé en français. 1893	"	85.
10. Petersen, Emil. Reaktionshastigheden ved Methylætherdannelsen. 1894	1.	50.
VIII, med 3 Tavler. 1895—98	12.	25.
1. Meinert, F. Sideorganerne hos Scarabæ-Larverne. Les organes latéraux des larves des Scarabés. Med 3 Tavler. Résumé et explication des planches en français. 1895	3.	30.
2. Petersen, Emil. Damptryksformindsættelsen af Methylalkohol. 1896	1.	"
3. Buchwaldt, F. En matematisk Undersøgelse af, hvorvidt Vædske og deres Dampe kunne have en fælles Tilstandsligning, baseret paa en kortfattet Fremstilling af Varmetheoriens Hovedsætninger. Résumé en français. 1896	2.	25.
4. Warming, Eug. Halofyt-Studier. 1897	3.	"
5. Johannsen, W. Studier over Planternes periodiske Livsyttringer. I. Om antagonistiske Virksomheder i Stofskiftet, særlig under Modning og Hvile. 1897	3.	75.
6. Nielsen, N. Undersøgelser over reciproke Potenssummer og deres Anvendelse paa Rækker og Integraler. 1898	1.	60.
IX, med 17 Tavler. 1898—1901	17.	"
1. Steenstrup, Japetus, og Lütken, Chr. Spolia Atlantica. Bidrag til Kundskab om Klump- eller Maanedskene (<i>Molidæ</i>). Med 4 Tavler og en Del Xylografer og Fotogravurer. 1898	4.	75.
2. Warming, Eug. Familien Podostemaceae. 5 ^{te} Afhandling. Med 42 Figurgrupper. Résumé en français. 1899	1.	60.
3. Meyer, Kirstine. Om overensstemmende Tilstande hos Stofferne. En med Videnskabernes Selskabs Guldmedaille belønnet Prisaafhandling. Med en Tavle. 1899	2.	60.
4. Jørgensen, S. M. Om Zeise's Platosemiæthylen- og Cossa's Platosemiamminsalte. Med 1 Tavle. 1900	"	75.
5. Christensen, A. Om Overbromider af Chinaalkaloider. 1900	1.	"
6. Steenstrup, Japetus. Heteroteuthis Gray, med Bemærkninger om Rossia-Sepiola-Familien i Almindelighed. Med en Tavle. 1900	"	90.
Gram, Bille. Om Proteinkornene hos oliegivende Frø. Med 4 Tavler. Résumé en français. 1901	2.	50.
8. Meinert, Fr. Vandkalvelarverne (<i>Larvæ Dytiscidarum</i>). Med 6 Tavler. Résumé en français. 1901	5.	35.
X, med 4 Tavler. 1899—1902	10.	50.
1. Juel, C. Indledning i Læren om de grafiske Kurver. Résumé en français. 1899	2.	80.
2. Billmann, Einar. Bidrag til de organiske Kvægsølvforbindelsers Kemi. 1901	1.	80.
3. Samsee Lund og Rostrup, E. Marktidsele (<i>Cirsium arvense</i>). En Monografi. Med 4 Tavler. Résumé en français. 1901	6.	"
4. Christensen, A. Om Bromderivater af Chinaalkaloiderne og om de gennem disse dannede brintfattigere Forbindelser. 1902	1.	40.
XI, med 10 Tavler og 1 Kort. 1901—03	15.	05.
1. Warming, Eug. Familien Podostemaceæ. 6 ^{te} Afhandling. Med 47 Figurgrupper. Résumé en français. 1901	2.	15.
2. Ravn, J. P. J. Molluskerne i Danmarks Kridtfaejringer. I. Lamellibranchiater. Med 1 Kort og 4 Tavler. 1902	4.	"
3. Winther, Chr. Rotationsdispersionen hos de spontant aktive Stoffer. 1902	2.	"
4. Ravn, J. P. J. Molluskerne i Danmarks Kridtfaejringer. II. Scaphopoder, Gastropoder og Cephalopoder. Med 5 Tavler. 1902	3.	40.
5. Winther, Chr. Polarimetriske Undersøgelser II: Rotationsdispersionen i Opløsninger	1.	60.
6. Ravn, J. P. J. Molluskerne i Danmarks Kridtfaejringer. III. Stratigrafiske Undersøgelser. Med 1 Tavle. Résumé en français. 1903	3.	85.
XII, med 3 Tavler og 1 Kort. 1902—04	10.	50.
1. Forch, Carl, Knudsen, Martin, og Sørensen, S. P. L. Berichte über die Konstantenbestimmungen zur Aufstellung der hydrographischen Tabellen. Gesammelt von Martin Knudsen. 1902	4.	75.
2. Bergh, R. Gasteropoda opisthobranchiata. With three plates and a map. (The Danish expedition to Slam 1899—1900, I.) 1902	3.	45.
3. Petersen, C. G. Joh., Jensen, Søren, Johansen, A. C., og Levinson, J. Chr. L. De danske Farvandes Plankton i Aarene 1898—1901. 1903	3.	25.
4. Christensen, A. Om Chinaalkaloidernes Dibromadditionsprodukter og om Forbindelser af Alkaloidernes Chlorhydrater med højere Metalchlorider. 1904	1.	35

Botaniske Skrifter

udgivne af det Kgl. danske Videnskabernes Selskab

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	Kr.	Ore
Borgesen, F. An ecological and systematic account of the Caulerpas of the Danish West Indies. 1907.	1.	75.
Christensen, Carl. Revision of the American species of Dryopteris of the group of <i>D. opposita</i> . 1907.	2.	85.
Ørejer, S. Symbolæ caricologicæ, med 17 Tavler. 44. fol.	6.	•
Gottsche, C. M. De mexikanske Levermosser, efter Prof. Liebmans Samling, m. 20 Tavler. 67	9.	25.
Liebmans, F. Mexicos Bregner. 49	4.	•
— Mexicos Halvgræs og Philetæria, m. 1 Tavle. 50	2.	30.
— Mexicos og Central-Americas neldeagtige Planter. 51	1.	15.
Petersen, O. G. Undersøgelser over Trærnes Aarringe. 1904.	1.	60.
Schouw, J. Fr. De italienske Naaletræers geographiske og historiske Forhold, m. 1 Kort. 44	1.	25.
— Ege- og Birkefamiliens geographiske og historiske Forhold i Italien, m. 1 Kort. 47	1.	•
— Om en Samling Blomstertegninger i den kgl. Kobberstiksamling. 49	•	65.
Warnung, Eug. Forgreningsforhold hos Phanerogamerne, betragtede med særligt Hensyn til Kløvning af Væxtpunktet, m. 11 Tavler og mange Træsnit. Résumé en français. 72	6.	45.
— Bidrag til Vaternes, Sandenes og Marskens Naturhistorie. 1904.	1.	75.
Ørsted, A. S. Centralamericas Gesneraceer, m. 12 Tavler. 58	4.	•
— Om en særegen Udvikling hos visse Snyltesvampe, navnlig om den genetiske Forbindelse mellem Sevenbommens Bævrerust og Pæretetræets Gitterrust, m. 3 Tavler. 68	1.	25.
— Bidrag til Kundskab om Egefamilien i Fortid og Nutid, m. 8 Tavler og 1 Kort. Résumé en français. 71.	6.	•

